The Equilibrium Controversy

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Guidobaldo del Monte's Critical Notes on the Mechanics of Jordanus and Benedetti and their Historical and Conceptual Background

With an Appendix by Oliver Hahn and Timo Wolff on the Analysis of Iron Gall Inks

Jürgen Renn and Peter Damerow

Communicated by
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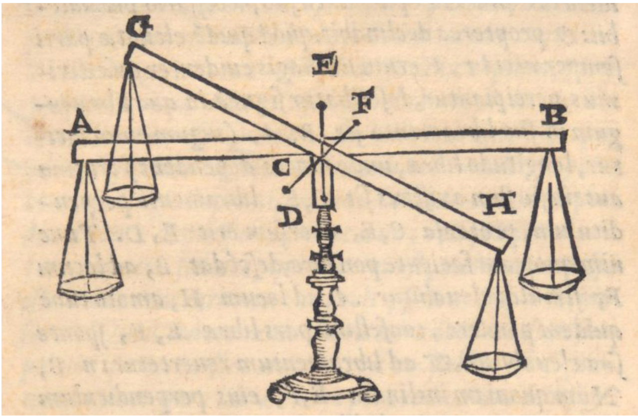

Figure 1.1: Illustration of an equal-arms balance, both in equilibrium and in a deflected position. From Piccolomini (1565).

1.1 On this book

Since ancient times, scales have symbolized justice and equilibrium. Balance and equilibrium in this wider sense are fundamental to the human condition, but what about the real, physical balance and its equilibrium? This book is not concerned with the balance between humanity and its natural environment, or with an equilibrium of power or of the human mind, but rather a seemingly innocent question concerning the real balance: the question of whether a balance in equilibrium, after having been deflected from its normal horizontal position, remains in its deflected position, returns to its original position, or tilts to the vertical. This question is of no immediate practical relevance – although it may affect the reliability of a balance – and is certainly not of fundamental significance to the human condition. Nevertheless, it captured the attention of philosophers, scholars, engineers, and scientists for almost two millennia, from Greek antiquity to the sixteenth century when this question became central among scholars and the subject of what we call here the *equilibrium controversy*. However, a conclusive answer was not found until the firm establishment of classical physics, and even then there were still aspects that provoked controversial discussions.

But why should anyone be interested in such a seemingly trivial and irrelevant issue? One can hardly avoid the impression that scientific investigations – and historical scholarship in particular – dedicated to such issues are inconsequential and detached from human endeavors. Today, of course, science is surely not removed from the human condition, but is actually critical for human survival. But this science is concerned with the grand challenges of humanity such as deciphering and interpreting the human genome, solving the energy problem, or overcoming the climate crisis. On closer inspection, however, all such pursuits are rooted in scientific knowledge that originated in intellectual concerns as remote from immediate practical applicability as the equilibrium controversy. There would be no scientific understanding of energy without the pivotal role once played by the balance and its equilibrium in understanding this concept.

The question of whether or not an equilibrated balance, deflected from its standard position, would return to the horizontal position, appears to be a typical textbook problem. In real life, most balances do return to their original position, so clearly we must be talking about an idealized balance that can be visualized, for instance, by an immaterial beam, suspended at its center, with two equal weights attached to its ends. Or we may imagine the more complex case in which the beam is extended, has itself weight, and is suspended above or below its center. All such cases can be solved with the techniques of classical mechanics, for instance, by introducing the concept of *center of gravity*, by conceiving of one arm of a deflected balance as an angular or *bent lever*, and by using the concept of *torque* in order to establish the effects that the *forces* exerted by the *weights* have in different positions along the balance.

Why then was it so difficult to resolve this question? Can a few simple experiments not settle the issue? The answer to such elementary questions about the progress of physics can only be found if we take into account the role that the historical development of fundamental concepts such as *force*, *weight*, *center of gravity*, and *torque* have played for the understanding of seemingly simple physical problems, such as those that formed part of the equilibrium controversy. The nature of the historical evolution of mechanical knowledge, as the subject of an *historical epistemology*, can only be understood if one realizes that this evolution is not a linear process, but rather involves extensive restructurations of knowledge accompanied by *concept development* in the sciences that deal with this knowledge.

The fundamental concepts of mechanics have a very long history. They have roots in antiquity and were – or so it seemed – definitively clarified in the classical physics of the eighteenth and nineteenth centuries. But then it turned out that in the course of the relativity and quantum revolutions of the early twentieth century, even such apparently basic concepts were subject to further profound modifications. Against the background of these conceptual revisions, even a simple problem such as that of the bent lever, for instance, could in fact once again become a challenging issue, although it had apparently been firmly established in the course of the equilibrium controversy of the sixteenth century. A genuine understanding of the bent lever in relativity could not simply be accomplished with the help of the classical concepts of *force* and *torque*, but actually necessitated a reconceptualization of the relation between energy and momentum from a relativistic perspective.[1]

Nevertheless, concepts with roots in antiquity such as *force*, *weight*, *center of gravity*, and *torque* continue to serve as important points of reference, even for modern physics. How did these concepts emerge? Did they perhaps result from the establishment of a definitive scientific method, which is often associated with the Scientific Revolution, an era in which the equilibrium controversy culminated and classical mechanics emerged? But why then did these concepts undergo further changes in the subsequent revolutions of physics? The present book aims to contribute to the understanding of the fundamental role of concept development for science by focusing on one particular example, by providing some relevant historical contexts, and by highlighting the specific role that scientific controversies and challenging objects played in this development.

Classical mechanics is often considered to be the most pure, abstract, and rational of the physical sciences.[2] It is hence natural to assume that its historical development must also have been essentially a history of linear progress, or at least of the steady accumulation of knowledge. This may have suffered interruptions and aberrations, but it nevertheless tended to reach clear conceptual foundations based on the consideration of idealized objects such as the balance described above. One aspect that will be-

[1] For historical discussion, see Janssen (1995).
[2] Truesdell (1968).

come particularly evident in this book is the role historical contingencies played for conceptual development at the heart of mechanics. There is, first of all, the contingency of those aspects of the material culture that become the object of scientific enquiry. These could include the balance, the pendulum, an elixir, or even the shadow of a gnomon. Then there is the contingency of the social and cultural conditions under which knowledge is recorded, transmitted, and appropriated, including the losses and transmutations occurring in such processes. Here we will show that such losses and transmutations not only acted as disturbances in an otherwise linear progress toward clarity, but that they also determined, to a large extent, the very nature of concept development in mechanics. Naturally, there is also the contingency inherent in the very processes of knowledge generation as knowledge itself is characterized by a complex cognitive architecture that gives rise to unpredictable twists in its development, as is familiar from other aspects of cultural evolution. And finally, there is the contingency that itself becomes the object of scientific reflection, for instance, in the form of the question of which material processes can be studied with scientific rigor and which must be excluded from the realm of mechanics because they are not subject to precise mathematical laws.

No doubt, science is a deeply human affair, historical down to its very core concepts. Perhaps there is no need for further study to illustrate that nothing that is human is alien to science. But is not the question of how the specific features that distinguish science from other social activities still a challenging intellectual problem? In particular, how can science ensure the long-term proliferation of knowledge while undergoing, at the same time, profound conceptual transformations as we have indicated? What role is played by the material culture underlying science as a social activity, and what significance does the long-term transmission of theoretical traditions have, and how is concept formation affected by different social and cultural contexts? In order to answer such questions, we propose that it makes sense to imitate science itself, that is, to start not necessarily from what are perhaps the most interesting, complex, and advanced forms of the phenomenon to be studied, but rather to take a simple but characteristic model case, such as the concept development taking place in the context of the equilibrium controversy.

Finally, a personal note: When Peter Damerow and I began to work on this book some three years ago, questions such as these were pertinent, but did not present the primary goal of our enterprise. Our principal aim was rather to make available to a wider public an important new source on the history of mechanics, a source that illuminates a critical phase of the equilibrium controversy: the handwritten marginal commentaries by one important early modern author on the book of another contemporary author. Our intention was to reserve theoretical issues of *historical epistemology* such as the questions listed above for a comprehensive study of the evolution of mechanical knowledge. It was only in the course of our efforts to understand the pre-history and contexts of the sources that the equilibrium controversy presented itself as a perfect illustration of some of the theoretical insights into the nature of the evolution of the knowledge under consideration.

This book began as an edition of new historical sources, but was transformed over time into a case study of the long-term history of mechanical knowledge. Yet, in accordance with the scope of the series, the sources remain at center stage, while the theoretical passages take more the form of commentaries and excursions than of systematic studies. Wherever feasible and useful, we have included quotations both in the original language and in English translation. We furthermore extensively used, as in the other volumes of this series, the digital library of the ECHO initiative, which makes many of the relevant historical materials freely accessible on the Web. We also included hitherto unpublished results of earlier work, pursued in the context of the research project Mental Models in the History of Mechanics at the Max Planck Institute for the History of Science.

As in other joint projects, Peter and I worked closely on every sentence until the book was almost finished. Even when Peter's struggle with his illness became ever more hopeless, he continued to work with the greatest intensity on this project. He carefully economized his last resources to be able to make final revisions and improvements. I have never known anyone so dedicated, so serene and cooperative, and so ingenious, even in the face of death. His mind and heart were stronger than any bodily weakness. He literally worked on this joint project until his very last moment and it hurts to think how much he would have liked to see it completed. It is therefore with the greatest respect for my friend and co-author that I have tried to implement the final amendments as far as possible and rush in making this, his last work, available in the open-access series that Peter initiated.

This would not have been possible without the continuing support of several colleagues. I am therefore particularly grateful to Lindy Divarci who looked after this project from the beginning, to Beatrice Gabriel who kept encouraging us and carefully copy-edited the final version, to Urs Schoepflin who helped to track down many of the sources, to Sabine Bertram for her help with the ink analysis, as well as to the development team, including Jörg Kantel and Kai Surendorf, for realizing the online version. Some of the transcriptions and translations have been corroborated by Eleonora Renn, Volkmar Schüller, and Stefan Trzeciok. We are particularly grateful to Antonio Becchi, Carlo Maccagni, and Pietro Daniel Omodeo for their extensive help with checking transcriptions and translations and for acting as critical and helpful submitters and reviewers, to Gideon Freudenthal for important suggestions on some of the theoretical passages, to Alexander S. Blum for his help in presenting the physics, and to Jochen Büttner, Peter McLaughlin, Matthias Schemmel, and Matteo Valleriani for many discussions in the framework of our joint research on the history of mechanics. Discussions with Horst Bredekamp always provided a strong motivation to see the project to its end. We would also like to thank Martin Frank, Enrico Gamba, and Pier Daniele Napolitani for many productive discussions of this material and for their collaboration in the context of the project Archimede nel Rinascimento: Laboratorio Urbino 1500. Parts of this work were written during a stay of one of us (J. Renn) at the Einstein Papers Project at the California Institute of Technology in Pasadena. We especially acknowledge the constant support of Rivka Feldhay and Paolo Galluzzi as members of the Scientific Advisory Board of the Max Planck Institute for the History of Science, and also the support received by the German Israeli Foundation (GIF) in the framework of the Project From Knowledge and Faith to Science and Religion: The Jesuit Way to Modernity. And finally we gladly acknowledge the stimulating context that the Collaborative Research Center Transformations of Antiquity under the direction of Hartmut Böhme and the project TOPOI – The Formation and Transformation of Space and Knowledge in Ancient Civilizations have provided for this work.

Jürgen Renn, December 2011

1.2 A first glimpse at a new document

In 2006 the library of the Max Planck Institute for the History of Science acquired a copy of the first edition of Giovanni Battista Benedetti's[3] *Diversarum speculationum mathematicarum et physicarum liber.*[4] The book

[3]Giovanni Battista Benedetti, 1530–1590.
[4]Benedetti (1585).

was purchased from an American bookseller who had acquired it at auction somewhere in Europe.[5]

Benedetti's book comprises six treatises: on arithmetical theorems,[6] on perspective,[7] on mechanics,[8] on certain opinions of Aristotle[9] (in particular concerning his theory of motion),[10] on the fifth book of Euclid's[11] *Elements*,[12] and on physical and mathematical problems dealt with in letter form.[13] The mathematical part concerns arithmetical principles, proving them geometrically; it also includes a discussion of perspective. The mechanical part contains a critique of sections of the Aristotelian *Mechanical Problems*[14] and also investigates issues of hydrostatics.

While Benedetti's book is itself an important source for understanding the struggles of early modern engineer-scientists with the ancient heritage of mechanical knowledge due to Aristotle, Archimedes[15] and others, this specific copy is of special value since it contains handwritten marginal notes by the leading expert on mechanics of the generation before Galileo,[16] Guidobaldo del Monte,[17] himself the author of the most influential Renaissance text on mechanics, the *Mechanicorum liber*.[18]

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[5]We would like to warmly thank Enrico Giusti for bringing our attention to the possibility of purchasing this work for the library of the Max Planck Institute for the History of Science in Berlin and Urs Schoepflin, its director, for pursuing the acquisition so efficiently.

[6]Benedetti (1585, 1–118).

[7]Benedetti (1585, 119–140).

[8]Benedetti (1585, 141–167), pages 329–355 in the present edition.

[9]Aristotle, 384–322 BCE.

[10]Benedetti (1585, 168–197).

[11]Euclid of Alexandria, fl. ca. 300 BCE.

[12]Benedetti (1585, 198–203).

[13]Benedetti (1585, 204–426), partly reproduced on pages 356–376 in the present edition.

[14]Aristotle (1980). The attribution of this work to Aristotle is controversial. While in the early modern period it was widely considered to be an original work of Aristotle, later philologists have questioned his authorship, ascribing it to one of his immediate followers; see Krafft (1970) and Rose and Drake (1971, 72). For more extensive discussions, see section 3.4.1.

[15]Archimedes, around 287–212 BCE.

[16]Galileo Galilei, 1564–1642.

[17]Guidobaldo del Monte, 1545–1607; often formerly referred to as Guido Ubaldo.

[18]DelMonte (1577), see the first volume of the present series (Renn and Damerow, 2010). The volume contains a complete facsimile reproduction of Guidobaldo's publication. Guidobaldo's authorship of the marginal notes is indubitably evidenced not only by the style of handwriting, but also by some references in the notes to his own publications. The indications in the notes, which allow the identification of Guidobaldo as their author, were listed in the description of the book in the auction catalog (*Martayan Lan Catalogue 38*). This description is based on work by Anthony Grafton, who also provided a transcription of some key passages.

Part 3 of this book presents facsimile images of those chapters of Benedetti's work that contain the handwritten marginal notes by Guidobaldo del Monte. Similar marginal notes by Guidobaldo have been identified by Martin Frank in a copy of the first printed edition of a work by Jordanus de Nemore,[19] the *Liber de ponderibus*[20] edited by Petrus Apianus;[21] these notes are reproduced in Part 2. With two exceptions the notes in Benedetti's book were written in the margins of the chapter on mechanics, covering 26 of the 428 pages of the whole book. The two remaining notes are added to two letters contained in a later part of the book. The second of these letters again deals with mechanics. This shows clearly that Guidobaldo was interested mainly in Benedetti's theory of mechanics. The contents of the notes indicate a strong criticism of Benedetti's theory, which is evidently related to objections raised by Guidobaldo also against Jordanus' work.

This criticism in Guidobaldo's notes concerns the central question of the equilibrium controversy: Does an equilibrated balance, if deflected into an oblique position of its beam, spontaneously return to the horizontal or does it remain in the deflected position? It will be shown, however, that this controversy only scratched the surface of a deeper-going conceptual crisis, indicated by the introduction – based on medieval sources – of a new, but ambiguous concept, the concept of *positional heaviness.* This crisis of the conceptual foundations of mechanics helped establish fundamental insights on which Galileo eventually built his theory of mechanics, as well as his theory of motion. More precisely, they concern the various controversial attempts to replace the ancient concepts of force and heaviness in the context of the causal interpretation of motion by modified concepts which were used to address the more complex technical experiences of the early modern period. We will show that the controversial opinions of Guidobaldo and Benedetti – as reflected in Guidobaldo's marginal notes on Benedetti's systematic treatment of the concepts of force and heaviness – concern core issues dealing with the problem of reorganizing the conceptual framework of ancient mechanics. In particular, Galileo's theory of motion along inclined planes, as well as many other of his characteristic themes, such as the motion of a pendulum, projectile motion, the motion of fall, and even his Copernicanism, were, as we shall argue, directly or indirectly related to this *equilibrium controversy.* In fact, Galileo's new science of motion would probably not have developed as it did without the

[19] Jordanus de Nemore (also Jordanus Nemorarius), fl. early thirteenth century.
[20] de Nemore (1533).
[21] Petrus Apianus, 1495–1552.

insights he gained from Benedetti, or rather, from the conflictual encounter between Benedetti's and Guidobaldo's perspectives on mechanics.

The conceptual and historical background of the controversy will be extensively presented and analyzed in Part 1 of this volume. It will be shown that the *equilibrium controversy* was part of a long-term development which can only be understood against the background of the multi-layered architecture of human knowledge. In the case of mechanical knowledge, this architecture comprises first basic intuitive insights gained from everyday experience of the behavior of material bodies. Second, this knowledge architecture comprises the knowledge of practitioners who use, elaborate, and improve mechanical devices, thus extending the general intuitive mechanical knowledge by developing specific professional skills. Third, it comprises scholarly expertise represented by written manuals, innovative constructions, and scientific theories. Moreover, a profound analysis of the long-term development, from antiquity to the early modern period, of the basic concepts of mechanics involved in this controversy requires the exploration of the mechanisms of the synchronic and diachronic transmission of knowledge with regard to the different layers of its architecture. In the case of mechanics, an exploration of this kind reveals substantial differences between these layers regarding the conditions and the outcome of the knowledge transfer. While the basic intuitive mechanical knowledge depends on general environmental challenges and is thus widely available, practical knowledge requires transmission using historically created and reproduced mechanical instruments and devices. In turn, theoretical knowledge requires the transmission of external representations and the reflective reconstruction of their meaning by stable scholarly communities, which are thus often fragile due to contingent social conditions.

The analysis of the *equilibrium controversy* offers an opportunity to study the interaction of various components of mechanical knowledge, such as the Archimedean theory of the center of gravity and Aristotelian dynamics, and to investigate the consequences of the incomplete transmission of this knowledge through various transmission paths, from antiquity to the Arabic and Latin Middle Ages, and finally to the early modern period.[22]

1.3 Scientific controversies and challenging objects

Scientific controversies are ubiquitous; the field that is sometimes called *rational mechanics* is no exception. A famous example is the so-called *vis*

[22] The timeline in chapter 9 provides an overview of this long-term development.

viva controversy which arose toward the end of the seventeenth century and was concerned with the question of which agent produces certain mechanical effects and which physical magnitude is conserved in mechanical interactions. This debate eventually helped to arrive at the insight that there are conservation laws for both energy and momentum. But what exactly is a scientific controversy?[23] At first glance, it may seem that a scientific controversy does not distinguish itself substantially from any other controversy, for instance, in politics or in religion. On closer inspection, however, the very existence of scientific controversies may seem a puzzling but perhaps irrelevant fact. If science essentially concerns the pursuit of truth, based on solid facts and guided by logical principles, then scientific controversies should arise only in unfortunate circumstances or when errors and misunderstandings occur. Ultimately, one if not all of the positions defended in the controversy would then be simply erroneous. In other words, scientific controversies may seem to reflect only the human aspect of science and its function as a social activity, undertaken by beings that do not always follow rational procedures. In this understanding, controversies in science are simply an accident of rationality, or even an indication that the alleged rationality of science does not exist at all and that science can be better understood without even making reference to it.

In view of the fact that controversies in science are so common and so closely related to its conceptual development, it seems, however, more plausible to assume that they are not simply a social or psychological phenomenon, but rather constitute an essential epistemic element of science and a medium of its historically developing rationality. A scientific controversy is understood here, using a definition proposed by Gideon Freudenthal, as a persistent antagonistic disagreement concerning a substantial scientific issue that cannot be resolved by the standard means available to science in the given period. In such a situation, the participants of the controversy attempt to defend their positions by showing that their understanding agrees with widely accepted explanations, that it allows them to explain certain challenging problems that may be objects of the controversy and to extend their interpretation to novel situations, hence broadening the empirical range of their approach. On the other

[23] Here we closely follow the arguments and the definition of a scientific controversy suggested by Gideon Freudenthal in the context of a jointly developed epistemological framework, see Freudenthal (2000, 2002). His contributions also offer relevant examples from the history of mechanics as well as comments on the literature about scientific controversies; for the jointly developed epistemological framework, see Damerow et al. (2004).

hand, they attempt to show that their adversaries fail on some or all of these accounts. In reality, we may also encounter controversies that are not marked by an ongoing exchange between two antagonistic positions, but rather by a chain of criticisms among different protagonists or by a complex interaction involving different partners, issues and arguments. Nevertheless, the essential features emphasized in the following apply to these cases as well.

Scientific controversies are often triggered by *challenging objects*.[24] These are artifacts or other parts of the material culture that confront existing theoretical frameworks with explanatory tasks that cannot be accomplished with the available conceptual means, thus triggering their further development and ultimately their transformation. They typically embody other forms of knowledge, for instance, the practical knowledge of artisans to invent, produce, or make use of such objects. The development of the theoretical knowledge of mechanics in the early modern period can to a large extent be accounted for by the increasing attention scholars and engineer-scientists of the period paid to new objects of study which they investigated by means of the extant conceptual frameworks. These objects and phenomena had their origin predominantly in the rapidly developing technology of the day such as the pendulum and the flywheel used in machine technology or the projectile trajectory relevant to artillery. The practical experience gathered in the application of these objects in a technological context became one of the points of departure for related theoretical considerations. Before the onset of the experimental method, these objects were a key source of empirical knowledge which accounts for one aspect of their fundamental role in the conceptual reorganization of mechanical knowledge in the sixteenth and seventeenth centuries. Thus, Galileo's new science of motion, for instance, can be conceived as resulting from a struggle with the challenges represented by the pendulum and the motion of a projectile, both addressed on the basis of attempts to understand another challenging object, the inclined plane.

Challenging objects served as *shared knowledge resources*.[25] In combination with the theoretical frameworks employed in their investigation, these objects largely determined the possible theoretical questions and answers. Focusing on such objects thus allows for an understanding of congru-

[24]The idea to approach the history of early modern mechanics from the perspective of its challenging objects was first launched in Renn (2001). It has since been widely taken up, see in particular Büttner et al. (2004), Bertoloni Meli (2006), Büttner (2008), Büttner (2009) and Valleriani (2010). The notion as it is here presented has been jointly formulated with Jochen Büttner.

[25]For the concept of shared knowledge, see Renn (2001) and Büttner et al. (2004).

ent theoretical developments – so characteristic for the period – which can hardly be accounted for by oral and textual transmission alone. Another stimulus for the development of the theoretical knowledge of mechanics is the fact that the theoretical accounts given for these challenging objects often sought to mirror and account for the complex relations of these objects in their technological context to other mechanical objects and phenomena of interest, and in particular to the so-called simple machines. In many cases this led to the integration of previously disparate mechanical knowledge. This evidently represents an important mechanism for the unification of mechanical knowledge. A key example that will be discussed below is Galileo's identification of the inclined plane with a bent lever and his attempt to explain the former in terms of the latter.

Scientific controversies are possible because they refer to the *shared knowledge* of its participants and they presuppose a common structuring of this knowledge by shared conceptual systems. They arise because the discussants adopt different interpretations of the same framework and draw different conclusions from it. For example, it may turn out that the same phenomenon can be conceived in two alternative manners. This may happen even within a single conceptual system, but is all the more likely as the relevant shared knowledge typically involves diverse conceptual systems or alternative options for choosing foundational concepts that may then serve as starting points for conceptualizing the given phenomenon. In any case, in the course of the exploration of the shared knowledge, partial differences of meaning may arise. The fact that these differences are only partial allows a meaningful exchange over the course of the controversy, while the very existence of these differences makes the controversy itself unavoidable.

Typically, scientific controversies are not resolved by victory, but rather by a further development and subsequent transformation into a new conceptual framework in which the original question often changes or even loses its meaning. Thus, the original issue of the *vis viva controversy* concerned two alternative proposals for the causal agent of certain effects, as well as the question of what physical magnitude is conserved. Eventually this alternative gave way to the understanding that there are actually two distinct magnitudes, momentum and energy, which are effective and conserved in physical interactions. But even when no party prevails, one of the opposing positions may have a greater impact on the emergence of a novel conceptual system. In any case, both antagonistic positions can be recognized, in hindsight, as alternative interpretations of the same underlying conceptual basis. This is precisely the hallmark of a more developed conceptual framework, that it allows a reconstruction of the previous positions, while it cannot be expressed itself in terms of the previous framework.

From this epistemological perspective, a number of typical features may be recognized that characterize a scientific controversy, such as the multiplication of examples in one's favor, attempts to reconstruct the adversary's position from one's own perspective, but also the unavoidable occurrence of misunderstandings, and a shift toward a more reflective stance, following the lineage of the premises of the argument defended. All of these moves effectively constitute a further exploration of the limits of the conceptual framework available to the historical actors. This conceptual framework is in fact never given from the outset in its entirety, that is, in all of its potential conclusions and applications, but actually only unfolds with the unfolding of the real scientific practice in which it is embedded. Conceptual development in this sense is hence the development of the shared knowledge of the community of practitioners, and their controversies are one essential form in which this development takes place. Its effectiveness may therefore depend on the specific historical conditions, be they material, social, or intellectual, favoring or impeding the possibility of controversies. As a result, some controversies may be resolved, in the sense outlined above, in a very short time, while others, such as the one treated here, may extend over centuries.

1.4 The physical background of the *equilibrium controversy*

The first balances were constructed in the grand early civilizations of antiquity, in Babylonia, in Egypt, and in China. They are attested since about the third millennium BCE. The introduction of balances was associated with the establishment of a quantified concept of weight. The balance also attained a symbolic significance from very early on, but its functioning did not become the subject of any written accounts before Greek antiquity. In fourth-century Greece, balances with unequal arms were invented, an invention which opened up the possibility of equilibrating different weights on one side of a balance with a single weight in different positions on the other side. This invention eventually gave rise to the formulation of the law of the lever as we find it in the works of Aristotle, Archimedes, Heron, and others. In modern terms, the law of the lever can be expressed as the equality of the product of weight and lever arm at each side of a balance in equilibrium. The invention of the balance with unequal arms also gave rise to the insight that the effect of a weight may depend on its position, the central subject of this book.

Numerous studies have been dedicated to the balance in practically all times and cultures, in particular, in the Arabic and Latin Middle Ages, as well as in the early modern period. The understanding of the balance became, more than that of any other instrument, the paradigm of mechanical knowledge, the core of a *science of weights.* For more than two millennia the balance served as a generator or touch stone for physical concepts, from the law of the lever to the principle of the conservation of energy. How could this happen? Here we deal not with the fascinating history of the balance as a key model of physical science, but focus rather on a specific aspect of this history which is related to the positional effect of a weight placed on a balance.

A balance with equal arms and equal weights is in equilibrium. We normally imagine such a balance in its default position, its arms aligned along the horizontal, perhaps with the two equal weights placed in two scale pans, appended to the beam of the balance at equal distances from its suspension point. But what happens when the balance is deflected from this horizontal position? The following study will show that historically the default expectation was – and is probably the case even today – that it returns to this position, which indeed happens for most balances in practical use. If we are taught by a modern physicist that this question can be answered, due to classical physics, within a theory of mechanics that is based on a network of causes involving concepts such as center of gravity, torque, bent lever, the position of the fulcrum, and friction, we are, of course, willing to concede that the equilibrated balance may not be as simple a physical device as it initially appeared. We may also learn that an ideal balance does not actually return to the horizontal position, but that it will rather stay in whatever position it is brought, thus illustrating the concept of an *indifferent equilibrium.*

Apparently, more sophisticated knowledge is required to answer even the seemingly simple question of whether a deflected balance will return to its original position. But from where did this sophisticated knowledge come? Modern physical theories such as theoretical mechanics no longer carry with them easily recognizable traces of the origin of their concepts. Physicists tend to assume that the correct answer to a problem such as that of the deflected balance must have ultimately emerged from careful experiments, the results of which were integrated into a proper theoretical framework. Without such a framework, even the most sophisticated experimental explorations will in fact yield nothing but an accumulation of data on single cases without a basis for generalization.

But how could such a theoretical framework emerge when observations concerning even the most elementary case of a deflected balance remained inconclusive, as they apparently did for more than two millennia? The question of whether or not an equilibrated balance would return to its horizontal default position or remain indifferent in whatever position it is brought was not in fact settled before the early modern period when it became the explicit subject of a controversy between Guidobaldo del Monte and other protagonists of *preclassical mechanics*,[26] in particular Benedetti.

There is, of course, the possibility that the knowledge necessary to decide this *equilibrium controversy* developed from sources other than the study of balances, as it certainly did to some extent. Without confronting other *challenging objects* such as the so-called simple machines, the lever, the wheel and axle, the pulley, the inclined plane, the wedge, and the screw, theoretical mechanics would not have evolved as it did. From its beginning in Greek antiquity, the aim of mechanics was to explain the communality of a set of mechanical devices and, in particular, of how they made it possible to achieve a large effect with a small force. Nevertheless, in much of the history of mechanics up to the early modern period, it was the balance that kept its paradigmatic role in forming and exploring basic concepts such as equilibrium and the positional effect of a weight. A study of the history of the equilibrium controversy, culminating in the confrontation between Guidobaldo and Benedetti, therefore also offers the opportunity to analyze this pivotal role of the balance more closely.

An ideal balance with equal arms and equal weights, that is, a balance with a weightless beam suspended from its center of mass in a homogeneous gravitational field, will indeed be in an indifferent equilibrium. When it is brought into any position compatible with its mechanical constraints, it will stay there. The weights at the end of the beam of the balance exert forces on the beam. Together with the arms at each side of the beam these forces form a *torque*, also called *moment* or *moment of force*, causing a tendency to rotate the balance. In the case of the ideal balance described above the torques are equal, which explains the equilibrium of the balance from a modern point of view.

[26]For the concept of *preclassical mechanics*, see Damerow et al. (2004); see also Renn et al. (2001), Büttner et al. (2004); Valleriani (2010); Damerow and Renn (2010); Büttner (2008); furthermore see the broad discussion in Bertoloni Meli (2006).

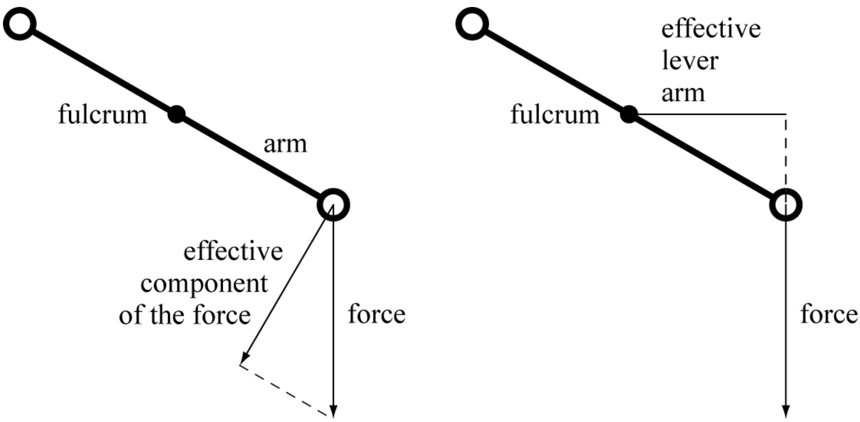

Figure 1.2: The *torque* of a weight attached to one side of a balance is given by the product of the arm and the *effective component of the force* acting perpendicularly to it, or, which is equivalent, by the product of the force and its *effective lever arm*, which is the component of the real lever arm perpendicular to the force.

The torque is given, more specifically, by the product of the arm and the *effective component of the force* acting perpendicularly to it. Alternatively, the torque can be expressed as the product of the force and its *effective lever arm*, which is the component of the real lever arm perpendicular to the force (see figure 1.2).[27] While the equivalence of both definitions is immediately clear if the algebraic notation of modern physics is used, it needs a more sophisticated argument to prove it purely geometrically using the mathematics of the time of preclassical mechanics.

If the balance is in a horizontal position, the torque at one side is simply the product of the weight and the balance arm at that side. If the balance is in an oblique position, the torque can be found by projecting the arm of the balance on the horizontal and multiplying the length of this projection, that is, the *effective lever arm*, with the weight. In modern understanding, the torque is actually a vector product, a concept not established before the nineteenth century. The requirement that the torques are equal for a balance in equilibrium can be conceived of as a generalization of the law of the lever. This generalization is applicable not

[27] Halliday and Resnick (1977, 232); Halliday et al. (2007, 225–226).

only to balances with unequal arms, but also to balances with arms that are not aligned or, in other words, to the so-called bent lever. If one arm of a balance is aligned with the horizontal, while the other arms is bent at an angle, it is again the projection of the bent arm on the horizontal that acts as the effective lever arm. This way of determining the effective lever arm by a projection on the horizontal was emphasized – as we will see below – by Benedetti and opposed by Guidobaldo.

The concept of *torque* is not the only modern concept to explain the equilibrium of a balance or the behavior of a bent lever. They can also be analyzed with the help of the concept of *mechanical work*. This concept is more generally applicable to physical systems than the concept of torque and, in a sense, even more intricate as a glance at its definition in classical physics makes clear. In general, the work performed by a force on a body along a trajectory is given by the line integral of the scalar tangential component of the force along the trajectory. Generally, the force is changing along the path, as it does in the case of a balance arm moving from the horizontal into an oblique position. The line integral of the changing force is given by the infinite sum of infinitely small displacements on the trajectory multiplied with the forces acting in these infinitesimal displacements. The changing force may have arbitrary directions, but according to the definition of mechanical work only the component of the force acting along the path of the displacement or, which is the same, the component of the path traversed along the direction of the force has to be taken into account. In modern terminology, this is the time integral of the scalar product of the force vector and the velocity vector along the trajectory, in contrast to the vector product of the lever arm and the force relevant to the consideration of the torque. Thus, the concept of work is even more demanding than the concept of torque because it involves the infinitesimal calculus so that the historical actors considered here, living before its introduction in the age of Newton[28] and Leibniz,[29] had no chance to fully master this concept.

Only in the most simple cases can mechanical work be thought of as the product of a force and the distance over which it acts. In particular, the work performed by the force of gravity does not depend on the path taken by a body. The vertical component of the force of gravity can be considered as practically constant and the varying horizontal component requires no force and thus has no influence on the mechanical work of the motion along the trajectory. Therefore, the work can be determined in this specific case by just considering the vertical distance traversed, for

[28]Isaac Newton, 1642–1726.
[29]Gottfried Wilhelm Leibniz, 1646–1716.

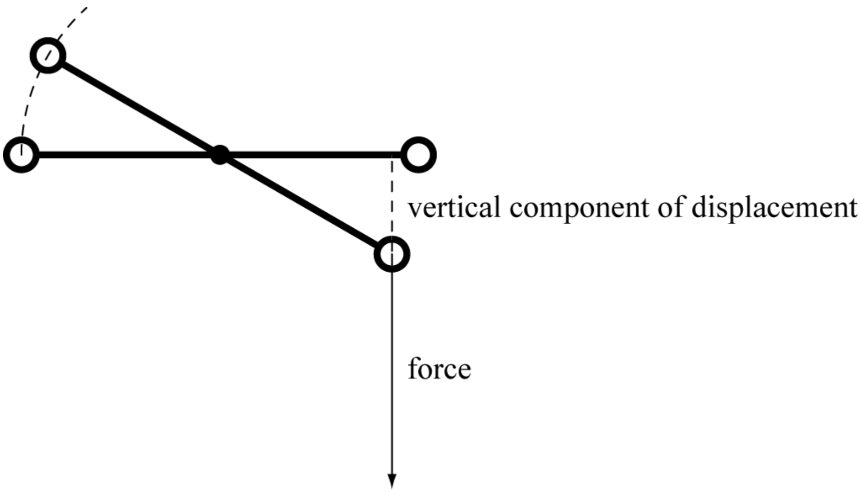

Figure 1.3: In the specific case of a balance in a homogeneous field of gravitation the *mechanical work* of a weight attached to one of its arms is given by the product of the constant gravitational force and the vertical component of its displacement.

instance, by a weight on a balance beam (see figure 1.3). The equilibrium of a mechanical constellation can then be characterized by the demand that the total work performed by a displacement of the entire constellation in agreement with the mechanical constraints is zero. Now the work performed by one weight is simply given by the product of the vertical component of its displacement by the force of weight. In the case of a balance with unequal arms, the vertical components of the displacements of the two weights are proportional to the lengths of the arms of the balance. Hence the total work performed by a displacement, for instance, by bringing the balance from the horizontal into some oblique position, is zero if the products of weight and length of the arm are equal on both sides. Then the work performed by lifting the weight on one side of the balance will be equal to the work gained by lowering the weight on the other side of the balance. In this way, the inverse proportionality of weights and arms follows, as stated by the law of the lever.

This way of determining mechanical equilibrium can be also illustrated by the case of an inclined plane: Let a weight be placed on the inclined plane and connected via a weightless rope through a wheel at the top to another weight hanging down on the other side, moveable along the vertical such that when the weight along the plane is moved upwards, the other weight is moving perpendicularly downward. In equilibrium, the total work performed by the system must be zero. For the weight moving along the vertical, the work performed is simply the product of the weight and the total displacement which happens to be along the vertical. For the weight moving along the inclined plane, the work performed is the product of the weight with the vertical component of the displacement along the plane. Given the connection through the rope, the total displacement along the plane must be the same as the total displacement of the weight moving along the vertical. In the case of equilibrium, the product of the weight along the vertical with the total displacement must be the same as the product of the vertical component of the displacement along the plane with the weight on the plane. But this vertical component is to the total displacement as is the height of the plane to its length. It hence follows that the product of the weight in the vertical and the length of the plane must be the same as the product of the weight on the plane and the height of the plane, or in other words, the weight along the vertical is to the weight on the plane as is the length of the plane to its height. This is the famous law of the inclined plane, as it was first stated by Jordanus. Remarkably, his proof also works with the consideration of vertical displacements which is why it has often been associated with the principle of work (see figure 3.13).

Such associations are in fact merely the product of anachronistic projections. They suppose an essentially teleological development of the history of scientific concepts in which earlier concepts are merely embryonic forms of the mature ones. A closer look at the equilibrium controversy makes it evident that such a perspective is of little help in understanding the nature of the historical process. None of the protagonists of this controversy strove for the introduction of novel physical concepts such as torque or work. They were actually merely concerned with a better understanding of the concept of weight and of the way a weight acts under given mechanical circumstances. They also did not attempt to differentiate between those physical aspects captured by the modern concepts of torque and work. While the modern concept of work indeed covers all mechanical devices, the concept of torque only applies to cases in which a lever arm is involved. Also, while the torque refers to the way a force acts at a particular point in space and time, the concept of work, being represented by an integral, refers to the action of a force along a certain path.

The participants in the controversy attempted instead to find a universal way in which the effectiveness of a weight varying with the mechanical circumstances could be described so as to be applicable to all mechanical devices. The concept of *positional heaviness*, introduced by Jordanus in the thirteenth century, represents such an attempt. One key aspect of the controversy was, as we shall extensively discuss in the following, the question of whether positional heaviness is to be measured by the projection of the lever arm on the horizontal (as the torque) or by the vertical component of a displacement of the weight on the lever arm (as the work). From a modern perspective this alternative makes little sense as it refers, as we have seen, to two distinct physical concepts that cannot always be applied to the same situations. How hopeless the attempt was to capture both aspects by a single, modified concept of weight becomes clear if one considers that the modern concept of torque refers to a point, while the concept of work refers to a displacement. Nevertheless, the equilibrium controversy contributed significantly to preparing the ground for the emergence of the later conceptual distinctions of classical physics, in particular, with the ambiguities and paradoxes that surfaced as it unfolded.

One of these paradoxes was the perplexing difficulty in establishing a stable answer to the simple question of whether an ideal balance deflected from its horizontal default position would return to it or not. By referring, as we did above, to an ideal balance we have actually introduced tacit premises, in particular, that the lines of force are parallel and that the gravitational force remains the same even when the balance is displaced from its default position. In short, we have neglected the *cosmological context* of the balance, the fact that the weights carried by its beam tend to fall not along parallel lines, but along lines meeting at the center of the earth and that the force of gravitational attraction may vary with distance. Clearly, the effects introduced when these circumstances are taken into account must be vanishingly small and practically irrelevant. They can, however, not be as easily dismissed as for instance the role of friction as they directly pertain to the understanding of the very concept of weight at the center of the analysis of the balance. Moreover, the cosmological context of a physical problem like that of the equilibrium of the balance was particularly important to the debates of the early modern period where such problems were inevitably related to larger issues of the physical world view. In fact, Aristotelian natural philosophy was the dominant conception of nature at the universities and, beginning with the Council of Trent, had been embraced as the official world view of the Church. It is therefore not surprising that, in this period, the cosmological dimension played a critical role for the equilibrium controversy.

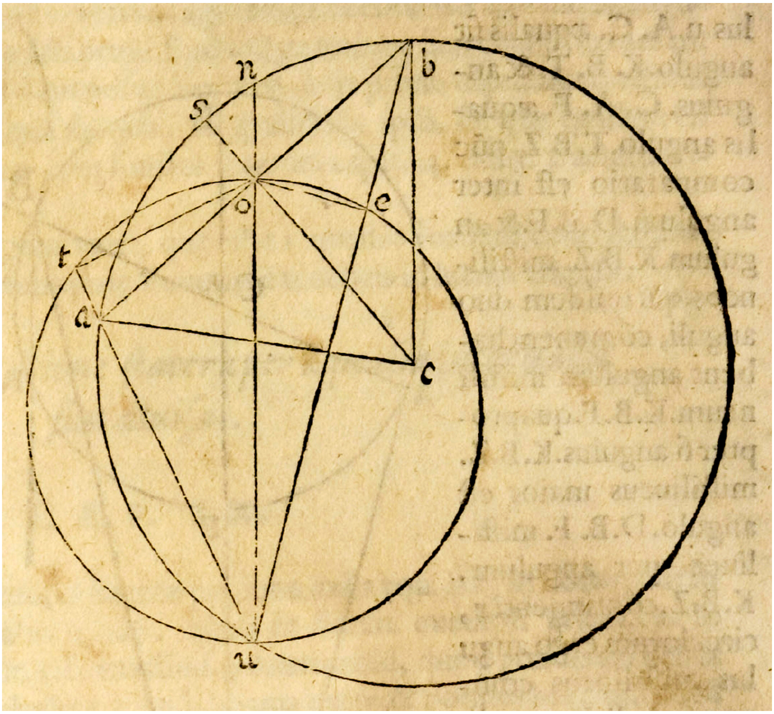

Figure 1.4: A diagram from Benedetti's book representing a balance AB supported at O in a cosmological context where U is the center of the earth. The weights in A and B have a tendency to fall toward U. The lines OT and OE drawn from the point of suspension are perpendicular to the lines AU and BU connecting the weights with the center of the earth.

It is not difficult to reconsider the question of the behavior of an equilibrated balance in a context in which the lines of force connect the weights to the center of the earth and in which the forces themselves may vary with the distance from that center (see figure 1.4). How does such a *cosmological balance* behave from the perspective of later classical physics?

The torque of one of the weights placed at the end of the beam is given by the projection of the respective balance arm onto a line that now plays the role that the horizontal had played in the ordinary balance. This line is obtained by drawing, from the point of suspension, a perpendicular onto the line connecting the weight with the center of the earth. Similarly, the torque of the weight on the other side of the balance can again be determined by constructing a perpendicular onto the line connecting this weight with the center of the earth. It now turns out that the ratio between the two lines measuring the torques at the two sides of the balance is the inverse of the ratio of the two lines connecting the weights to the center of the earth, if it is assumed that the gravitational force does not change with distance. As a consequence, the torque on the side of the balance which has descended is larger than the torque on the side which has been raised so that the cosmological balance would neither stay indifferent nor return to the default position, but rather align itself along the vertical. This was also the conclusion that Benedetti reached and to which Guidobaldo violently objected. If it is furthermore assumed that the gravitational force does vary with distance, becoming weaker as the distance increases (as in reality), the effect that the balance turns into the vertical is even strengthened. Under the given circumstances, the only way to obtain an indifferent equilibrium is to assume that the gravitational attraction behaves the other way around, becoming stronger as the distance increases, as if it were a rubber band by which bodies are confined to their center of attraction.

Guidobaldo, the most influential writer on mechanics of the sixteenth century, was proud to have reconciled the Archimedean theory of equilibrium, based on the concept of center of gravity, with the Aristotelian understanding of weight as tending to the center of the world. This reconciliation was embodied by what was evidently, in his view, the greatest discovery he had made: the insight that both an ideal balance and also what we have called a cosmological balance remain in indifferent equilibrium. His adversary Benedetti claimed that, while such an indifferent equilibrium holds under terrestrial circumstances, it is impossible for a cosmological balance. He thus challenged Guidobaldo's grand synthesis. While Benedetti's conclusion is in accordance with later classical physics, the controversy could hardly be settled definitively with the arguments available at the time. It was the equilibrium *controversy* more than its *resolution* that spurred the further development of physics.[30]

[30] The equilibrium controversy was not settled during the period under consideration here. Also, we have not attempted to be exhaustive in dealing with all pertinent con-

1.5 Editorial remarks

Latin and Italian texts are transcribed using orthographic normalization, that is, punctuation, accents, and capital letters have been revised according to standard Latin and Italian. As a rule, abbreviations and symbols have been expanded or resolved. Arabic names are romanized. Translations are adapted from standard references, when available, and modified whenever necessary. All further translations, unless otherwise indicated, are by the authors.[31]

Some of Guidobaldo's marginal notes to Benedetti's text were deleted and are now unreadable. Some notes have been cut off by a bookbinder. Such passages have been amended as far as possible.

The considerable number of deleted passages raised the question of whether it would be possible to read the text underneath the deletions by applying special analytical methods and also whether the deletions have been performed by the same author who wrote the notes. In order to answer these questions the composition of the ink has been examined by means of an X-ray fluorescence analysis (XRF), performed at the Federal Institute for Materials Research and Testing in Berlin.[32] The result of the preliminary analysis is that the deletions were made in the same ink and hence most probably by Guidobaldo himself and that the text underneath cannot be rendered legible by a non-destructive analysis.

The copy of DelMonte (1577), reproduced in the first volume of the present series, (Renn and Damerow, 2010), is itself a testimony to the equilibrium controversy. Passages relevant to the controversy have been underlined using an iron gall ink from the period but distinct from that used by Guidobaldo himself in his marginal notes to Benedetti's work. The composition of the ink has also been analyzed by means of an X-ray fluorescence analysis, performed at the Federal Institute for Materials Research and Testing in Berlin (see figure 1.5).

tributions. For glimpses of other aspects of the controversy and its historical sequel in the recent literature, see Roux (2004, 36–39) and Bertoloni Meli (2006, 31–32). A particularly interesting case is the treatise on mechanics from 1597 by Colantonio Stigliola, 1546–1623, see Gatto (2006). For a discussion of the role of the bent lever as a challenging object in the relativity revolution, see Janssen (1995).

[31] Translations of Benedetti, Guidobaldo, and Tartaglia, for example, are taken from Drake and Drabkin (1969), sometimes with slight modifications.

[32] See the appendix.

Figure 1.5: X-ray fluorescence analysis of the ink used in underlinings found in a copy of Guidobaldo's mechanics. The underlinings were written with an iron gall ink from the period. They concentrate on the foundational aspects of Guidobaldo's approach, his comments on Aristotle, and the critique of his adversaries.

Chapter 2
The Authors and their Critic

2.1 The author Jordanus de Nemore

Jordanus de Nemore, or Jordanus Nemorarius as he is called in some manuscripts, was the author of several treatises completed before 1260. Nothing specific is known about his personal life. The historical period and the range of his scholarly activities are only circumscribed by the inclusion of his works in the *Biblionomia*, a catalog of the library of Richard de Fournival, the chancellor of the Amiens Cathedral,[1] compiled between 1246 and 1260.[2]

Codex 43 of this catalog lists several works attributed to Jordanus:
Philotegni or *De triangulis*,
De ratione ponderum,
De ponderum proportione, and
De quadratura circuli.

Codex 45 lists:
Practica or *Algorismus*,
Practica de minutiis, and
Experimenta super algebra.

Codex 47 refers to:
Arithmetica.

Codex 48 lists:
De numeris datis,
Quedam experimenta super progressione numerorum, and
Liber de proportionibus.

Codex 59 refers to:
Suppletiones plane spere.

[1] Richard de Fournival, ca.1201–1260
[2] For the following, see Brown (1967); Høyrup (1988); Grant (2008).

Many more later manuscripts are attributed to Jordanus, but this may have been due to the close association of his name with the subjects he established in the Latin tradition, in particular also the *science of weights*. It is therefore often difficult to assess which works he actually authored, which works he adopted himself from an earlier tradition and just commented upon, which works were partly authored by him but then extended by later commentaries, and which works were simply ascribed to him because of his role as an authority in a particular field.

Jordanus thus emerges as one of the most important mathematicians of the Middle Ages. His extant mathematical writings deal with geometry, algebra, and arithmetics. His work clearly draws both on ancient and on Arabic sources, as is the case for his contributions to mechanics. He thus represents for the mathematical sciences, in a sense, a parallel figure to his contemporary Albertus Magnus,[3] who established Aristotelianism as a frame of reference for theological and philosophical discourse, benefitting from the Arabic-Latin translation movement of the preceding century.[4] More specifically, Jordanus flourished in a period in which Latin Europe was about to establish its own institutional and intellectual structures for absorbing the rich knowledge inherited from the Arabic world. By bringing subjects such as the science of weights, known to him through the Arabic tradition, into a more rigorous, Euclidean form, he elevated them to the scientific standards of the emerging scholastics, a transformation that did not take place without leaving traces on the contents with which it was concerned.[5] Here we claim, in particular, that the concept of *positional heaviness* which Jordanus introduced in order to distinguish between a weight and its positional effect was exactly such a trace of the framework of emerging scholastics (see page 60).

The starting point of Jordanus' work on mechanics was probably the *Liber karastonis*, ascribed to Thābit Ibn Qurra[6] in a translation that may go back to Gerard of Cremona,[7] as well as the *Liber de canonio*,[8] probably a Latin translation of a text going back to a Greek source. Both texts deal with the balancing of the steelyard and take into account the fact that a balance has a material beam which itself possesses weight. The *Liber karastonis* provides a proof of the law of the lever from an Aristotelian foundation and the *Liber de canonio* focuses on the material beam. Taken

[3] Albertus Magnus, ca.1200–1280
[4] For an overview, see Abattouy et al. (2001) and Speer and Wegener (2006).
[5] See also the discussion in Høyrup (1988).
[6] Thābit ibn Qurra, died in 901.
[7] Gerard of Cremona, 1114–1187.
[8] Moody and Clagett (1960, 55–76).

together, they constituted the challenge of constructing a sophisticated theory of the balance on an Aristotelian foundation in a Euclidean form, in other words, just the kind of challenge that Jordanus also addressed in his other works.

Three major groups of manuscripts on the *science of weights* attributed to Jordanus can be distinguished.[9] The *Elementa super demonstrationem ponderum* contain seven postulates and nine propositions together with extensive proofs. They also contain a reference to one of Jordanus' mathematical works. The *Elementa* are often found in medieval manuscripts together with the *Liber de canonio.* They may thus be considered as providing a theoretical foundation, anchored in an Aristotelian framework, for the treatment of the material beam in the *Liber de canonio.*

The *Liber de ponderibus* begins with precisely the same postulates and propositions, albeit their wording is partly different.[10] Furthermore the postulates are preceded by a prologue, explicitly introducing the term *gravitas secundum situm*, i.e. *positional heaviness*; at the end four additional propositions are appended, which stem from the *Liber de canonio.* This treatise, however, does not contain the extended proofs of the *Elementa* but instead, in the various forms in which it is extant, two types of explanatory commentaries to the propositions, one short in a scholastic style, one longer involving mathematical arguments as well.

Finally, the *Liber de ratione ponderis* is a much longer treatise divided into four parts containing ten, twelve, six and seventeen propositions respectively. The text begins with seven postulates that are similar to those of the other two treatises. The first postulate adds reference to the "virtus," i.e. "force" of tending downward and resisting motion. The last postulate explains the horizontal equilibrium position of the beam in terms of angles with the vertical. Also, most of the propositions of the first part are similar to those of the other treatises, with a reference to upward motion omitted in the first proposition, a reference to unequal weights added in the second proposition, with proposition 3 of the *De ratione ponderis* taking the place of proposition 4 of the *Elementa* and vice versa, proposition 5 remaining identical, with proposition 6 of the *De ratione ponderis* taking the place of proposition 8 of the *Elementa*, and with proposition 7 of the *De ratione ponderis* taking the place of proposition 9 of the *Elementa.* Propositions 6 and 7 of the *Elementa*, dealing with the bent lever in a

[9] See the edition of the manuscripts and the commentaries by Moody and Clagett (1960); see also Brown (1967).
[10] Compare the propositions in de Nemore (1533) with the corresponding propositions in Moody and Clagett (1960, 119–142).

way that is problematic from a modern perspective, however, are replaced by the "correct" proposition 8 of the *De ratione ponderis*. Propositions 9 and 10 of the *De ratione ponderis* deal with the descent of a weight along a rectilinear path and with the inclined plane. Part 2 of the *De ratione ponderis* treats the material beam, Part 3 further cases of the bent lever, while Part 4 adresses various subjects of motion.

As mentioned above, Jordanus' work became known in the sixteenth century through printed editions of his *Liber de ponderibus* by Petrus Apianus[11] and of the *De ratione ponderis* by Tartaglia.[12] The latter comprises the two kinds of commentaries mentioned above, the longer one obviously based on knowledge of other manuscripts by Jordanus as well.

The present volume deals mainly with Apianus' edition of the *Liber de ponderibus*, that is, the edition annotated by the marginalia of Guidobaldo. In manuscripts of the *Liber de ponderibus*, but not in the Apianus edition, the text concludes with the formula:

> Explicit tractatus de ponderibus magistri Jordanis.
>
> Here ends the treatise on weights of Master Jordanus.[13]

The authorship of Jordanus de Nemore is nevertheless controversial, even for the postulates and the theorems. Some manuscripts ascribe the postulates and the first nine theorems not to Jordanus but to Euclid. Indeed, the final sentence of the second comment to the ninth theorem of the Apianus edition reads:

> Hic explicit secundum aliquos liber Euclidis de ponderibus.
>
> Here ends, according to some, Euclid's book on weights.[14]

Since in medieval manuscript traditions, propositions, proofs, and ascriptions of authorship led a life of their own, rather independently from each other, there is little one can conclude with certainty from these circumstances.[15]

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[11] de Nemore (1533).
[12] de Nemore (1565).
[13] Moody and Clagett (1960, 164–165).
[14] de Nemore (1533, D i verso), page 320 in the present edition.
[15] Moody speculated that the theorems were transmitted independently from the proofs and traditionally ascribed to Euclid. From this view, Jordanus can neither be the author of the concept of *positional heaviness* nor of the theorems, but rather emerges as a commentator who developed the technical proofs found in the *Elementa super demonstrationem ponderum* (Moody and Clagett, 1960, 146–147) as well as those of the improved and extended version of this treatise, the *De ratione ponderis* (Moody and Clagett, 1960, 167–227).

The postulates and the first nine theorems in fact form a rather closely knit deductive system centered on the notion of *positional heaviness* that could have hardly arisen without some explicit technical proofs in the first place.[16] For this reason, we will treat the core theory of the various treatises, the *Elementa*, the *Liber de ponderibus*, and the *De ratione ponderis* as the work of Jordanus. We have to leave open, in particular, whether the prologue of the *Liber de ponderibus* with its apparent or real echoes of the Aristotelian *Mechanical Problems*, or the substantial improvements found in the *De ratione ponderis* are the accomplishment of Jordanus himself or of a later commentator. For most of our arguments it is sufficient to associate them with the paradigm he created.

2.2 The author Giovanni Battista Benedetti

Giovanni Battista Benedetti was born in Venice on August 14, 1530 and died on January 20, 1590 in Turin.[17] He belonged to a patrician family and was educated in philosophy, music, and mathematics by his father, who, according to Gaurico, was a Spaniard interested in philosophy and the natural sciences.[18] At the age of 23 Benedetti published his first scientific treatise, the *Resolutio omnium Euclidis problematum*,[19] offering the solution to geometrical problems using a compass with a fixed opening. The work reacted to a challenge that emerged from a controversy between Niccolò Tartaglia[20] and Ludovico Ferrari[21] in the years 1546–1548. The letter of dedication, addressed to Gabriel de Guzman, a Spanish priest, contains some autobiographical remarks by Benedetti. According to these remarks he did not receive any formal education, nor did he have a master. However, he acknowledged that Tartaglia had introduced him to the

[16]While this still leaves the speculative possibility that such proofs once existed, were then lost, and finally reconstructed by Jordanus, such a reconstruction remains without any specific historical evidence in the sources.
[17]There is little historical evidence concerning Benedetti's personal life. For biographical accounts, see the pioneering study from 1926 by Giovanni Bordiga, with a commented bibliography by Pasquale Ventrice of 1985 (Bordiga, 1985). See also Drake and Drabkin (1969, 31–41), Cappelletti (1996), and Drake (2008). A comprehensive review of his work and historical context may be found in Manno (1987). For a reconstruction of Benedetti's European network of correspondents, see Cecchini (2002). For a detailed presentation of Benedetti's mechanical theories we benefitted from Maccagni's studies, in particular, Maccagni (1967).
[18]Luca Gaurico, 1476–1558, in *Tractatus astrologicus* (Gaurico, 1552).
[19]Benedetti (1553).
[20]Niccolò Tartaglia, 1500?–1557.
[21]Lodovico Ferrari, 1522–1565.

first four books of Euclid's *Elements*, probably between 1546 and 1548.[22] Tartaglia may also have familiarized the young Benedetti with the problems of mechanics as he had treated them in his own book, *Quesiti, et inventioni diverse* of 1546.[23] Benedetti later also became acquainted with the edition of a work by Jordanus that Tartaglia had prepared and that contained an analysis of the bent lever by the principle that Benedetti was using in his own work to determine the *positional heaviness* of a body.[24]

According to an epigraph preserved in Turin, Benedetti had a daughter who died in childbirth in 1554 at the age of 26. In the same year he published another work, the *Demonstratio proportionum motuum localium.*[25] Here he developed a theory of the motion of fall, first proposed in the dedicatory letter of the *Resolutio* of 1553.[26] According to this theory, bodies of the same material fall through a given medium with the same speed and not with speeds in proportion to their weights, as Aristotle had claimed. Benedetti thus tried to overcome the fallacies of the Aristotelian theory of fall by employing the Archimedean concept of buoyancy, assuming that the motion of fall depends on the specific rather than the absolute weight. The use of Archimedean notions to correct Aristotle's physics was probably stimulated by Tartaglia's Italian translation of the first book of Archimedes' treatise on bodies in water in 1543.[27] Benedetti's challenge to Aristotle apparently raised considerable discussion. In his *Demonstratio* he discussed Aristotle's views at length and responded to his critics. In the second edition of the *Demonstratio*, also published in Venice in 1554,[28] Benedetti argued that the resistance incurred by a falling body in a medium depends not on its volume, but on its surface area. This is also the view that Benedetti presented in *Diversarum speculationum mathematicarum et physicarum liber*, published in Turin in 1585 and issued again under slightly different titles in Venice in 1586 and in 1599.[29] He explained the acceleration of the motion of fall in terms of an increasing *impetus* of the falling body. Such examples show how he dealt with new challenging problems, which were difficult and sometimes impossible

[22] In the letter to the reader of Benedetti's *Diversarum speculationum mathematicarum et physicarum liber* (Benedetti, 1585, first page of *Ad lectorem*) the author referred to Tartaglia as his main mathematical source.
[23] Tartaglia (1546).
[24] de Nemore (1565). See the discussion in chapter 3.9.
[25] Benedetti (1554).
[26] Benedetti (1553).
[27] Archimedes (1543a).
[28] Benedetti (1555); see Benedetti (1985).
[29] Benedetti (1585, 1586, 1599).

to treat using the mainstream theories of his time, by bringing forth and promoting new ideas. In spite of Benedetti's efforts to secure priority for his ideas by repeated publication, they were plagiarized by Jean Taisnier[30] in 1562 and spread without recognition of his authorship.[31]

In 1558 Benedetti joined the court of Ottavio Farnese, the Duke of Parma,[32] as "lettore di filosofia e matematica."[33] There, he performed astronomical observations and built sundials whose construction he later described in his own book on the subject in 1574.[34] In two letters to Cipriano da Rore, choirmaster at the Court of Parma, Benedetti explained the musical consonance and dissonance of two tones by the ratio of oscillations of waves of air generated by the strings of musical instruments. He claimed that the frequency of two strings of equal tension must have an inverse ratio to the lengths of the strings, and thus proposed to mathematically describe the degree of consonance or dissonance of two tones. These letters were only published much later in Benedetti's comprehensive *Diversarum speculationum mathematicarum et physicarum liber*. In January 1567 Benedetti left Parma with a letter of recommendation from the Duke.

In the same year Benedetti was invited by the Duke of Savoy, Emanuele Filiberto,[35] to the Court in Turin.[36] The Duke, after the invasion and devastation of his territory by French and Spanish troops, was engaged in a renewal of the civic and military infrastructure that included political and economic reforms, but also an increased support for education and the sciences.[37] Benedetti became involved in this renewal as an advisor to the Duke, as a court mathematician, and as an engineer-scientist. In Turin he constructed mathematical instruments such as sundials, calculated horoscopes, built a fountain, and executed other public tasks.[38] Concurrently, he possibly taught at the new University of Turin and educated the son of the Duke, the later Carlo Emanuele I, in mathematics. In recognition of his services to the court he was made a nobleman in 1570.

[30] Jean Taisnier, 1508–1562.
[31] Taisnier (1562), see the discussion in Drake (2008).
[32] Ottavio Farnese, 1524–1586.
[33] Bordiga (1985, 593 ff.).
[34] Benedetti (1574). Drake (2008) and others following him suggest the year 1573. This seems to be an error.
[35] Emanuele Filiberto, 1528–1580.
[36] Benedetti (1585, first page of the dedication to the Duke).
[37] See Ricuperati (1998).
[38] See Mamino (1989) and Roero (1997).

In his book *De gnomonum umbrarumque solarium usu liber* of 1574[39] he dealt at length with the construction of sundials with faces of varying inclinations and also with cylindrical and conical surfaces. His treatise *De temporum emendatione opinion* of 1578 aimed at correcting and reforming the calendar. In 1578 the Duke initiated a public disputation at the University of Turin, at which Benedetti argued with Antonio Berga on whether there was more water or more land covering the surface of the earth. The views which Benedetti brought forth against Berga in this debate were published in Turin in 1579 under the title *Consideratione di Gio. Battista Benedetti, filosofo del sereniss. S. Duca di Savoia, d'intorno al discorso della grandezza della terra e dell'acqua del eccellent. sig. Antonio Berga.*[40] In 1580, after the death of Emanuele Filiberto, Benedetti was confirmed in his position by Carlo Emanuele I. There is evidence that, by 1585, he was married. In 1581 he wrote a lengthy letter in which he reacted to a treatise questioning the reliability of astrology and ephemerides, later published in *Diversarum speculationum mathematicarum et physicarum liber.*[41] Benedetti was, as this book shows, an admirer of Copernicus and developed cosmological views of his own, which were remarkably close to the views of his correspondent Francesco Patrizi (the fluidity of space and the infinity of the universe outside the sphere of the fixed stars) and to Giordano Bruno (Copernicanism and plurality of worlds) who visited Turin and Chambéry around 1578.[42]

In astrological accounts, Benedetti predicted his own death for the year 1592, as one reads in the conclusion of the *Diversarum speculationum mathematicarum et physicarum liber.* As he lay on his deathbed in January 1590, he tried to account for his premature death with a calculational error of four minutes that he must have made in his horoscope.

2.3 The critic Guidobaldo del Monte

Guidobaldo del Monte was born on January 11, 1545 in Pesaro, in the territories of the Duke of Urbino and died on January 6, 1607 in nearby Montebaroccio (today Mombaroccio).[43] He studied mathematics at the

[39] Benedetti (1574).
[40] Benedetti (1579).
[41] Benedetti (1585, 228–248).
[42] See Seidengart (2006) and Omodeo (2009).
[43] For the following short biography, see the volume on Guidobaldo's *Mechanicorum liber* in this series; see also Rose (2008) and Gamba and Andersen (2008). For extensive discussions of Guidobaldo's science and historical context, see Gamba and Montebelli (1988), Biagioli (1990), Bertoloni Meli (1992), Gamba (1998), Micheli (1992),

University of Padua in 1564. After serving in the army for some time and participating in the campaign of the Holy Roman Emperor Maximilian II[44] against the Turks, he joined the circle of Federico Commandino[45] in Urbino. Commandino was a key figure of a scientific humanism that aimed at restoring the ancient mathematical sciences by editing and translating works of Euclid, Archimedes, Pappus,[46] and others. In later life Guidobaldo pursued his studies, writing several books and constructing and producing scientific instruments at the family castle in Montebaroccio.

In his own work Guidobaldo built on the restoration of ancient science inaugurated by Commandino and, in 1577, published the comprehensive and influential *Mechanicorum liber*.[47] The book focuses on Heron's[48] five simple machines – the lever, the pulley, the axle in a wheel, the wedge, and the screw – complemented by the balance as a sixth one. Following Heron and Pappus, Guidobaldo claimed that every mechanism can be reduced to one of these machines and that their properties can in turn be derived from those of the balance and the lever. He saw himself as pursuing an approach that could be traced directly to Archimedes. In fact, the latter's concept of center of gravity plays a key role in his treatise. But Guidobaldo also followed the Aristotelian tradition by attaching great importance to the concept of the center of the world, deriving mechanical properties from the mutual relation of the three centers: the point of suspension or support of a body (its fulcrum), its center of gravity, and the center of the world. In 1581 Guidobaldo's book was published in Italian,[49] translated and introduced by Filippo Pigafetta.[50] In 1588 Guidobaldo published a commentary on Archimedes' book on the equilibrium of planes,[51] followed in 1600 by a major treatise on perspective.[52]

The Urbino school of engineer-scientists to which Guidobaldo belonged was characterized by a strict focus on classical antiquity as the only legitimate model for science as well as by an *esprit de corps* that found

Henninger-Voss (2000), Bertoloni Meli (2006), van Dyck (2006a), van Dyck (2006b) and Becchi et al. (2012).

44 Maximilian II, 1527–1576.

45 Federico Commandino, 1509–1575.

46 Pappus of Alexandria, ca. 290–350.

47 DelMonte (1577), Renn and Damerow (2010).

48 Heron (or Hero) of Alexandria, ca. 10–70.

49 DelMonte (1581); see the discussion in Henninger-Voss (2000).

50 Filippo Pigafetta, 1533–1604.

51 DelMonte (1588), see the discussion in Frank (2007).

52 DelMonte (1600), see the discussion in Gamba and Andersen (2008) and Marr (2011).

its most prominent expression in Bernardino Baldi's[53] posthumously published *Cronica de' Matematici*.[54] Baldi was Guidobaldo's friend and a fellow-disciple of Commandino. His *In mechanica Aristotelis problemata exercitationes*, also published posthumously,[55] built on Guidobaldo's mechanics and constituted another attempt to demonstrate the harmony between Archimedean and Aristotelian approaches to mechanics and thus of the integrity of the ancient tradition. The members of the Urbino school were unanimous in their rejection of what they considered medieval contaminations of the ancient tradition by writers such as Jordanus and his early modern followers Tartaglia and Cardano.[56] Accordingly, the judgement on authors such as Benedetti who was considered a proponent of this tradition was harsh. This is evident from Guidobaldo's marginal comments presented in this volume, but also from the short biography of Benedetti in Baldi's *Cronica de' matematici*:

> Gio: Battista Benedetti Venetiano attese alle Matematiche, nelle quali servì i Duchi di Savoja. Scrisse un libro di Gnomonica, il quale toccò molte cose appartenenti alle dimostrationi della detta disciplina, se non che viene ripreso da più esquisiti di non haver'osservato quel metodo, e quella purità nell'insegnare, che ricercano le Matematiche, ed è stato osservato da gl'ottimi Greci, e da gl'Imitatori loro. Scrisse anco alcune altre cose leggiere, e di non molto momento.
>
> The Venetian G.B. Benedetti occupied himself with mathematics, a field in which he served the Dukes of Savoy. He wrote a book on gnomonics which deals with many themes belonging to the proofs of this discipline. It is, however, reproached by more distinguished scholars for not having followed that method and that purity in teaching which mathematics requires and which has been observed by the great Greeks and those who followed them. He furthermore wrote some other light things of little import.[57]

In 1589 Guidobaldo became Visitor General of the fortresses and cities of the Grand Duke of Tuscany. A year earlier he had come in contact with

[53] Bernardino Baldi, 1553–1617.

[54] Baldi (1707). The manuscript version is preserved at University of Oklahoma Libraries, History of Science Collections. See also Nenci (1998).

[55] Baldi (1621); see volumes 3 and 4 on Baldi's treatise in this series Nenci (2011a,b).

[56] Gerolamo Cardano, 1501–1576.

[57] Baldi (1707, 140). Baldi himself was keenly interested in gnomonics on which he wrote an extensive manuscript that remained, however, unpublished.

the young Galileo. They frequently exchanged letters about mechanical subjects and probably met for the first time when Guidobaldo visited Tuscany in the late Spring of 1589,[58] and again in 1592 in Montebaroccio. Guidobaldo became Galileo's mentor and patron, securing him university positions first in Pisa (1589) and later in Padua (1592). One link between them was Galileo's Pisan friend and colleague Jacopo Mazzoni in whose work Guidobaldo was interested.[59] Galileo's initial scientific interests, concerning problems of static equilibrium analyzed in the style of Archimedes, were well matched with those of Guidobaldo. Later Galileo also emulated Guidobaldo's activities as an engineer-scientist, setting up a workshop for producing scientific instruments and writing treatises on fortification and mechanics.[60] However, in the course of time, significant differences emerged in their approach to the developing mathematical science of nature in which Galileo took a position closer to that of Benedetti. In contrast to Guidobaldo, Galileo was convinced, in particular, that also phenomena of motion such as projectile motion, the oscillations of a pendulum, or motion along an inclined plane were amenable to an exact mathematical treatment. Like Benedetti, but not Guidobaldo, he furthermore developed a keen interest in the Copernican world system. In 1592, the year of Galileo's move to Padua, Guidobaldo was visited at Montebaroccio by Galileo with whom he performed the experiments on projectile motion that led to the discovery of the law of fall.[61] On that occasion, and probably even earlier, they must have discussed foundational issues of mechanics as well, including the relation between Guidobaldo's and Benedetti's approach, possibly using the very copy of Benedetti's book, parts of which are reproduced here. Galileo's early intellectual career thus unfolded in the midst of the tension between Guidobaldo and the *classicist* Urbino school, on the one hand, and Benedetti's more open–minded attitude to tradition, on the other.

[58]This information is based on recent studies by Francesco Menchetti, subsequently extended by Martin Frank, see Menchetti (2012).

[59]Jacopo Mazzoni, 1548– 1598. Guidobaldo's interest in Mazzoni has recently been stressed by Martin Frank (personal communication).

[60]For an extensive historical discussion, see Valleriani (2010).

[61]See Renn et al. (2001).

Chapter 3
The Context

3.1 The long-term transmission of mechanical knowledge

The context of this discussion of the *equilibrium controversy*, based on Guidobaldo's marginalia, is the long-term development of mechanical knowledge, in particular from the origin of theoretical mechanics in antiquity to the dawn of classical mechanics in the late Renaissance, when the controversy first became a central issue of contemporary discussions. It may be helpful therefore to begin with a brief survey of the history of mechanics, a history that extends over more than two millennia. This long period can be divided into six more or less coherent periods:

- The first period may simply be called the *prehistory of mechanics*; it comprises the long period in which human cultures accumulated practical mechanical knowledge without documenting this knowledge in written form and without developing theories about it. Although the origin of other sciences such as mathematics and astronomy can be traced back to ancient urban civilizations such as those of Babylonia and Egypt, this, surprisingly, is not the case for mechanics. Although there are numerous sources testifying to the large construction projects of these civilizations, there is in fact not one single document referring to the mechanical knowledge that must have been involved in these endeavors.

- The next period properly merits the label *origin of mechanics*. It saw, in particular, the formulation and proof of the law of the lever. More generally, it is characterized by the appearance of the first written treatises dedicated to physics and mechanics in ancient Greece, associated in particular with names such as Aristotle, Euclid, Archimedes, and Heron.[1] These works had an enormous impact on the subsequent development of mechanical knowledge. Aristotelian *physics* focused on the role of forces on moving bodies, and Aristotelian and Archimedean *mechanics*, based

[1] For a discussion of the parallelism between the emergence of mechanics in Greece and in China, see Renn and Schemmel (2006).

on the law of the lever, provided two fairly independent points of reference for the development of theoretical mechanics. They finally merged into one common conceptual basis for a new coherent theory of mechanics only with the advent of classical physics in the late seventeenth and eighteenth centuries.

- The third period is, at the beginning, characterized by the transformation of mechanics into a *science of balances and weights* and the enrichment of Aristotelian physics by a *theory of impetus* and a mathematical extension of the Aristotelian doctrine of generation and corruption to a *theory of changing qualities*. This period covers the Arabic and Latin Middle Ages, which saw the production of a substantial literature on mechanics focused on a relatively small range of subjects, in particular, the behavior of the balance and the justification of the law of the lever, and the changing qualities of mechanical bodies such as heaviness and velocity.

- The fourth period is that of preclassical mechanics, ranging from the sketches of Renaissance engineers such as Leonardo da Vinci[2] to the mature works of Galileo Galilei. In contrast to the preceding period, it deals with an increasingly large number of subjects, including the inclined plane, the pendulum, the stability of matter, and the spring, in attempts to integrate the science of weights with the effect of forces on moving bodies, which necessarily transformed the inherited theoretical building blocks.

- The fifth period is that of the *rise of a mechanistic world view*. The successful integration of earlier traditions into the fundament of classical physics appeared without alternatives. From the first comprehensive vision of a mechanical cosmos, such as that of Descartes, via the establishment of classical and later analytical mechanics, this process led to the attempts of nineteenth-century scientists to build physics on an entirely mechanical foundation, which was conceived as an ontological basis of the natural sciences.

- The sixth period comprises the decline of the mechanistic world view and the disintegration of mechanics at the turn of the nineteenth to the twentieth century, resulting in the emergence of modern physics and its conceptual revolutions represented by the relativity and quantum theories.

[2]Leonardo da Vinci, 1452–1519.

This schematic overview of the long-term development of mechanics raises a number of puzzling questions. For example: How did theoretical mechanics originate in ancient Greece and why did this not happen earlier? What kind of knowledge made the formulation of the law of the lever possible, and what knowledge was required for its proof? What accounts for the remarkable differences between the medieval science of weights and preclassical mechanics? What kind of empirical knowledge made the emergence of classical mechanics possible and what accounts for its remarkable stability over the more than 200 years of classical physics? What explains the steady development, consolidation, and stabilization of Aristotelian physics over more than 2000 years? How can one explain the disintegration of mechanical concepts around the turn from the nineteenth into the twentieth century and the creation of revolutionary theories such as general relativity and quantum theory, which proved to be an adequate foundation for today's physical and cosmological knowledge, although that knowledge was not available when they emerged? How did the law of the lever survive all these changes? And finally the question that will come under closer scrutiny in the present volume, how did the concept of *positional heaviness* emerge and, under varying labels, become an integral part of preclassical and, after a substantial transformation, of classical mechanics?

3.2 The ancient roots of mechanics

The development of mechanical knowledge sketched here was a non-linear and multi-layered historical process. In particular, the following analysis of the specific process of the transmission of mechanical knowledge from antiquity to the Arabic and Latin Middle Ages and finally to the Renaissance makes it clear that the development of mechanics was anything but a successive accumulation and theoretical integration of a growing body of mechanical knowledge. Theoretical mechanics had a twofold root in the ancient Greek reflection on practical experiences. On the one hand, there was the intuitive experience that in order to move a body a certain effort is required depending on its weight. This experience became the basis of the concept of force as the cause of terrestrial and celestial motion in Aristotelian dynamics. On the other hand, the reflection on the potential of mechanical devices to reduce the force required to move a body became the basis of the Archimedean theory of equilibrium and its generalization in the concept of the center of gravity.

In ancient Greece the development of these basic concepts of mechanics was an issue of highly personalized communication between the members of a relatively small group of experts, for which the correspondence of Archimedes is typical.[3] There was a strong relation between authors, their theoretical biases, and their specific subjects. Following the model of Euclid's compilation of the mathematical knowledge of his time, Archimedes tried to present his insights in the form of deductive treatises.[4] Thus, the concept of *center of gravity* became the core concept of deductive mechanics. Aristotle, in contrast, presented his account of natural phenomena in a network of categorizations linked by syllogisms. Originally, there was no intimate relation between Aristotelian dynamical explanations of motion and gravity, on the one hand, and the deductive mathematical method, on the other. Similarly, Aristotelian explanations of mechanical devices merely followed the tradition of the literature presenting issues in form of problems and their solutions, without exposing an immanent necessity constituted by its subject. Finally, the shift from a personalized to a more institutionalized representation of knowledge in late antiquity contributed much to the historical transfer of the ancient heritage, but resulted in a compilation rather than an integration of the various elements of mechanics developed in the ancient Greek tradition.

3.3 Preclassical mechanics

Preclassical mechanics of the sixteenth and early seventeenth centuries was characterized by an elaboration of the knowledge resources available in light of *challenging objects* such as labor-saving machinery, ballistics, the stability of architecture, or ship-building provided by contemporary technology. Preclassical mechanics was a historical stage in its own right in the development of mechanics. It was pursued by a class of engineer-scientists who addressed these technical challenges by drawing on heterogeneous bodies of knowledge, which comprised the growing set of recovered ancient scientific and technical texts. The heterogeneity as well as the fragmentary nature of the *shared knowledge* of early modern science, especially with regard to the heritage of ancient science and its subsequent transformation, is well illustrated by the conflictual integration of Aristotelian and Archimedean knowledge resources on mechanics as it can be traced in the works of Tartaglia, Cardano, Guidobaldo, Benedetti, Galileo, and many

[3] See the discussion in Netz (1999).
[4] Dijksterhuis (1956).

others, and in particular also in the conflicting approaches of Guidobaldo and Benedetti documented in this volume.

No simplistic division of the protagonists of preclassical mechanics into followers of Aristotelian dynamics and Archimedean statics, into northern and central Italian schools, into Aristotelians and anti-Aristotelians, will be able to do justice to the complex interlocking of the diverse components of the shared ancient heritage.[5] For instance, the works of Archimedes on statics and hydrostatics provided not only a model for a deductive theory of physics emulating the structure of Euclid's *Elements*, but also assets to modify specific explanations within an Aristotelian conceptual framework, in particular, the Aristotelian explanation of free fall in media with the help of the Archimedean concept of extrusion. Conversely, Aristotelian natural philosophy provided a physical underpinning to the Archimedean theory of equilibrium of planes, fostering its extension to a more comprehensive treatment of mechanics. Although discussions about mechanics in the early modern period were often shaped by questions of the superiority or compatibility of the diverse bodies of knowledge inherited from antiquity, in the end everyone drew from the same sources so that Aristotelian and Archimedean elements are found alongside each other, albeit in different constellations, in the works of authors as diverse as Tartaglia, Cardano, Guidobaldo, Benedetti, and Galileo.

The integration of these elements took place under the new conditions of the early modern period for the development of mechanical knowledge that were given not only by the emergence of challenging objects, but also by an intellectual context that involved many more actors intervening simultaneously than had ever been the case since antiquity. Accordingly, the inherent potential of the traditional bodies of knowledge was much more intensely exploited than ever before – in directions shaped, but not uniquely determined, by the concrete material at hand, that is, by the challenging objects that represented focal points of attention.

As a consequence, a multiplicity of different pathways developed, sometimes leading to the same insights into a given problem, sometimes to diverging opinions on it. At the same time, intrinsic tensions within a given traditional body now emerged more clearly, due to the fact that it was no longer, as was typically the case in antiquity, one single author or a string of authors separated by generations who were involved in its elaboration. Instead, one and the same problem was now often addressed from

[5] For a still very helpful survey of different sixteenth-century knowledge traditions, see Drake and Drabkin (1969).

distinctive perspectives, thus becoming a borderline problem of different knowledge traditions and catalyzing their conflictual integration.[6]

The traditional image of this period, which still often lurks behind even the most sophisticated historical reconstructions, whereby single authors studied single problems with greater or lesser success as judged in hindsight, thus preparing the eventual combination of the pieces of a puzzle into one coherent whole, is fundamentally mistaken. In fact, strictly speaking, *all* solutions proposed in preclassical mechanics are untenable from the viewpoint of classical mechanics since they were articulated in conceptual frameworks incompatible with those of later science, making use of alien concepts such as *positional heaviness*, *natural* and *violent tendencies*, or *impetus*.

Thus, the emergence of classical mechanics from preclassical mechanics cannot have been an essentially cumulative process of selecting from ancient sources isolated pieces of knowledge that were deemed valuable, separating the wheat from the chaff, and then gradually adding new insights. Rather, the emergence of classical mechanics from its ancient roots must have amounted to the structural transformation of a *system of knowledge*, which involved a reorganization of conceptual systems on the basis of results achieved within traditional frameworks. Such a transformation would not have been possible without the interaction of the authors and a confrontation of their proposed solutions to mechanical problems, and can neither be described in terms of a linear accumulation of knowledge nor in terms of a competition between distinct schools or traditions.

When was a problem actually solved? With hindsight, we may claim, for instance, that Aristotle was the first to analyze the equilibrium of the balance, that Archimedes proved the law of the lever, that Jordanus solved the problem of the inclined plane, and that Galileo discovered the law of fall. With hindsight, it may indeed seem that these insights could have been achieved as isolated contributions, independent of the establishment of a larger, stable conceptual framework. From the perspective of preclassical mechanics, however, it is remarkable how far these solutions actually were from being evident or uncontroversial among the contemporaries. Some of these became the topic of heated debates, such as the question of the equilibrium of the balance. In particular, as will be seen in the following, the question of whether or not a balance with equal arms would, when deflected from the horizontal position, return to its original state, was controversially discussed by authors such as Tartaglia, Cardano, Guidobaldo, and Benedetti. The issue arose because partly different con-

[6] For the notion of borderline problem, see Renn (2007, 30).

ceptual frameworks seemed to suggest different answers, since the exact relation between concepts such as *center of gravity* and *positional heaviness* had not yet been definitively established.

Solutions that appear to come close to the correct solution in classical mechanics, such as Jordanus' analysis of the inclined plane, were rather isolated and in part disregarded by contemporary authors. But an isolated solution, a solution that is not taken up, discussed, contradicted, reformulated, supported by new arguments, or used in novel contexts, fails to be a convincing solution. A broadly shared acceptance of the solution to a particular problem typically presupposes controversy and only results from embedding the solution within an extended network of arguments, eventually connecting it with everything else within a system of knowledge. A progressive accumulation of established results remained, however, illusionary as long as the argumentative networks developed in preclassical mechanics were neither comprehensive in the sense of being adequate to deal with the entire scope of shared mechanical knowledge nor coherent in the sense of yielding an unambiguous solution, at least to those problems within their range of extension. The eventual emergence of more or less stable solutions to such basic mechanical problems only emerged in the course of the transformation of preclassical into classical mechanics, and represented an outcome rather than a precondition of this development.

From a traditional perspective, it may come as a surprise that not only new discoveries of dynamics such as the law of fall and the parabolic shape of the projectile trajectory, but even elementary insights from statics such as the indifferent equilibrium of a balance suspended from its center of gravity, the equilibrium of a body on an inclined plane, or the principle of the bent lever were still not definitively established toward the end of the sixteenth century, although it seems that these results could easily have been inferred from known ancient and medieval sources. But from the perspective sketched above, the seemingly fruitless contemporary debates about such problems take on a new significance. Rather than representing encounters between blind men sometimes hitting the mark and sometimes not, they were a medium of the dialogical transformation of a system of knowledge.[7]

But how did this dialogical transformation of knowledge actually take place and how in the end did more or less stable solutions to basic mechanical problems result? Although we have claimed that this stabilization did not happen in a piecemeal fashion, but rather in the context of a more holistic process of conceptual reorganization, it is also clear that the trans-

[7] See the discussion in section 1.3; compare also the approach developed in Beller (1999).

formation of preclassical into classical mechanics did not happen in one fell swoop. Rather, within the developing network of mechanical arguments, some more or less stable nodes gradually emerged which did not correspond to the original roots from which the network was growing, such as the principles of Aristotelian physics and of Archimedean mechanics. These nodes resulted instead from an elaboration of the consequences of the traditional framework triggered by the confrontation with challenging objects. Some of these consequences constituted indeed the starting points for the reorganization of the accumulated knowledge, eventually yielding the new principles of classical mechanics, such as the principle of inertia, the principle of work, or the understanding of the directional character of force and of the relation between force and torque. Thus, the principle of inertia, for instance, could be obtained by reflecting on Galileo's results concerning projectile motion which had still been achieved within a preclassical conceptual framework.[8] In turn, these results were related to a stabilization of the knowledge on mechanical devices such as the inclined plane, the bent lever, and the deflected balance, also resulting from a tedious process of the elaboration and integration of different knowledge resources. This process was typically accompanied by controversies over the conceptual foundations of mechanics, for instance about the role of such concepts as that of *positional heaviness*, *center of gravity*, or *momento* as it can be traced in the works of Guidobaldo, Benedetti, and Galileo, and as they are illustrated in an exemplary way by Guidobaldo's annotation of Benedetti's book.

3.4 The ancient and medieval origins of the *equilibrium controversy*

Whether and under what circumstances an equilibrated balance deflected into an oblique position returns into the horizontal is a question that goes back to the second problem of the Aristotelian *Mechanical Problems.* This, however, deals with balances having a beam of finite thickness so that it makes a difference whether they are supported from above or from below. But the seemingly simple case of the beam being supported at its center of gravity is not discussed. The concept of a *center of gravity* in fact was unknown to the Aristotelian author. As far as we know, this concept, which immediately suggests one stance in the later controversy, was not intro-

[8]See the extensive discussion of the preclassical framework of Galileo's work in Renn et al. (2001) and Damerow et al. (2004).

duced before Archimedes.[9] As will become clear below, it was Guidobaldo who revived and applied this concept in the context of the controversy about the deflected balance.

An understanding of the positional dependence of the effect of a weight under the constraints of its attachement to the beam of a balance was, as far as we know, expressed in geometrical terms for the first time in the Aristotelian *Mechanical Problems.*[10] The issue was clarified by Archimedes who formulated the *law of the lever*, supplying a convincing deductive proof.[11]

The question of why the deflected balance nevertheless caused a vivid controversy in the early modern period is obviously related to the question of which parts of the ancient mechanical knowledge were transmitted when, where, and how to early modern scholars.[12] As will be shown below, the ignorance of the concept of the *center of gravity* in fact led to the introduction of the concept of *positional heaviness.*

[9] For an argument that Archimedes only elaborated an already existing intuitive concept, see Dijksterhuis (1956, 298–313).

[10] Aristotle (1980). See the discussion in Krafft (1970), Damerow et al. (2002), and Renn and Damerow (2007).

[11] The *law of the lever* is also contained in the Aristotelian *Mechanical Problems.* However, the answers to the problems in this treatise are not based on this law but – as far as they deal with the force-saving potential of mechanical devices which is the case for approximately half of the problems – on a *basic explanatory principle* discussed in the following section 3.4.1. Notwithstanding the debated attribution of the treatise to Aristotle, this principle can be considered as a precursor of the *law of the lever.* This law itself occurs in the whole treatise only once, pointlessly inserted into the answer to the third problem which is based on the basic principle as all other related problems (Aristotle, 1980, 352–353). It is therefore likely that the inserted phrase stating the *law of the lever* is a later addition based on the erroneous inclusion of a marginal note in a Byzantine manuscript from which all surviving copies are derived, see section 3.4.2 below.

[12] Caverni (1972, vol. 4, 190 ff.) discusses extensively the problem of the deflected balance and the opinions of Jordanus, Leonardo, Tartaglia, Cardano, Guidobaldo, and Benedetti, as well as the subsequent discussions of this issue in the seventeenth century. Duhem (1991) also refers frequently to the problem. In both accounts Leonardo is considered a pivotal figure for having found or identified the correct solution to this problem as well as to the related problem of the bent lever. Both authors essentially assume that the transmission of such insights was unproblematic, supposing in particular that Benedetti's treatment was based on knowledge of Leonardo's manuscripts. Clagett (1959, 159) assumes that Galileo knew the correct solution to the bent lever problem because he was familiar with Jordanus' *De ratione ponderis* in the Tartaglia edition. We will come back to the challenges of knowledge transmission below. The role of the *equilibrium controversy* for Galileo's *Mechanics* is also discussed in the introductory essay of Galilei (2002). The role of the controversy for Guidobaldo's work has been discussed extensively in Palmieri (2008). For a general survey of early modern mechanics, see also Laird (1986).

3.4.1 The Aristotelian context

Throughout the entire period under consideration, from antiquity via the Arabic and Latin Middle Ages to the early modern period, core ideas of Aristotelian dynamics were evidently known to scholars dealing with mechanical problems.

According to Aristotle the velocity of a body in natural descent is proportional to its weight while that of a body in violent motion is proportional to the moving force. In his *Physics* he wrote:

> [...] as to differences that depend on the moving bodies themselves, we see that of two bodies of similar formation the one that has the stronger trend (ῥοπή) downward by weight (βάρος) [...] will be carried more quickly than the other through a given space in proportion to the greater strength of this trend.[13]

A passage from *On the Heavens* dealing with the effect of a moving force sheds light on what he means by the term *more quickly*:

> [...] if there is a moving body which is neither light nor heavy (βάρος), its motion must be enforced, and it must perform this enforced motion to infinity. That which moves it is a force (δύναμις), and the smaller, lighter body will be moved farther by the same force. [...] For as the greater body is to the less, so will be the speed of the lesser body to that of the greater.[14]

This passage contains in essence the core idea of the Aristotelian dynamics of violent, that is, enforced motion. The same force exerts a greater effect on a lesser than on a greater body. The effects are measured by the speeds in the sense of distances traversed in the same time which are inversely proportional to the sizes of the bodies.

The proportionality between force and effect, however, seems to contradict experiences gained from levers and balances. Applied to such tools, the same force has different effects depending on the position where it acts on a beam. The Aristotelian *Mechanical Problems* can be interpreted as an attempt to avoid this contradiction. The resolution of this contradiction in the *Mechanical Problems* relies on a *basic explanatory principle* interpreting the balance in terms of motions of weights along circles of different radii.[15] This principle states that the part of the radius of a circle that is

[13] Aristotle (1995, 216a, 12–17).
[14] Aristotle (1986, 301b, 4–14).
[15] Damerow et al. (2002, 94–95).

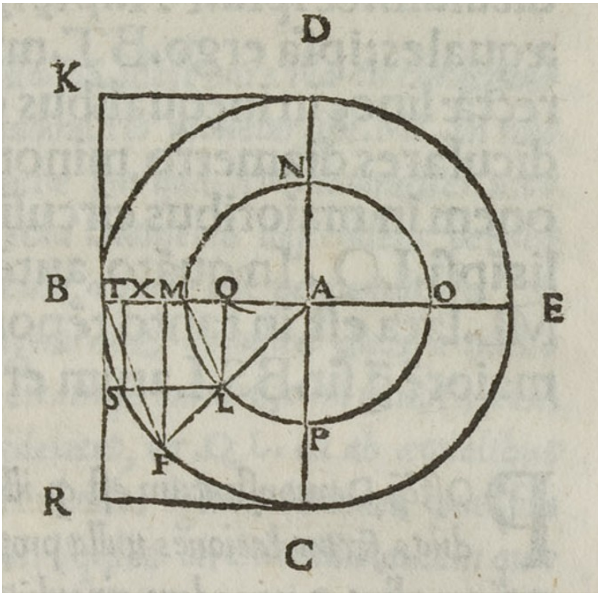

Figure 3.1: Drawing by Tomeo in his 1525 translation *Aristotelis quaestiones mechanicae* illustrating that the arc *BS* of the larger circle is less bent than the arc *ML* of the smaller circle. According to Aristotle, the interference of the violent constraint on the natural downward motion must thus be greater on the smaller than on the greater circle.

farther from the center moves more quickly than the part that is closer to the center being moved by the same force. The greater effect of a weight moving on the greater circle described by a larger beam is explained by the lesser interference of that violent constraint with the natural motion downward when compared to the motion along a smaller radius. In the Latin translation by Tomeo, the relevant passage reads:

> Si autem duobus ab eadem potentia latis hoc quidem plus repellatur, illud vero minus, rationi consentaneum est tardius moveri id quod plus repellitur eo quod repellitur minus: quod videtur accidere maiori et minori illarum quae ex centro circulos describunt: quoniam enim proprius est manenti, eius quae minor est extremum, quam id quod est maioris, veluti

> retractum in contrarium ad medium tardius fertur minoris extremum. Omni quidem igitur circulum describenti istuc accidit: ferturque eam quae secundum naturam est lationem secundum circunferentiam: illam vero quae praeter naturam, in transversum et secundum centrum: maiorem autem semper eam quae praeter naturam est ipsa minor fertur: quia enim centro est vicinior quod retrahit vincitur magis.
>
> Now if of two objects moving under the influence of the same force one suffers more interference, and the other less; it is reasonable to suppose that the one suffering the greater interference should move more slowly than that suffering less, which seems to take place in the case of the greater and the less of those radii which describe circles from the centre. For because the extremity of the less is nearer the fixed point than the extremity of the greater, being attracted towards the centre in the opposite direction, the extremity of the lesser radius moves more slowly. This happens with any radius which describes a circle; it moves along a curve naturally in the direction of the tangent, but is attracted to the centre contrary to nature. The lesser radius always moves in its unnatural direction; for because it is nearer the centre which attracts it, it is the more influenced.[16]

The author thus introduced the idea of explaining the dependence of the effect of a weight on its position by considering factors such as the natural and violent components of the motion and the lesser or greater deviation of the motion from its natural course.

The Aristotelian analysis left much room for interpretation. In any case, the Aristotelian *Mechanical Problems* could have become the starting point for formulating a concept of *positional heaviness* and were indeed brought into connection with it by early modern writers on this subject such as Tartaglia.[17] Positions taken in the early modern period on this issue such as those of Guidobaldo and his adversaries can be considered as elaborating one or the other alternative implicit in the Aristotelian analysis.[18] However, it is rather unlikely that this was how the concept of

[16] Tomeo (1525, 27r–27v). Translation in Aristotle (1980, 341–342).
[17] See, for instance, how Tartaglia's virtual interlocutor reminds him in his *Quesiti* of the connection between the claim that the equilibrated balance deflected into an oblique position will return to the horizontal and the treatment of the balance in the Aristotelian *Mechanical Problems*; see Tartaglia (1546, 88v), cf. Drake and Drabkin (1969, 124).
[18] See section 3.8.

positional heaviness actually came into being in the thirteenth century, since at that time the text of the Aristotelian *Mechanical Problems* had probably not yet circulated in the Latin West.[19] However, once it became available it was printed, translated, and commented upon by numerous early modern scholars and became a standard point of reference for mechanical arguments in the sixteenth century.[20]

3.4.2 The transmission of ancient mechanics

The concept of *positional heaviness* was, as discussed in section 2.1, explicitly introduced by the medieval scholar Jordanus de Nemore. Its historical roots can only be determined, however, by a closer look at the milestones of ancient mechanics represented in the works of Aristotle, Archimedes, and Heron, and at the vexing history of the transmission and transformation of this heritage by scholars in the Arabic world.[21]

From the viewpoint of the transmission of ancient knowledge, the *first milestone* of the development of the science of mechanics is the work of Aristotle. As mentioned above, the backbone of the long-term transmission of mechanical knowledge was Aristotelian dynamics, known in the Arabic world, in the medieval Latin period, and in early modern times. It was used throughout as a point of reference for arguments on balances and other mechanical devices.

The Aristotelian *Mechanical Problems* had a somewhat less continuous history as they were probably unknown when the *science of weights* first emerged in the Latin Middle Ages.[22] The treatise has been transmitted as part of the Aristotelian corpus, but its attribution to Aristotle has been called into question although there is a consensus that it dates back to the third century BCE and has emerged from the immediate context of his work.[23] With the exception of an earlier Arabic epitome,[24] all extant Greek manuscripts and later printings are based on one archetype Byzantine codex, the codex Z.Gr.214 of the Biblioteca Marciana.[25] Altogether

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[19] The arguments for its availability are indeed rather weak, see Clagett (1957).
[20] Rose and Drake (1971).
[21] For an overview of Arabic science and technology, see Hill (1993, 59–70).
[22] It is sometimes assumed that Jordanus must have been familiar with the Aristotelian *Mechanical Problems*. However, Jordanus wrote his works in the first half of the thirteenth century whereas there are no indications that any manuscript reached the Latin West at that early date.
[23] Rose and Drake (1971, 72).
[24] Abattouy (2001, 180 and 195–199).
[25] Rose and Drake (1971, 73).

twenty-nine Greek manuscripts survived.[26] The three oldest manuscripts, that is, all extant manuscripts written before the fifteenth century, were written in Byzantium and only later transferred to Italy, whereas the majority of later manuscripts were certainly written in the West. The fifteenth century is marked by an extreme increase in copying activities.[27] Thus twenty-one of the later manuscripts were written in the fifteenth century, four in the sixteenth, and one in the seventeenth century.[28] The situation is further confused by the widely overlooked fact that the treatise was also covered by Pachymeres' *Philosophia* which contains paraphrases and commentaries on most of Aristotle's works. Pachymeres[29] was a Greek scholar who spent most of his life in Constantinople. The extant manuscripts of *Philosophia* show a similar distribution as the "normal" manuscripts with a peak in the sixteenth century.[30]

Thus, the Aristotelian *Mechanical Problems* only became widely available to the scholars of the Latin tradition at the beginning of the early modern period through Greek manuscripts and their Latin translations, all of which derived from a single Byzantine source from the late eleventh or early twelfth century.[31] In the Arabic context, however, the core text of the Aristotelian *Mechanical Problems* was known, but not the deductive justification of its *basic explanatory principle*[32] which in the early modern period served as an important background for arguments concerning positional heaviness. The *Mechanical Problems* contain with this principle a precursor formulation of the law of the lever, knowledge of which

[26] One must assume that the actual number at the time was much higher, but due to their long and changeful history, not all manuscripts that had once existed came down to us. Rose and Drake (1971, 72–76) list twenty-nine manuscripts. The codices Ms. Vat. Gr. 2231 and Vat. Gr. 905 have to be added to this list while Ms. Phi 1,10 of the Biblioteca de El Escorial has to be dropped.

[27] At the end of the fifteenth century, between 1495 and 1498, the Aristotelian corpus was printed for the first time by Aldus Manutius (1449–1515). The *Mechanical Problems* are contained in the second volume, printed in 1497.

[28] All data on the manuscript tradition are based on unpublished work by Paul Weinig who joined the project on the history of mechanical knowledge at the Max Planck Institute for the History of Science for several years.

[29] Georgios Pachymeres, 1242–ca.1310.

[30] Thirteenth century: three copies; fourteenth century: three copies; fifteenth century: nine copies; sixteenth century: twenty copies, among them Ms. Phi 1,10 of the Biblioteca de El Escorial which was erroneously listed by Rose and Drake (1971, 73) as an original Greek copy; seventeenth century: one copy; eighteenth century: one copy.

[31] This way of transmission, however, is untypical for the transmission of Greek manuscripts in general. Byzantine scholars tended to neglect sources of the mathematical science and natural philosophy; see Krumbacher (1897, 605–638) for an overview of the scarcity of technical literature.

[32] Abattouy (2001, 195–199).

was hence transmitted with the text. They also constitute, as mentioned above, a bridge between Aristotelian dynamics and the characteristics of mechanical devices to save force, a bridge that is sustained mainly by the proof of the basic explanatory principle.

The *second milestone* is the work of Archimedes on mechanical problems, without doubt a culmination of ancient mechanical knowledge. However, this was only partially transmitted. In particular, his writings on mechanics, apart from a fragment of *On floating bodies*, became known to the Arab world only indirectly, e.g., through the works of Heron and Pappus. In the Latin world they only became known through the translations from the Greek by Willem of Moerbeke[33] after 1269.[34] However, the law of the lever and the tradition of deductive proofs associated with his work were known in all the periods in question. But his key concept of a *center of gravity* seems to have been unknown in the Latin Middle Ages until the translations of Moerbeke and those produced later.

The *third milestone* is the work of Heron of Alexandria. Heron of Alexandria evidently knew all of the Greek sources representing mechanical ideas but used them eclectically in his reduction of mechanical devices to a classification of simple machines. In particular, Heron's *Mechanics* refers to parts of Archimedes' works on mechanics, some of which have been lost. Heron introduced the concept of *center of gravity* and applied it several times. Moreover, he used the concept implicitly when he dealt with the equilibrium of arbitrarily shaped beams of balances.[35] Furthermore, in an added proposition concerning a beam in form of a pulley, he introduced a from a modern viewpoint correct solution which covered the bent lever as a special case (see figure 3.2).[36]

Heron's *Mechanics* as a whole was, as far as we know, only transmitted to the Arabic world, while excerpts relating mainly to the simple machines were also transmitted in Greek to the West by Pappus' selection, in particular by the Latin translation of Pappus of Alexandria (1588).

Further achievements of ancient mechanics regarding the balance are known to us only through fragments, probably either transmitted directly from the Greek or indirectly to the Arabic and subsequently to the Latin world. One example is a proof of the law of the lever in a *Book of the Balance* ascribed to Euclid. This proof is preserved only in Arabic and

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[33] Willem of Moerbeke, c.1215–1286.

[34] Clagett (1984), see the introduction to volume 1.

[35] Heron of Alexandria (1900, 88).

[36] Heron of Alexandria (1900, 90).

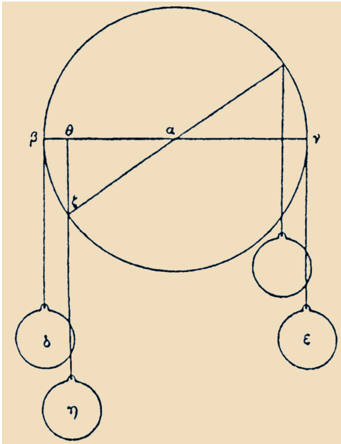


Figure 3.2: The law of the bent lever is an implicit consequence of a theorem in Heron's *Mechanics.* Two ropes are fixed at points β and γ to the border of a pulley. If unequal weights are attached to the ropes, the rope of the smaller weight will be rolled up while the greater weight will move downwards to a certain point ζ until the weights are in equilibrium. Thus the pulley with the ropes acts as if it were a bent lever ζαγ.

its ascription to Euclid is somewhat doubtful.[37] The most remarkable aspect of Euclid's proof, if compared to that of Archimedes, is the fact that it proceeds without involving the concept of center of gravity, using instead a concept characterizing the positional effect of a weight on a balance, designated as the *force of heaviness.* Another achievement of ancient mechanics with unclear origins is the treatment of the material beam of the balance in the *Liber de canonio.*[38]

A *fourth milestone* in the transmission of the ancient heritage of mechanics was its reception and transformation by scholars in the Arabic world. As mentioned earlier, the Arabic world had access to Aristotelian dynamics as well as to the Aristotelian *Mechanical Problems*, but without the proof of the main principle. In addition, the full text of Heron's

[37]See (Clagett, 1959, 24–30). Here as in other passages we make use of a text written by the authors jointly with Peter McLaughlin (Renn et al., 2003).
[38]Moody and Clagett (1960, 55–75); see also the discussion in Knorr (1982, 15–39).

Mechanics as well as the selection made by Pappus were available.[39] Furthermore, the *law of the lever* was known, including a proof in a text ascribed to Euclid,[40] and on this basis the material beam was correctly treated.[41]

Arabic treatises, probably composed on the basis of Greek material by authors such as Thābit and al-Isfizari,[42] focused on the proof of the law of the lever on the basis of Aristotelian dynamics, including a treatment of the material beam, and knowledge about the bent lever.[43] In particular, a scholium to the treatise of Thābit contains the idea, crucial for an understanding of the bent lever, that the effect of a weight suspended not directly from the beam of a balance but from the end of a rod that is rigidly connected to the beam at an oblique angle, will be as if it were suspended at the foot of the perpendicular drawn from the weight to the beam.[44] On the other hand, Thābit, contrary to later Arabic authors, did not make use of the concept of the center of gravity, a circumstance which may have motivated his attempt to justify the equilibrium of the balance instead on the basis of Aristotelian dynamics.

In his *Book on the Balance of Wisdom*, completed 1121–1122, al-Khāzini[45] treated the question of what happens if the balance beam is supported from above or from below, which was raised in the Aristotelian *Mechanical Problems*. He explicitly considered the case in which the balance is supported at the *center of gravity* of the beam and claimed correctly that it remains at rest in whatever position it is left.[46]

In the course of the translation movement of the twelfth century, only a fraction of the Arabic material was transmitted to the Latin world. In particular, a treatise by Thābit entitled *Liber karastonis* was transmitted in a Latin version, probably translated by Gerard of Cremona[47] from a lost Arabic version. However, this treatise did not contain a treatment of the bent lever and states in contrast to al-Khāzini's *Book on the Balance of Wisdom* that the deflected balance returns to the horizontal:

[39] Jackson (1970).
[40] Clagett (1959, 24–30).
[41] Knorr (1982, 15–39).
[42] Al-Muzaffar al-Isfizari, 1048–1116.
[43] See Moody and Clagett (1960, 81–82) for the occurrence of such knowledge in a treatise of Thābit, and Abattouy (2001, 227) for its occurrence in a treatise of al–Isfizari.
[44] For a discussion of the manuscript sources, see Knorr (1982, 75–87).
[45] Abu al-Fath Khāzini, fl. 1115–1130).
[46] Abattouy (2001, 191).
[47] Gerard of Cremona, 1114–1187.

> Dico ergo quod linea sit veniens super equidistantiam orizontis, ita quod si nos inclinemus punctum A ad punctum T et elevetur punctum B ad punctum D, sufficiet pondus a donec redeat linea AB ad locum suum ex equidistantia orizontis.
>
> Then I say the line is in horizontal equilibrium, so that if we incline point A to point T and elevate point B to point D, the weight a is sufficient for line AB to return to its place of horizontal equilibrium.[48]

When Jordanus took up the subject of the science of weights at some point in the thirteenth century, the main sources he probably had at his disposal were, apart from Aristotelian dynamics, in particular the *Liber karastonis* and the *Liber de canonio.* Most likely he did not yet have access to Archimedes' achievements, nor to any part of Heron's work. In a sense, Jordanus was thus in a position similar to Thābit in the ninth century, confronted with the challenge to provide a deductive treatment of the science of weights on the basis of Aristotelian dynamics, without having the concept of *center of gravity* or other Archimedean achievements at his disposal. The further elaboration of the framework he built was evidently guided by taking into account new challenging objects beyond the balance such as the inclined plane.

A more extensive access to the ancient heritage did not become possible before the fifteenth century. Among the first to refer to the Archimedean concept of *center of gravity* was Leonardo da Vinci who used it in his work on mechanical devices. In particular, he applied the concept to the problem of the bent lever (see figure 3.3):

> S'e centri de' pesi saranno equidistanti al loro centro comune, essi pesi staranno equali in equilibra. S'e perpendiculari de' centri de' pesi saranno equidistanti al perpendicolare del lor centro commune, essi pesi staranno equali in equilibra, se essi pesi sieno equali. Per tal ragione il centro del mondo è sempre mobile per la mutazione della inondazione dell'Oceano.
>
> When the centers of the weights are equally distant from their common center, these weights will be equal in equilibrium. When the perpendiculars of the centers of the weights are equidistant from the perpendicular of their common center, these weights will be equal in equilibrium, if these weights are

[48] Moody and Clagett (1960, 94–95).

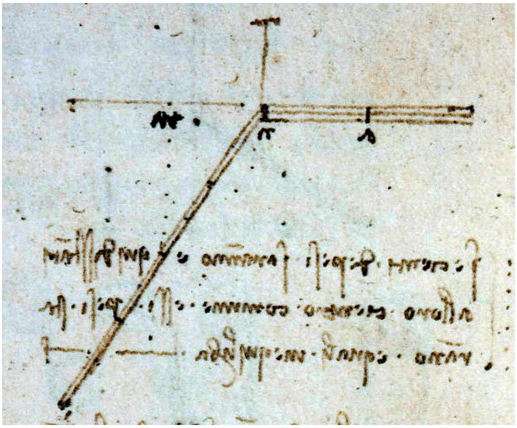


Figure 3.3: Leonardo argued correctly that a bent lever will be in equilibrium if the weights of the two parts of the beam are equal and their centers of gravity have the same distance from the vertical through their common center of gravity.

> equal. For this reason, the center of the world is always mobile because of the change of the tides of the ocean.[49]

Leonardo also treated the behavior of an equilibrated balance deflected into an oblique position (see figure 3.4). He argued that the observable fact that such a balance tends to return to the horizontal is a consequence of the imprecision of the fulcrum:

> La gravità è tutta per tutta la lunghezza del suo sostentaculo e tutta in ogni parte di quello. Per che causa accade in isperienzia che quando l'aste istando per obbliqua linia e restando colle sue parti equalmente distante a la linia centrale, essa non resta obliqa, anzi si fa equidiacente e componente colla detta linia centrale con 4 angoli retti? Risponda nascere dalla imperfezzione del polo.
>
> The heaviness is whole for the whole length of its carrier and whole in each part of it. Why does it happen in experience that, when the beam is along an oblique line and with its parts

[49]Leonardo da Vinci (1992, folio 126v).

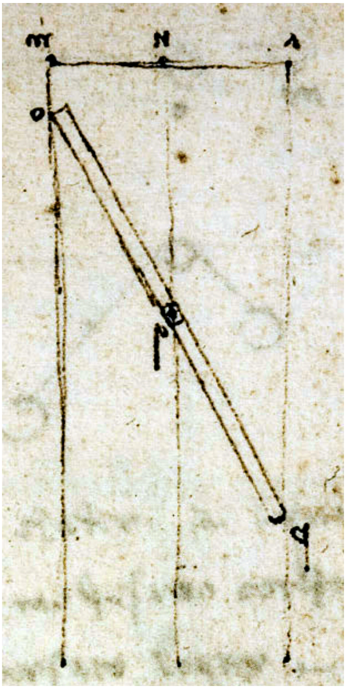


Figure 3.4: Leonardo correctly assumed that an equilibrated balance deflected into an oblique position will not return into the horizontal. He explained the common experience that the balance nevertheless seems to return into the horizontal as resulting from the difficulty in constructing a balance whose fulcrum matches precisely the center of gravity.

> equally distant from the central line, it does not remain oblique, but rather makes itself horizontal and forming with the said central line 4 right angles? Answer that this comes from the imperfection of the fulcrum.[50]

As is well known, the impact of such insights found in Leonardo's manuscripts on the subsequent scientific development is difficult to assess. From our point of view, there can be no question of singular *discoveries*

[50]Leonardo da Vinci (1992, folio 128r).

that may have been lost or found in a scientific relay race as long as such insights are not integrated into a wider network of knowledge.[51]

Another striking instance in which significant contributions to a further development of mechanics based on Archimedean principles can be attributed to an author in hindsight, while in fact the contemporary impact remained rather limited, is the extraordinary work of Francesco Maurolico.[52] By early 1548 Maurolico completed a major work composed of four books, entitled *De momentis aequalibus*.[53] In this work he systematically defined the concept of *momentum* as the positional effect of weights responsible for their equilibrium. His work was first published, however, more than a century later when knowledge of Archimedes' work had become widely available.[54]

3.4.3 The unreconciled ancient heritage

The preceding overview shows that the transmission of the ancient knowledge about mechanics was neither cumulative nor a linear process. From the Hellenistic world there were essentially two pathways of transmission to the Latin scholarly tradition. The first was the transmission to the Arabic world and from there through the boundary areas of Arabic and Latin cultures in Spain and Sicily to the rest of Europe. The second was the transmission to the Eastern Roman Empire centered around the city of Constantinople and from there first to Italy and later to other Westeuropean regions.

The result was a patchwork of partly incompatible conceptual networks of mechanical knowledge, embedded in quite distinct cultural and social settings. Consequently, the intermittent and scattered transmission of the concept of *center of gravity* led in particular to the emergence of an alternative conceptualization of the way in which equilibrium results from the functioning of weights depending on different mechanical constellations, and this focused on the concept of *positional heaviness*.

Using the concept of *center of gravity* the equilibrium can be conceptualized in terms of the relation of the center of gravity and the point of suspension. Such a conceptualization leads directly to the concept of *torque* in classical mechanics if the equilibrium is expressed as an equality of physical magnitudes. However, as shown above, the concept of *center of gravity*, introduced by Archimedes and taken up by Heron and Pappus,

[51] See Renn et al. (2001); Büttner et al. (2004).
[52] Francesco Maurolico, 1494–1575.
[53] Maurolico (1685a).
[54] See Clagett (1974); Napolitani (1998, 2001).

was known in principle in the Arabic scholarly community but evidently had only a limited and rather late impact on Arabic mechanical knowledge. In any case, it did not become part of the Arabic treatises on mechanics which were translated in the twelfth century into Latin. It became known to the Latin scholarly world only in the later thirteenth century through the translation of works of Aristotle by William of Moerbeke who had access to now lost Greek manuscripts probably transmitted from Byzantium.

Thus, the early medieval Latin scholars were familiar with Aristotelian dynamics through works such as Aristotle's *Physics* or *De caelo*, but with the tradition of ancient mechanics only through the selective translation of Arabic sources. In this situation the attempt to solve the problem of explaining the causes for the equilibrium of balances resulted in the idea that the actual weight of a body changes according to the mechanical context. This idea, however, is ambiguous in itself. If it is the *weight* that really changes, what then is the magnitude that has been determined since millennia using the balance as a weighing device? But if the real weight does remain the same, what kind of weight is it that changes according to the context? How can one explain that two bodies holding a balance in equilibrium may nevertheless have different weights?

This ambiguity is reflected in the conceptual fuzziness of the terms used to express the effect of a weight under different mechanical conditions. The most advanced attempt to eliminate this ambiguity was offered by Jordanus. In accordance with the growing role of Aristotelian methodology for structuring knowledge, his solution made use of Aristotelian logic in order to avoid the apparent fallacies related to the mechanical problems associated with weights in different positions on a balance. Aristotle had introduced the term *fallacia a dicto simpliciter ad dictum secundum quid* to denote the fallacy of ignoring a qualification such as the position of a weight, supposing that what is true under certain circumstances,[55] e.g. the equilibrium of equal weights on a balance, is true also in general, e.g. generalizing the statement of equilibrium for the positions of weights on an equal-arms balance to all positions of the counterpoise of a steelyard. Thus, Jordanus introduced the term *gravitas secundum situm* in contrast to the *pondus* of a body, translated here as *positional heaviness* and *weight* respectively. In this way a historically consequential concept of mechanics had been shaped by a reflection suggested by the intellectual context of early scholasticism (see section 2.1).

[55] For a discussion of this fallacy, see Überweg (1882, 418–422) and Schreiber (2003, 141–151).

With the revival of the work of Jordanus in the early modern period this solution was widespread, but now became confronted with the alternative conceptualization of the equilibrium of balances by means of the concept of *center of gravity* revived with the translation of works by Archimedes and Pappus and indirectly through quotations of Heron's work in Pappus' *Collectiones.*

Thus, Renaissance and early modern scholars stumbled upon an unreconciled part of the ancient conceptual heritage. This eventually led to the *equilibrium controversy* on which the present study is focused.

3.5 Jordanus' approach to positional heaviness

The treatise *Liber de ponderibus* is, as discussed in section 2.1, another representation of Jordanus' core theory with an extended set of propositions, but here the postulates are preceded by a prologue. This prologue introduces Jordanus' concept of *positional heaviness* as a new technical term together with a justification of its introduction:

> Quia si sumantur de circulo maiori et minori arcus equales, corda arcus maioris circuli longior est. Propterea possum ex hoc ostendere, quod pondus in libra tanto fit levius, quanto plus descendit in semicirculo. Incipiat igitur mobile descendere a summo semicirculi, et descendat continue. Dico tunc quod, cum maior arcus circuli plus contrariatur recte linee quam minor, casus gravis per arcum maiorem plus contrariatur casui gravis qui per rectam fieri debet, quam casus per minorem arcum. Patet ergo quod maior est violentia in motu secundum arcum maiorem, quam secundum minorem; alias enim non fieret motus magis contrarius. Cum ergo apparet plus in descensu adquirendum impedienti, patet quia minor erit gravitas secundum hoc. Et quia secundum situationem gravium sic fit, dicatur gravitas secundum situm in futuro processo.
>
> If equal arcs are taken on a greater circle, and on a smaller one, the chord of the arc of the greater circle is longer. From this I can then show that a weight on the arm of a balance becomes lighter to the extent that it descends along the semicircle. For let it descend from the upper end of the semicircle, descending continuously, I then say that since the longer arc of the circle is more contrary to a straight line, than is the shorter arc, the fall of the heavy body along the greater arc is more contrary to the

fall which the heavy body would have along the straight line, than is a fall through a shorter arc. It is therefore clear that there is more violence in the movement over the longer arc, than over the shorter one; otherwise the motion would not become more contrary (in direction). Since it is apparent that in the descent (along the arc) there is more impediment acquired, it is clear that the gravity is diminished on this account. But because this comes about by reason of position of the heavy bodies, let it be called positional heaviness in what follows.[56]

Figure 3.5: Title vignette of Apianus' edition of Jordanus' *Liber de ponderibus* displaying a scholar and a practitioner. The scholar explains the functioning of a steelyard according to Aristotelian principles.

The justification seems to echo an argument in the Aristotelian *Mechanical Problems* but could also have been inspired in a more general way by Aristotelian physics. At the very least, Aristotelian scholars must have faced a contradiction. On the one hand, according to Aristotelian dynamics, the moving force or weight of a body is proportional to the resulting

[56]Moody and Clagett (1960, 150), see also Apianus' edition of de Nemore (1533, A iii verso), page 300 in the present edition. Translation adapted from Moody and Clagett (1960, 151).

swiftness. On the other hand, according to the principle of Aristotelian mechanics, in circular motion the swiftness, as discussed in section 3.4.1, caused by equal forces does not stay the same but rather becomes proportional to the distance from the center (see figure 3.5). As shown above, the Aristotelian explanation for this principle was based on the assumption that the interference of the center forcing the motion into a circular path impedes the motion toward the center of the world in dependence on the degree to which the path is curved. This is similar to the argument used by Jordanus to justify for the seemingly changing weight the designation as *positional heaviness* (*gravitas secundum situm*), applying, as is claimed in section 3.4.3, an Aristotelian strategy for avoiding the *secundum quid fallacy* to the science of weights.

The term *positional heaviness* thus became the core concept in the postulates and the propositions of Jordanus' treatises on the science of weights. First he attempted in the postulates to provide a precise definition of the term on the basis of the Aristotelian assumptions. In Apianus' edition of the *Liber de ponderibus*, the first to the fifth postulate are formulated accordingly:

> Prima est: Omnis ponderosi motum ad medium esse.
>
> Secunda: Quanto gravius tanto velocius descendere.
>
> Tertia: Gravius esse in descendendo, quanto eiusdem motus ad medium est rectior.
>
> Quarta: Secundum situm gravius esse, quanto in eodem situ minus obliquus est descensus.
>
> Quinta: Obliquiorem autem descensum minus capere de directo, in eadem quantitate.
>
> The first is: The motion of every weight is toward the center [of the world].
>
> The second: The heavier it [the weight] is, the faster it descends.
>
> The third: It is heavier in descending, insofar as its movement toward the center [of the world] is straighter.
>
> The fourth: It is positionally heavier, insofar as its descent, in that same position, is less oblique.
>
> The fifth: But a more oblique descent partakes less of the straight [descent], for the same quantity [of the path].[57]

[57]de Nemore (1533, A iv recto), page 301 in the present edition.

The second postulate simply asserts the basic principle of Aristotle that the velocity of a moving body depends on the exerted force, in this case the heaviness of a falling body (see section 3.4.1). The third postulate also refers to the Aristotelian tradition, but now to the consequences of the relation between natural motions and what acts contrary to them. The fourth postulate introduces the term *positional heaviness* as resulting from the *obliqueness* of descent. Finally, the term *obliqueness* is explained in the fifth postulate by the amount of straight descent covered by it for equal quantities of the path. This fifth postulate had, as will be shown below, the greatest influence on all attempts to quantify the concept of *positional heaviness.*

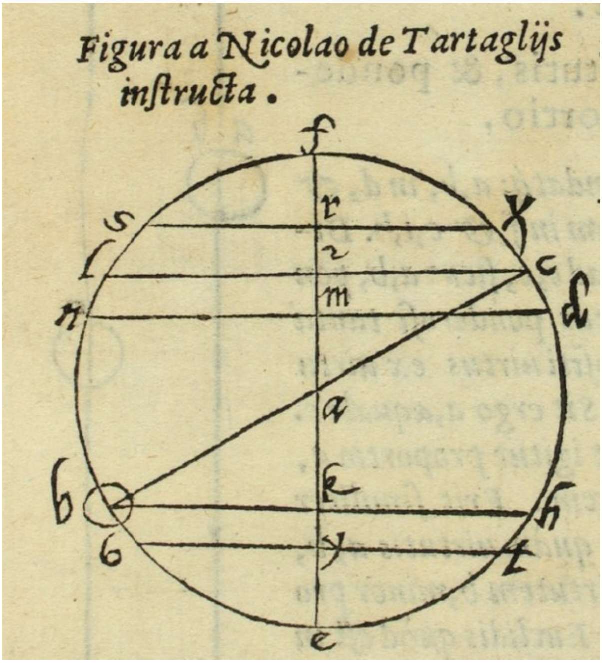


Figure 3.6: Figure added by Tartaglia to the proof of the second proposition of Jordanus' *De ratione ponderis.* Jordanus argued that the *positional weight* at C is greater than at B because the vertical descent ZM is greater than the vertical descent KY.

Having stated his postulates, Jordanus started with a proposition related to the Aristotelian doctrine that velocity is proportional to weight respectively to the exerted force:

> Propositio prima.
>
> Inter quaelibet duo gravia est velocitas descendendo proprie, et ponderum eodem ordine sumpta proportio, descensus autem, et contrarii motus, proportio eadem, sed permutata.
>
> First proposition
>
> Between any two heavy bodies, the proper velocity of descent is directly proportional to the weight; but the proportion of descent and of the contrary movement of ascent is the inverse.[58]

Then, with his second proposition, Jordanus presented the claim which later became the issue of the *equilibrium controversy* triggered by the inherent ambiguity of the concept of *positional heaviness* (see figures 3.6 and 3.7:

> Propositio secunda.
>
> Cum fuerit aequilibris positio aequalis, aequis ponderibus appensis, ab aequalitate non discedet, et si ab aequidistantia separetur, ad aequalitatis situm revertetur.
>
> Primum patet, quia sunt equae gravia. Secundum patet per suppositionem quartam, vocatur autem illud situs, quod circulus dicitur, sicut patet per praedicta.
>
> Second proposition.
>
> If an equilibrated [balance] is in horizontal position [*positio aequalis*], with equal weights suspended, it will not leave the horizontal position [*aequalitate*]; and if it is removed from the horizontal position [*aequidistantia*], it will return to the horizontal position [*aequalitatis situm*].
>
> The first [part] is evident because the weights are equally heavy. The second is clear from the fourth postulate; but it [the weight] is called positionally [heavy] because one speaks about the circle as is evident from the preceding.[59]

[58] de Nemore (1533, A iv verso), page 302 in the present edition. Translation in Moody and Clagett (1960, 155).

[59] de Nemore (1533, B ii recto), page 305 of the present edition. Translation by the authors, cf. Moody and Clagett (1960, 156–157).

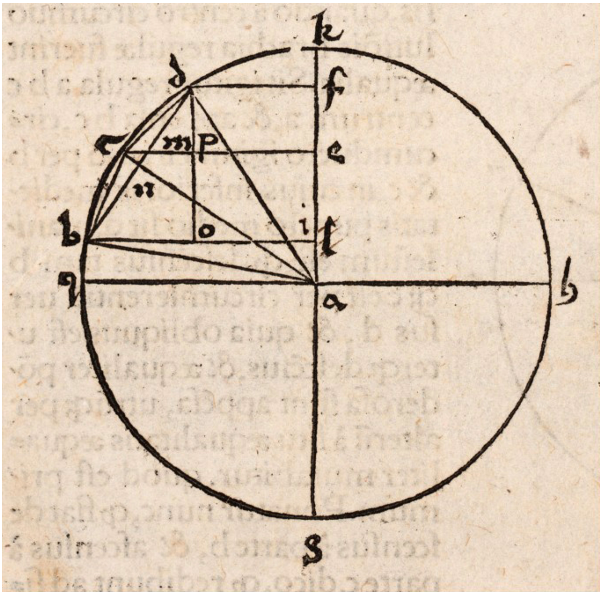


Figure 3.7: Figure added in Apianus' edition to the proof of the second proposition of Jordanus' *Liber de ponderibus.* The commentary argues that equal arcs along a circle correspond to unequal vertical descents depending on the distance from the horizontal. Thus, the upper arc *CD* corresponds to a smaller vertical descent, i.e. *FE*, than the lower but equal arc *BC* corresponding to the vertical descent *EI.*

The short argument in Apianus' edition of Jordanus' *Liber de ponderibus* as well as the detailed proof in Tartaglia's edition of Jordanus' *De ratione ponderis* make essential use of the fourth postulate. The argument specifies the *positional heaviness* of the weights at the deflected balance by identifying the vertical components corresponding to their respective descents. Jordanus' result is the erroneous claim that the deflected balance returns into the horizontal position because the upper weight acquires a greater *positional heaviness* than the lower weight.

Several of the following propositions are concerned with the influence of various circumstances on the positional heaviness of a weight. The fourth proposition of Apianus' edition of the *Liber de ponderibus*,[60] for example, concerns the dependence of the positional weight on the deflection of the beam of a balance:

> Propositio quarta.
>
> Quodlibet pondus in quamcumque partem discedat secundum situm sit levius.
>
> Fourth proposition.
>
> In whichever direction any weight departs [from the position of equality], it becomes positionally lighter.[61]

As in the case of the second proposition, the proofs of all further propositions that concern the magnitude of the positional heaviness are essentially based either on the fourth postulate, thus relating the positional heaviness to the obliqueness of the downward tendency or motion and, in consequence, to the vertical descent, or directly on the Aristotelian dynamics as it is formulated with some variations in the first theorem of all three treatises ascribed to Jordanus.

Accordingly, Jordanus presented as the eighth proposition[62] the *law of the lever* by expressing the equilibrium of two unequal weights as equality of their positional heaviness (see figure 3.8):

> Propositio octava.
>
> Si fuerint brachia librae proportionalia ponderibus appensorum, ita, ut in breviori gravius appendatur, aeque gravia erunt secundum situm.
>
> Eighth proposition.
>
> If the arms of the balance are proportional to the weights suspended in such a manner that the heavier [weight] is suspended on the shorter [arm], positionally they will be equally heavy.[63]

The proof of this proposition is based directly on Aristotelian dynamics as formulated in the first proposition. Jordanus made particular use

[60] The proposition is the same as the fourth proposition in Jordanus' *Elementa* and the third proposition in his *De ratione ponderis*.

[61] de Nemore (1533, B iii verso), page 308 in the present edition. Translation adapted from Moody and Clagett (1960, 157).

[62] The proposition is the same as the eighth proposition in Jordanus' *Elementa* and the sixth proposition in *De ratione ponderis*.

[63] de Nemore (1533, C iv recto), page 317 in the present edition.

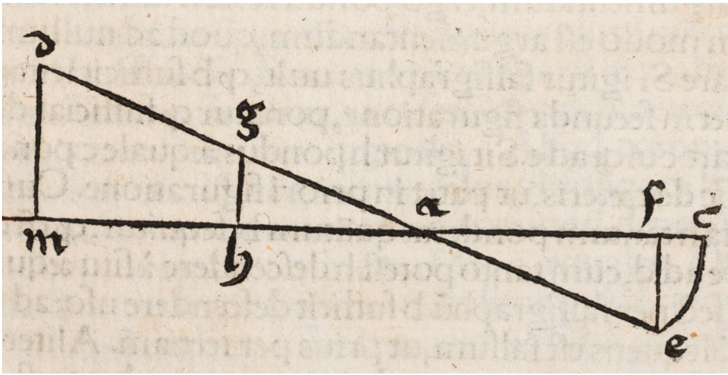


Figure 3.8: Jordanus' proof of the *law of the lever* and Apianus' commentary are based on the establishment of a relationship between the motion of unequal weights *D* and *E* on a balance and upward motions *MD* and *HG* of similar weights caused by the same force treated according to Aristotelian dynamics.

of the assumption, which is summarized succinctly in the passage from *On the Heavens* quoted above:[64] distances and weights are inversely proportional when the same force is applied to them. In order to apply this assumption to the equilibrium of a balance with unequal weights he argued that the descent of the heavier weight on one side of the balance can be considered as being equivalent to an upward motion of the same weight on the other side. He could thus compare ascents of different weights over different distances, inversely proportional to the weights, which according to Aristotelian dynamics can be achieved by the same force. This then serves to show that the balance is in equilibrium.

Apianus not only reported Jordanus' proof, but also extended it with commentaries. He made explicit use of the concept of *positional heaviness*, exploiting the vertical component of the path of the beam as a measure (see figure 3.7).

For our context it is important to note that both the *Elementa* and the *De ponderibus*, that is, the present text, are distinguished from *De ratione ponderis* by the fact that the latter omits two incorrect propositions on the bent lever, propositions 6 and 7. In Apianus' edition proposition 6 reads:

[64]See section 3.4.1.

> Propositio sexta.
>
> Cum unius ponderis sint appensa, et a centro motus inaequaliter distent, et si remotum secundum distantiam propinquius accesserit ad directionem, alio non moto secundum situm, illo levius fiet.
>
> Sixth proposition
>
> When equal weights are suspended at unequal distances from the center of movement, and if the longer arm is bent so that its end is at the same distance from the vertical as is the shorter arm, then, if the latter remains unmoved, the weight on the longer arm will become positionally lighter than the other weight.[65]

This incorrect proposition on the bent lever was replaced in Jordanus' treatise *De ratione ponderis* by a correct theorem (numbered as proposition 8) which indirectly states the measure of positional heaviness by means of vertical projections on the horizontal, and which later became central to Benedetti's work (see figure 3.15):

> Si inequalia fuerint brachia librae, et in centro motus angulum fecerint: si termini eorum ad directionem hinc inde aequaliter accesserint: aequalia appensa in hac dispositione aequaliter ponderabunt.
>
> If the arms of a balance are unequal, and form an angle at the axis of support, then, if their ends are equidistant from the vertical line passing through the axis of support, equal weights suspended from them will, as so placed, be of equal heaviness.[66]

It seems that the *De ratione ponderis* was an improved version of the *Elementa*, probably due to Jordanus himself.[67] In particular, the distance between the weight and the vertical through the point of suspension of the beam of a balance is used as a measure of its positional heaviness, also in other theorems.[68] But in spite of this improvement, other issues involving positional heaviness, such as the claim that a balance would always return to its horizontal position, still received the same problematic

[65] de Nemore (1533, C ii verso), page 314 in the present edition, cf. Moody and Clagett (1960, 158–159).
[66] de Nemore (1565, 6), cf. Moody and Clagett (1960, 184–187).
[67] Moody and Clagett (1960, 171–172).
[68] Moody and Clagett (1960, 205–206).

treatment as they had in the *Elementa*.[69] Thus, the new insight expressed in proposition 8 did not lead to a thorough conceptual revision of the theory of the balance expounded by Jordanus and in particular not to a revision of the concept of *positional heaviness*. A similar situation holds for the later treatise on weights by Blasius of Parma,[70] who also stuck to the erroneous assumption that a deflected balance returns to the horizontal. The bent lever, however, is correctly treated by taking the projections on the horizontal as a measure of positional heaviness.[71]

The erroneous claim that the deflected balance spontaneously returns to the horizontal was later criticized by Leonardo da Vinci with direct reference to the science of weights.[72] Leonardo critically discussed the explanation of the balance by "Pelacani," i.e., Blasius of Parma. According to Leonardo, Blasius had claimed that the longer arm of the balance will fall more quickly than the shorter arm because its descent traverses the quarter circle more directly than the shorter arm. Since the weights tend to fall along the perpendicular, the motion will be slower the more the circle is curved. In the proof of the seventh proposition of Part I of his treatise on weights, Blasius indeed claimed that a heavy body seeks to move along a straight line and that the slower it moves the more it deviates from its natural path.[73] Leonardo argued against this by considering a case in which the weights are attached by ropes and fall perpendicularly without being impeded by the curvature of the circle described by the balance. He concluded that what is more distant from its suspension will be carried less by it. Since it is carried less, it acquires more freedom, and since a free weight will always descend, the end of the beam which is more distant from the fulcrum will sink more quickly than any other part as it carries a weight.

It remains unclear whether Leonardo's insights into the behavior of a deflected balance had any impact on the scholarly discussion of this problem in the early modern period. With certainty we only know that the *De ratione ponderis* was published in 1565 in Venice by Curtius Trojanus at the instigation of Niccolò Tartaglia.[74] In this form it may have become one of the starting points for Benedetti's treatment of *positional heaviness*. He could have indeed taken the result that Jordanus had formulated in

[69]Moody and Clagett (1960, 176–179).
[70]Blasius of Parma, 1345–1416.
[71]Moody and Clagett (1960, 251).
[72]See Lücke (1953, 485) on Ms. 2038 Bib. Nat. 2 v. See also the discussion on pages 56–59.
[73]Moody and Clagett (1960, 243–245).
[74]de Nemore (1565).

proposition 8, transforming it into a general principle for analyzing the positional effect of weight. Remarkably, the controversy in the sixteenth century between Tartaglia, Cardano, Guidobaldo, and Benedetti on the notion of *positional heaviness* was triggered by a conundrum that had remained unsettled for centuries, as the different versions of Jordanus' work testify.

3.6 Tartaglia's approach to positional heaviness

Compared to the situation of Jordanus, the availability of sources on mechanics was significantly different in the early modern period (see sections 3.4.1 and 3.4.2). In particular, the Aristotelian *Mechanical Problems* had become widely known through the transmission and translation of the Byzantine manuscript. As mentioned above, part of Heron's work had become available through Pappus. In addition, Archimedes' work on the equilibrium of planes made available the knowledge of how to treat the problems of the balance in a deductive way on the basis of the concept of center of gravity.

This broad availability of ancient sources brought about a novel situation for discussions of the dependency of the effect of a weight on its position. In particular, it now became relevant to establish connections between the different conceptual frameworks embodied in these sources. One of the key protagonists to contribute both to the spread of ancient and medieval sources and to the creation of a new synthesis was the engineer-scientist Niccolò Tartaglia. Following Jordanus, he formulated the law of the lever in *Quesiti, et inventioni diverse*[75] in terms of positional heaviness:

> Se li brazzi della libra saranno proportionali alli pesi in quella imposti, talmente, che nel brazzo più corto sia appeso il corpo più grave, quelli tai corpi, over pesi seranno equalmente gravi, secondo tal positione, over sito.
>
> If the arms of the balance are proportional to the weights imposed on them, in such a way that the heavier weight is on the shorter arm, then those bodies or weights will be equally heavy positionally.[76]

Tartaglia's book became a point of reference – and a target of severe criticism – both for Guidobaldo and Benedetti and shall therefore be con-

[75] Tartaglia (1546).
[76] Tartaglia (1546, 92v). Translation in Drake and Drabkin (1969, 132–133).

sidered here in further detail. Tartaglia also followed Jordanus in claiming in his Third Petition that

> [...] un corpo grave esser in el discendere tanto più grave, quanto che il moto di quello è più retto al centro del mondo.
>
> [...] a heavy body in descending is so much the heavier as the motion it makes is straighter toward the center of the world.[77]

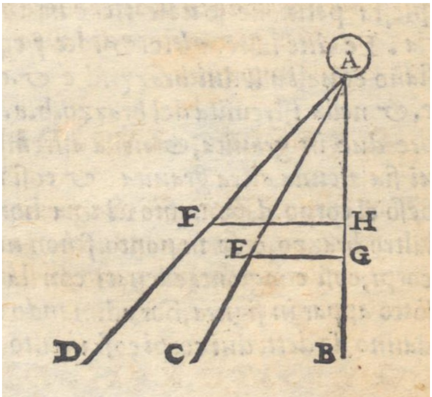


Figure 3.9: According to Tartaglia, the descent of a body from A to D is more *oblique* than the descent from A to C since the projection AH on the *line of descent to the center of the world* is shorter than the projection AG.

He substantiated the idea by the defining concepts of *line of direction to the center of the world* and *obliqueness*:

> La linea della direttione è una linea retta imaginata venire perpendicolarmente da alto al basso, e passare per il sparto, polo, over assis de ogni sorte libra, over bilancia.
>
> The *line of direction* is a straight line imagined to come perpendicularly from above to below and to pass through the support or axis of any kind of scale or balance.[78]

[77]Tartaglia (1546, 84v). Translation in Drake and Drabkin (1969, 118).
[78]Tartaglia (1546, 83r). Translation in Drake and Drabkin (1969, 115).

…

Più obliquo se dice essere quel descenso, d'un corpo grave, il quale in una medesima quantita, capisse manco della linea della direttione, overamente del descenso retto verso il centro del mondo.

The descent of a heavy body is said to be *more oblique* when for a given quantity it contains less of the line of direction, or of straight descent toward the center of the world.[79]

Thus, Tartaglia measured the straightness of the given descent by its projection on the vertical line of direction (see figure 3.9).

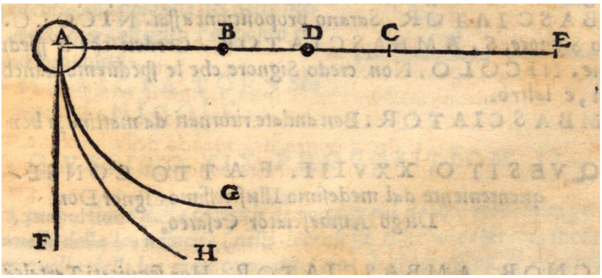


Figure 3.10: According to Tartaglia, the descent of a body from A to G is more *oblique* than the descent from A to H since its curvature is greater.

Alternatively, he measured the straightness of descent also with reference to the more or less acute angle with the path of straight and direct descent to the center of the world (see figure 3.10).[80] He thus followed a procedure introduced by Jordanus in *De ratione ponderis* which Tartaglia later edited,[81] a procedure, however, that was absent in Jordanus' other works. In the case of a curved descent in particular, for instance along a circular arc, Tartaglia determined straightness by the lesser or greater curvature of the path of descent, making use of the idea of *angles of contact* (also referred to as *curvilinear angles* or as *mixed angles* in the following),

[79] Tartaglia (1546, 83r). Translation in Drake and Drabkin (1969, 115).
[80] Tartaglia (1546, 84v–85r). Translation in Drake and Drabkin (1969, 118–119).
[81] de Nemore (1565, 3v–4r). Evidently, at the time Tartaglia wrote *Quesiti*, he must have been familiar with Jordanus' *De ratione ponderis*.

formed not by straight lines but by circles or by a straight line and a circle.[82]

Tartaglia treated the case of a scale or balance of equal arms with equal weights attached to them following Jordanus. Taking the latter's stance in the equilibrium controversy, he also concluded that, when the scale is moved from its initially horizontal equilibrium position by an external intervention so that one weight is above, the other below the horizontal, the scale will return to the horizontal position by itself because the weight that has been raised has become positionally heavier than the weight that has been lowered (see figure 3.11). On the other hand, he claimed that the greater positional heaviness cannot be compensated by adding a weight to the lower weight since even the smallest weight attached to this side would move the scale to a vertical position.

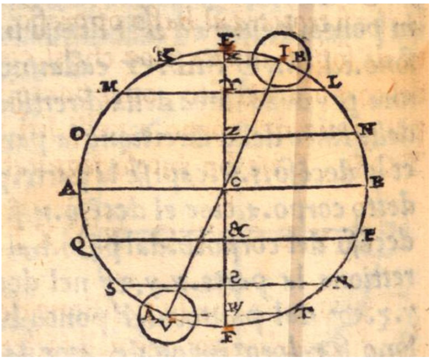


Figure 3.11: According to Tartaglia, the body at I is positionally heavier than the body at V since the projection XY is greater than WF.

To justify the first claim, Tartaglia considered the balance in any position outside the horizontal and now compared, following Jordanus, the descents of the two weights with the aim of establishing which of them is more direct. For this purpose, he compared descents through equal parts of the circle described by the arms of the balance, i.e. descents through equal angles taken downward from the given position of the beam. Due

[82] See Tartaglia (1546, 84v–85r), Drake and Drabkin (1969, 119). For a review of the contemporary controversial discussion of this mathematical concept, see Maierù (1992), see also Bordiga (1985, 627–628).

to geometrical reasons, it now turns out that the descent of the upper weight is always straighter than that of the weight that has been lowered so that the upper weight becomes, according to the definition, positionally heavier. As a consequence, the balance will return to its original horizontal position.[83]

In his discussion of this result Tartaglia actually employed two different measures of straightness, both of which were in agreement with his definition quoted above. In the proof of his proposition, he made use of the projection of a finite circular descent on the vertical line of direction, comparing those projections for descents of equal angles. Later, however, he compared instead more directly the angles between the curved path of descent and a straight perpendicular line to the center of the world.[84] For this purpose he actually compared *angles of contact*, just as Jordanus had done in *De ratione ponderis* (see figure 3.12). In this way Tartaglia concluded that the angle between the circular descent of the lower weight and the vertical line to the center of the world is larger than the angle between the circular descent of the higher weight and the said line.

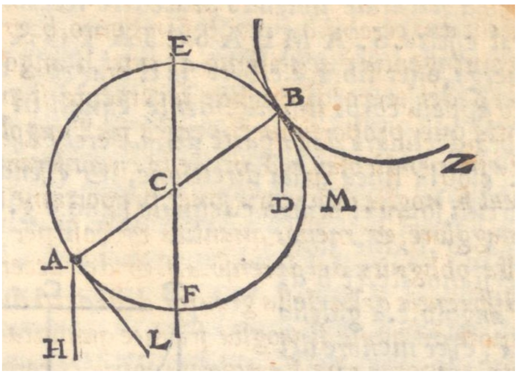


Figure 3.12: According to Tartaglia, the body at B is positionally heavier than the body at A since the *angle of contact* between BD and BF (taken along the periphery) is smaller than the angle between AH and AF (taken along the periphery).

He thus again obtained the result that the descent of this higher weight is more direct and the weight itself positionally heavier. Had he just com-

[83]Tartaglia (1546, 88v–90r), Drake and Drabkin (1969, 124–127).
[84]Tartaglia (1546, 90r–92r), Drake and Drabkin (1969, 130–131).

pared the ordinary angles between the tangents to the circular paths of descent and the vertical, the two angles would have simply been equal. It is thus the difference or ratio between the angles of contact, themselves *less than any difference or ratio you please which can occur between any large and small quantities*[85] that is responsible for the difference in positional heaviness. Therefore, this difference cannot be compensated by any finite weight placed on the side of the scale that happens to be positionally lighter.

Following and improving upon Jordanus, Tartaglia also treated the inclined plane with the help of the concept of *positional heaviness*.[86] He considered two adjacent inclined planes of different inclinations but of equal height (see figure 3.13). He then took two weights which may be imagined to be connected by a weightless rope making sure that if one weight moves up, the other moves down. He claimed that when the weights are in the same proportion as the lengths of these planes with the greater weight being placed upon the more oblique plane, equilibrium will result. This is, in fact, a correct proposition about bodies placed on inclined planes.

In his proof, Tartaglia, following Jordanus, managed to compare ascents of equal lengths along the differently inclined planes, but starting from the same height. From the larger vertical projection of the displacement along the steeper ascent he concluded that the corresponding weight must have a larger positional heaviness. By means of a geometrical argument he showed that the ratio between the positional heaviness of two weights equals the inverse relation between the lengths of the inclined planes. As a consequence, the weight on the steeper plane – due to its proportionally increased positional effectiveness – is able to equilibrate the larger weight on the more oblique plane.

Tartaglia thus employed the concept of *positional heaviness* in his proof of the law of the lever, in his problematic conclusion that a balance with equal arms always returns to the horizontal position although the infinitely small driving force cannot be compensated by any weight, and as well in demonstrating the equilibrium of an inclined plane. In each case, his analysis was based on evaluating the straightness of descent, either by determining the projection of the descent on the vertical, or by its angle with the line connecting a heavy body to the center of the world. Some of these achievements and the conceptual framework on which they depend were both further elaborated and criticized by Benedetti and Guidobaldo.

[85] Drake and Drabkin (1969, 130).
[86] Tartaglia (1546, 96v–97r), Drake and Drabkin (1969, 140–143).

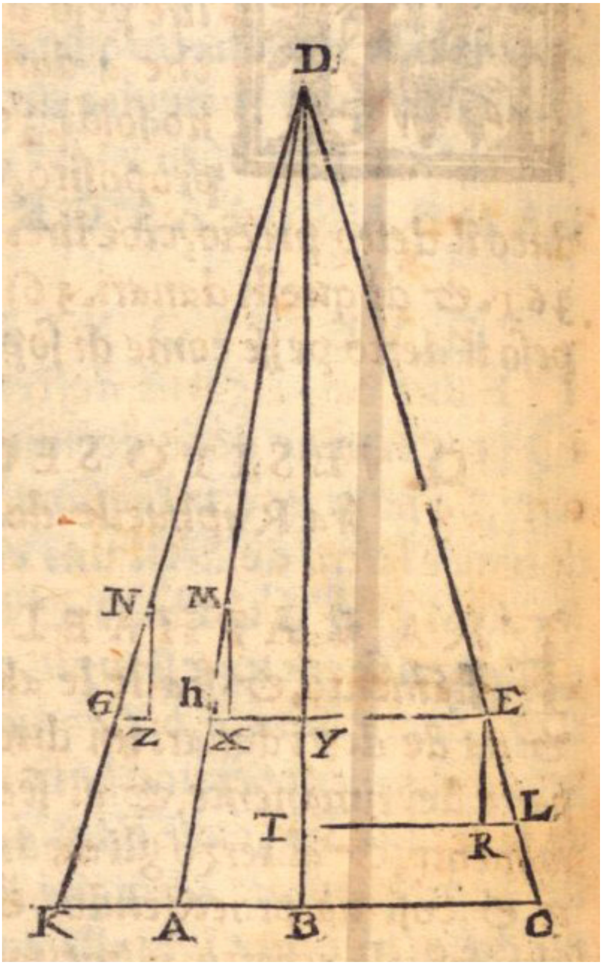


Figure 3.13: According to Tartaglia, given that *MH* equals *NG*, the lines *MX* and *NZ* represent the different positional heaviness of a body on the corresponding inclined planes. A body at *H* is thus positionally heavier than the same body at *G* in proportion to the length of the lines *MX* and *NZ* which for geometrical reasons equals the proportion between the lengths of the inclined planes *DK* and *DA*.

Tartaglia's systematic use of the concept of *positional heaviness* adopted from Jordanus became a starting point for numerous attempts to apply Archimedes' law of the lever to challenging new objects of preclassical mechanics. The way Tartaglia applied the concept already shows its inherent difficulties, which puzzled scholars in the early modern period. The concept was supposed to provide an answer to the problem that the effect of a weight depends somehow on material conditions which hindered its straight movement toward the center of the world. But precisely how this effect came about remained ultimately undetermined. In particular, Tartaglia was unable to convincingly eliminate the resulting ambiguity of the concept. As we shall see in the following, this ambiguity triggered several contradicting interpretations which became stumbling blocks of preclassical mechanics and resulted in acrimonious struggles between their adherents.

3.7 Cardano's approach to positional heaviness

Girolamo Cardano was born in Pavia in 1501. His father was a lawyer and a friend of Leonardo da Vinci. He studied and practiced medicine, a subject on which he published extensively. Later, he published also on mathematics, contributing significantly to the development of algebra. On the issue of solving third-degree equations he had an intense priority dispute with Tartaglia. He also made major contributions to mechanics. In 1570 he was imprisoned by the Inquisition for heresy, in particular for having casted the horoscope of Christ. In the same year, Cardano published his *Opus novum de proportionibus* in which he returned to a consideration of mechanical problems, in particular, of weights on a balance and their displacements along horizontal and vertical components.

Cardano first treated the balance on a few pages of the first book of *De subtilitate*, published in 1550.[87] At that time he may have been familiar with Jordanus' work through Apianus' edition of *Liber de ponderibus* printed 1533.[88] It is also possible that he knew the work of Jordanus through Tartaglia's *Quesiti*[89] published 1546, four years before his own publication. In any case, the first part of his text is substantially based on Jordanus's treatment of the medieval doctrine of the *science of weights*.

Cardano began his treatment of the balance with the figure of a balance deflected from the horizontal equilibrium into an oblique position (see

[87] Cardano (1550, 16–21).
[88] de Nemore (1533).
[89] Tartaglia (1546).

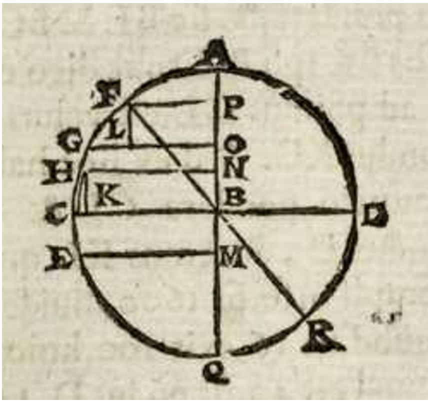


Figure 3.14: According to Cardano there are three ways to determine *positional heaviness*. The *positional heaviness* in point F, for instance, may be determined by the horizontal FP, by the vertical FL, or by the angle QBF.

figure 3.14). He claimed that a weight placed at the end of the beam of the balance will be heavier in the horizontal position than in any oblique position:

> Dico quod pondus in C constitutum erit gravius quam si lanx collocetur in quocunque alio loco, ut pote quod constitueretur lanx in F. Ut autem cognoscamus quod C sit gravius in eo situ quam in F, necessarium est ut in aequali tempore movetur per maius spacium versus centrum. Videmus enim graviora pari ratione in reliquis existente velocius ad centrum ferri.
>
> I say that the weight placed at C will be heavier than when the scale beam is placed in any other position, like when for instance the beam is located in F. But in order to recognize that C is heavier in this position than in F, it is necessary that it is moved in the same time through a greater distance toward the center. We see namely that the heavier bodies, everything else being equal, are more quickly carried toward the center.[90]

The claim that a weight will be heaviest if the beam is in horizontal position corresponds precisely to the way in which Jordanus introduced

[90] Cardano (1550, 16).

the technical term *positional heaviness* in the *Proemium* to the *Liber de ponderibus*[91] and how he formulated the claim in his fourth proposition using the term. In Tartaglia's *Quesiti*, which Cardano may have known, there is no explicit proposition with the same claim, but it is implicitly contained in the proof of his fifth proposition and explicitly formulated as the first corollary to this proposition:

> Dalle cose dette, et dimostrate di sopra, se manifesta qualmente un corpo grave in qual si voglia parte, che lui se parta, over removi dal sito della equalità lui si fa più leve, over leggiero secondo il sito, over luoco, et tanto più quanto più sara remosso da tal sito [...]
>
> From the things said and demonstrated above, it is manifest how a heavy body, whenever parted or removed from the position of equality, is made positionally lighter, and the more so, the more it is removed from that position.[92]

The first step of the justification of Cardano's claim refers to Aristotelian dynamics associating the heaviness of a body with the velocity of its descent as it was formulated in Jordanus' second comments and in a sequence of definitions and postulates of Tartaglia's *Quesiti* leading to his second postulate. Jordanus' postulate reads:

> Secunda, quanto gravius tanto velocius descendere.
>
> Second: That which is heavier descends more quickly.[93]

Tartaglia's postulate reads:

> Simelmente adimandamo, che nasia concesso quel corpo, ch'è di maggior potentia debbia anchora discendere più velocemente, et nelli moti contrarii, cioè nelli ascensi, ascendere più pigramente, dico nella libra.
>
> Likewise we request that it be conceded that that body which is of greater power should also descend more swiftly; and in the contrary motion, that is, of ascent, it should ascend more slowly – I mean in the balance.[94]

[91]de Nemore (1533, A iii verso), page 300 in the present edition.
[92]Tartaglia (1546, 90r). Translation in Drake and Drabkin (1969, 127).
[93]de Nemore (1533, A iv recto), page 301 in the present edition. Translation in Moody and Clagett (1960, 175).
[94]Tartaglia (1546, 83v). Translation in Drake and Drabkin (1969, 116).

Cardano then announced two reasons for his claim that positional heaviness at the end of a deflected beam is greater the closer the beam is to the horizontal (see figure 3.14):

> Quod autem hoc contingat magis pondere et libra in *C* collocata quam in *F*, ostendo duabus rationibus.
>
> Prima, quod si in aliquo tempore moveatur ex *C* in *E*, et sit arcus *CE* aequalis *FG*, quod tardius descendet ex *F* in *G*, quam ex *C* in *E*, et ita erit levius in *F*, quam in *C*.
>
> Secundo, quod posito quod in aequali spatio temporis moveretur ex *C* in *E*, ex et *F* in *G*, adhuc per arcum *CE* aequalem *FG*, magis appropinquaret centro quam per motum factum in arcu *FG*.
>
> Ideo ergo duplici ratione magis gravabit pondus lance posita ad perpendiculum cum trutina, quam in quoque alio loco.
>
> But that it is heavier when the balance is placed at *C* than at *F*, I demonstrate with two reasons.
>
> First, because, insofar as [the weight at the end of the beam] is moved in some time from *C* to *E*, the arc *CE* being equal to *FG*, it descends more slowly from *F* to *G* than from *C* to *E*, and so it will be lighter at *F* than at *C*.
>
> Second, because, insofar as it is assumed that [the weight at the end of the beam] is moved in the same amount of time from *C* to *E*, and from *F* to *G*, up to this point through the same arc *CE* equal to *FG*, it approaches the center more than through the motion made along the arc *FG*.
>
> Hence for this double reason the weight placed on the scale beam perpendicularly with respect to the support will be heavier than in any other position.[95]

Cardano's first reason considers the case that equal arcs are traversed in different times and infers that the speed through the arc further away from the horizontal must be smaller and thus that the weight in this position must be lighter. His second reason considers the case that equal arcs are traversed in equal times and infers that the weight closer to the horizontal approaches the center of the world more.

The following main part of the argument explains the two cases. The first explanation reads (see figure 3.14):

[95] Cardano (1550, 16–17).

> Primum igitur sic declaratur. Manifestum est in stateris, et in his qui pondera elevant, quod quanto magis pondus a trutina, eo magis grave videtur: sed pondus in C distat a trutina quantitate BC lineae, et in F quantitate FP, sed CB est maior FP, ex decimaquinta, tertii elementorum Euclidis: igitur lance posita in C, gravius pondus videbitur quam in F, quod erat primum.
>
> Ex hac etiam demonstratione manifestum est, libram quanto magis discendit versus C ex A, tanto gravius pondus reddere, et eo velocius moveri: at ex C versus Q, contraria ratione pondus reddi levius, et motum segniorem, quod et experimentum docet.
>
> The first is thus explained in the following way. It is manifest in steelyards and in those [instruments] that lift weights, that the more the weight is [removed] from the support, the heavier it appears: but the weight at C is removed from the support by the quantity BC of the line, and at F by the quantity FP, but CB is larger than FP, from the fifteenth [proposition] of the third [book] of Euclid: hence when the beam is placed at C, the weight will appear heavier than at F, which was the first.
>
> From this demonstration it is also manifest that the more the balance descends from A toward C, the heavier the weight will be rendered, and the quicker it will be moved: but from C to Q, for the contrary reason, the weight will be rendered lighter, and the motion slower, as also the experiment shows.[96]

This explanation is puzzling and not only because the last sentence refers to an experiment that relates the equilibrium controversy to the motion along a circular arc, as in the case of a pendulum. Cardano's text introduces an idea that cannot be found in Tartaglia's *Quesiti*, that is, the idea that if the beam is in a deflected position, the horizontal distance to the vertical through the center of the balance is a measure of the positional weight. It is, however, as discussed earlier, implicitly contained, among others, in the eighth proposition of Jordanus' *De ratione ponderis*, which is the proposition that has been corrected from proposition six of *Elementa* and *Liber de ponderibus* (see section 3.5). It is thus also contained in the eighth proposition of Tartaglia's later edition of Jordanus' corrected and extended treatise, which is supplemented by improved drawings (see figure 3.15).

[96] Cardano (1550, 17).

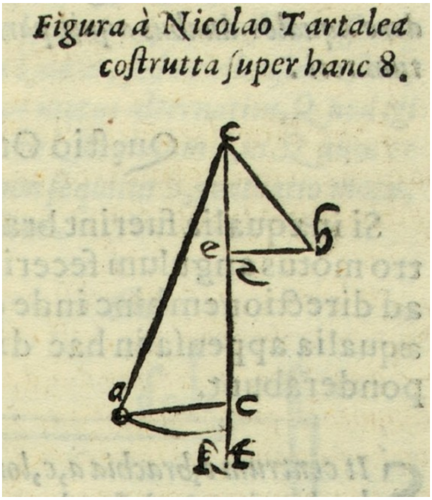


Figure 3.15: Tartaglia's figure added to the text of Jordanus in order to explain the idea that in the case of a bent lever *ACB* with equal weights attached, the equilibrium is characterized by equal horizontal distances *EB* and *AC*.

The conclusive part of Cardano's second explanation reads (see figure 3.14):

> Secundum vero sic demonstratur. [...] Dum igitur libra movetur ex *C* in *E* pondus descendit per *BM* lineam, seu propinquius centro redditur quam esset in *C*, et dum movetur per spatium arcus *FG*, descenditque per *OP*, et *BM*, maior est *OP*.
>
> Igitur suppositio etiam quod in aequale tempore transiret ex *C* in *E*, et ex *F* in *G*, adhuc velocius descendit ex *C*, quam ex *F*. Igitur gravius est in *C*, quam in *F*. Ex hoc autem demonstratur quod dicit Philosophus, quod si aequalia sint pondera in *F* et *R*, libra tamen sponte redit ad situm *CD*, ubi trutina sit *AB*. Nec hoc demonstrat Iordanus, nec intellexit.
>
> But the second will be thus demonstrated. [...] When thus the balance is moved from *C* to *E* the weight descends through the line *BM*, that is, it is rendered closer to the center than when

> it was in *C*, and when it is moved through the space of the arc *FG*, it will descend through *OP*, and *BM* [is] larger than *OP*.
>
> Hence also the supposition that when [the weight] traverses in the same time from *C* to *E*, and from *F* to *G*, it thus descends more quickly from *C* than from *F*. Hence, it is heavier at *C* than at *F*. But from this it is demonstrated what the Philosopher says, that when the weights at *F* and *R* are equal, the balance will nevertheless spontaneously return to the position *CD*, where the support is *AB*. And this Jordanus neither demonstrates nor understands.[97]

In this case Cardano again closely followed the fourth and fifth postulates of Jordanus by which the vertical descent is taken as a measure of the *positional heaviness* (see section 3.5). His argument also parallels Tartaglia's line of argument in the *Quesiti* where the twelfth in combination with the seventeenth definition completely determines his further reasoning (see section 3.6).

Tartaglia's twelfth and seventeenth definitions read:

> Diffinitione XII:
>
> Un corpo se dice essere più, over men grave d'un'altro nel descendere, quando che la rettitudine, obliquita, over dependentia del luoco, over spacio dove descende lo fa descendere più, over men grave dell'altro, et similmente più, over men veloce dell'altro, anchor che siano ambidui simplicemente eguali in gravità.
>
> Definition XII:
>
> A body is said to be more or less heavy in descent than another when the straightness, obliquity, or pendency of the place or space where it descends makes it descend more or less heav[il]y than the other, and similarly more or less rapidly than the other, though both are simply equal in heaviness.[98]

> Diffinitione XVII:
>
> Più obliquo se dice essere quel descenso, d'un corpo grave, il quale in una medesima quantità, capisse manco della linea della

[97]Cardano (1550, 17).
[98]Tartaglia (1546, 82v). Translation in Drake and Drabkin (1969, 114).

> direttione, overamente del descenso retto verso il centro del mondo.
>
> Definition XVII:
>
> The descent of a heavy body is said to be more oblique when for a given quantity it contains less of the line of direction, or of straight descent toward the center of the world.[99]

Consequently, Cardano, like Jordanus and Tartaglia, also arrived at the erroneous conclusion that an equilibrated balance, deflected into an oblique position, will spontaneously return to the horizontal. Strictly speaking, this conclusion does not follow from his first measure of positional weight, but Cardano evidently failed to notice the fact that his different measures have different implications. In particular, he failed to recognize the potential of the first measure to lead to the correct solution of later classical physics.

Remarkably, Cardano ascribed the traditional stance in the equilibrium controversy to Aristotle himself. While he disputed that the claim that a deflected balance returns to the horizontal had been properly demonstrated by Jordanus, he referred to the Aristotelian *Mechanical Problems* for further support of this claim and introduced, on this basis, a third measure of positional weight, the angle with regard to what he called the *meta*, the direction of the line of support in the sense that, when the support is from above, the *meta* is represented by the lower half line, and when the support is from below, the *meta* is represented by the upper half line (see figure 3.14):

> Aristoteles dicit hoc contingere, quum trutina est supra libram, quia angulus *QBF* metae, maior est angulo *QBR*. Et similiter quum trutina fuerit *QB*QB, erit meta *AB*, et tunc angulus *RBA*, maior erit angulo *FBA*, sed maior angulus reddit gravius pondus.
>
> [...]
>
> Generalis igitur ratio haec sit: pondera quo plus distant a meta seu linea descensus per rectam aut obliquum, id est, per angulum, eo sunt graviora.
>
> Aristotle says that this happens when the support is above the balance, because the angle *QBF* of the *meta* is larger than the angle *QBR*. And similarly when the support is *QB*, the *meta*

[99]Tartaglia (1546, 83r). Translation in Drake and Drabkin (1969, 115).

> will be AB, and thus the RBA will be larger than the angle FBA, but the larger angle will render the weight heavier.
>
> [...]
>
> The general reason is hence this: the more the weights are removed from the *meta* or from the line of descent along a straight or an oblique line, that is, [as measured] by an angle, the heavier they are.[100]

Many years later, in 1570, Cardano published his *Opus novum de proportionibus* where he once again returned to a consideration of mechanical problems, in particular also of weights on a balance and their displacements along horizontal and vertical components without, however, proposing a new approach to the question of the stability of the balance.[101]

3.8 Guidobaldo's approach to positional heaviness

In his *Mechanicorum liber* of 1577[102] Guidobaldo del Monte made considerable efforts to criticize the concept of *positional heaviness*, as introduced by Jordanus and revived by Tartaglia and Cardano, as well as some of the consequences drawn from it. As this criticism provides the background for many of his marginal notes to Benedetti's book, it shall be discussed in the following at some length.[103] Even in the preface to his book, Guidobaldo stressed the fundamental importance of his criticism of Jordanus and his early modern followers Tartaglia and Cardano:

> Verum quo facilius totius operis substructio ad fastigium suum per duceretur, nonnulla quoque de libra fuerunt pertractanda, et praesertim dum unico pondere alterum solum ipsius brachium penitus deprimitur: que in re mirum est quantas fecerint ruinas Iordanus (qui inter recentiores maximae fuit auctoritatis) et alii; qui hanc rem sibi discutiendam proposuerunt.
>
> Now, in order that my whole work might be more easily built up from its foundation to its very top, certain properties of the balance had to be treated, particularly the case when one arm

[100] Cardano (1550, 17–18).
[101] Cardano (1570, 100–102).
[102] DelMonte (1577), see Renn and Damerow (2010).
[103] See also the illuminating discussion of Guidobaldo's controversy on the same issue with the "Goto" in Gamba and Montebelli (1988). For a discussion of Guidobaldo's criticism of the concept of *positional heaviness*, see also Duhem (1991, 150–156).

> of the balance is depressed by a single weight. On this subject it is strange what disastrous errors were made by Jordanus (who had enjoyed the greatest authority among recent writers) and others who proposed this subject.[104]

Of particular importance to Guidobaldo was the claim that the balance, in any position also outside the horizontal, is in an indifferent equilibrium, in contrast to the opinion of Jordanus, Tartaglia, and Cardano, that it would return spontaneously to the horizontal position. The argument was of such importance to Guidobaldo that he spent the better part of some fifty pages of his book in dealing with it. In the Italian edition of 1581, he made the translator, Filippo Pigafetta, insert at the end of this discussion a lengthy comment,[105] actually written by himself, referring both to the theoretical novelty of Guidobaldo's treatment and to the evidence he had been able to offer for it by actually constructing balances that displayed indifferent equilibrium:

> Che questo autore è stato il primo a considerare esquisitamente la bilancia, ed intenderla dalla natura, e dal vero esser suo; però che egli il primiero di tutti ha manifestato chiaramente il modo del trattarla, e insegnarla, con proporre tre centri da essere considerati in questa speculatione; l'uno è il centro del mondo, l'altro il centro della bilancia, ed il terzo il centro della gravezza della bilancia, che in essa era un nascosto secreto di natura. Senza questi tre centri, chiara cosa è, che non si puote venire in conoscimento perfetto, ne dimostrare gli effetti varii della bilancia, i quali nascono dalla diversità del collocare il centro della bilancia in tre modi, cioè quando il centro della bilancia sta sopra il centro della gravezza di essa, overo quando è di sotto, o pure allhorche il centro della bilancia è nell'istesso centro della gravezza di lei;
>
> si come l'autore insegna nelle tre precedenti dimostrationi, cioè nella seconda, nella terza, e nella quarta propositione: perochè nella seconda mostra quando la bilancia torna sempre egualmente distante dall'orizonte; nella terza quando non solo non ritorna, ma si move al contrario; nella quarta, che essendo

[104] The preface of DelMonte (1577) is printed without page numbers, see the facsimile reproduction in Renn and Damerow (2010, 54). Translation in Drake and Drabkin (1969, 246).

[105] DelMonte (1581, 28v–29r). See the discussion in Micheli (1995), in particular, pp. 163–167.

la bilancia sostenuta nel suo centro dalla gravezza sta ferma dovunque ella si trova, il quale effetto in particolare non è più stato tocco, ne veduto, ne manco da niuno manifestato, fuor che dall'autore: anzi fin hora tenuto falso, ed impossibile da tutti gli predecessori nostri; i quali con molte ragioni si sono sforzati di provare non solamente il contrario, ma hanno etiandio affermato per certo, che la sperienza mostra la bilancia non dimorare già mai ferma se non quando ella è egualmente distante dall'orizonte.

La qual cosa in tutto è contraria alla ragione prima, per essere la dimostratione della sudetta quarta propositione tanto chiara, facile, e vera, che non so, come se le possa in modo alcuno contradire: e poi all'esperienza conciosia che l'autore habbia fatto sottilissimamente lavorare bilancie giuste a posta per chiarire questa verità, una delle quali ho io veduto in mano dell'Illustre Signor Gio. Vicenzo Pinello, mandatagli dall'istesso autore, la quale per essere sostenuta nel centro della su a gravezza, mossa dovunque si vuole, e poi lasciata, sta ferma in ogni sito dove ella vien lasciata. Ben è egli vero, che non bisogna, nel fare cotesta esperienza, correr così a furia, per essere cosa oltra modo difficile, come dice l'autore di sopra, il fare una bilancia, la quale sia nel mezo delle sue braccia sostenuta à punto, e nel centro proprio della sua gravezza.

Per la qual cosa egli è da por mente, che qual'hora alcuno si mettesse a far cotale esperienza, e non gli riuscisse, non perciò si deve sgomentare, anzi dica pur fermamente di non haver bene operato, ed un'altra volta ritorni a farne la sperienza, fin che la bilancia sia giusta, ed eguale, e venga sostenuta a punto nel centro della gravezza sua.

Et benche da altri siano state tocche le altre due predette speculationi, cioè quando la bilancia ritorna sempre egualmente distante dall'orizonte, e quando si move al contrario di questo sito, tuttavia non si è più intesa questa verità già mai apertamente, se non dall'autore nostro; peroche gli altri non hanno co'l senno penetrato in ciò tanto avanti, che habbiano saputo con distintione considerare il centro della bilancia in tre modi, come ho narrato.

Now our author is the first to have considered the balance in detail and to have understood its nature and true quality. For he is the first of all to have shown clearly the way of dealing with it and teaching about it, by propounding three centers to be considered in its theory: one is the center of the world, another the center of the balance, and finally the center of gravity of the balance: for in this was a hidden secret of nature. Without these three centers, it is clear that one could not come to a perfect knowledge or demonstrate the various effects of the balance which emerge from the diversity of placing the center of the balance in three ways, that is, when the center of the balance is above the center of gravity, or below, or when the center of the balance is in its very center of gravity.

These the author shows in the three preceding demonstrations, that is, in the second, third, and fourth propositions. In the second, he shows when the balance returns to a horizontal position, in the third when it not only does not return but moves in the contrary direction, and in the fourth that, when a balance is sustained at its center of gravity, it remains at rest wherever it is left. This last effect in particular has not been dealt with before, or seen, or even suggested by anybody besides this author. Indeed, until the present time it has been held to be false and impossible by all our predecessors, who not only have given many arguments attempting to prove the contrary, but have even assumed it to be certain that experience shows the balance never to remain fixed except when parallel to the horizon.

This is quite contrary to reason, first because the demonstration of the above fourth proposition is so clear, easy, and true that I do not know how it could be contradicted in any way; and second, their view is contrary to experience, inasmuch as our author has arranged for precise balances to be manufactured in a very sophisticated way for the purpose of showing this truth, one of which I saw in the hands of the illustrious Giovanni Vicenzo Pinelli,[106] sent to him by the author himself, and, because it was sustained at its center of gravity, it could be moved to any position and would rest at any place where it was left. True it is that in performing this experiment one might not act hastily, for it is an extremely difficult thing (as the author says above) to make a balance which is sustained

[106] Giovanni Vincenzo Pinelli, 1535–1601.

> precisely at the center of its arms and at its precise center of gravity.
>
> For this reason it is good to remember that, when anyone tries to perform such an experiment and does not succeed, he should not be discouraged, but rather should say that he had not been careful enough, and should try repeatedly until the balance is just and equal and is sustained precisely at its center of gravity.
>
> And though others have touched on the other two propositions (that is, when the balance always returns to a horizontal position, and when it moves to the contrary side), yet the truth of this has never been understood except by this author, and others have not gone far enough to have made a distinct consideration of the center of the balance in three ways, as I have explained.[107]

This inserted text higlights the centrality of the reconcialiation of Aristotelian and Archimedean approaches for Guidobaldo's work, embodied in the relation between the three centers: the center of the world, the center of the balance, and the center of gravity of the balance.[108] Inachievable as this synthesis was because of the impossibility of an indifferent equilibrium with gravitational forces acting toward a center, it did create a challenging problem driving the further development of mechanics, including experimental endeavors as described by Pigafetta.

Guidobaldo started the main part of his book with a long chapter on the balance. After some propositions about balances that are not suspended from the center of gravity, he turned with his fourth proposition to his major concern mentioned above, that is, the indifference of a balance suspended from the center of gravity against displacements. He actually gave a rather concise demonstration of the statement formulated in this proposition:

> Libra horizonti aequidistans aequalia in extremitatibus, aequaliterque a centro in ipsa libra collocato, distantia habens pondera; sive inde moveatur, sive minus; ubicunque relicta, manebit.

[107] DelMonte (1581, 28v). Translation in Drake and Drabkin (1969, 294–295), with modifications.

[108] For the role of this argument in Guidobaldo's work, see van Dyck (2006a) and van Dyck (2006b); for the general role of cosmological considerations in preclassical mechanics, see Damerow and Renn (2010); Büttner and Renn (2007).

> A balance parallel to the horizon, having its center within the balance and with equal weights at its extremities, equally distant from the center of the balance, will remain stable in any position to which it is moved.[109]

The gist of his proof consists of a simple idea. The proof directly follows from his definition of the concept of center of gravity of a body, adopted from Commandino's Latin translation of Pappus' *Collectiones*:[110]

> Centrum gravitatis uniuscuiusque corporis est punctum quoddam intra positum, a quo si grave appensum mente concipiatur, dum fertur, quiescit; et servat eam, quam in principio habebat positionem: neque in ipsa latione circumvertitur.
>
> The *center of gravity* is a certain point within it, from which, if it is imagined to be suspended and carried, it remains stable and maintains the position which it had at the beginning, and is not set to rotating by that motion.[111]

If the balance is suspended from its center of gravity it must – according to this definition – remain stable in any position. From a modern point of view of the necessary rigor of demonstrations, this of course is a tautology rather than a proof. What is missing is a justification of the implicit assumption that a center of gravity meeting the requirements of Pappus' definition always exists.

Guidobaldo also gave another definition of the center of gravity which he adopted from Federico Commandino:

> Centrum gravitatis uniuscuiusque solidae figurae est punctum illud intra positum, circa quod undique partes aequalium momentorum consistunt. si enim per tale centrum ducatur planum figuram quomodocunque secans semper in partes aequeponderantes ipsam dividet.
>
> The center of gravity of any solid shape is that point within it around which are disposed on all sides parts of equal moments [*partes aequalium momentorum*], so that if a plane is passed

[109] DelMonte (1577, 5r), Renn and Damerow (2010, 65). Translation in Drake and Drabkin (1969, 261).

[110] See Commandino's edition (Pappus of Alexandria, 1660, 450), first published in 1588 (Pappus of Alexandria, 1588).

[111] DelMonte (1577, 1r), Renn and Damerow (2010, 57). Translation in Drake and Drabkin (1969, 259).

> through this point cutting the said shape, it will always be divided into parts that are in equilibrium [*partes aequeponderantes*].[112]

If the concept of *moment* is understood in a modern sense as describing the effect of a weight depending on its position, that is, the vector product of the weight and the lever arm, the definition is essentially correct (see section 1.4). It should be noted, however, that the concept of *momento* or *momentum*, later revived by Galileo,[113] was employed by Guidobaldo neither here nor in any of his other demonstrations.

Guidobaldo must have felt that his proof of the indifference of a balance suspended from the center of gravity could not easily convince his contemporaries. Thus he noted:

> Quoniam autem huic determinationi ultimae multa a nonnullis aliter sentientibus dicta officere videntur; idcirco in hac parte aliquantulum immorari oportebit; et pro viribus, non solum propriam sententiam, sed Archimedem ipsum, qui in hac eadem esse sententia videtur, defendere conabor.
>
> But with regard to this last conclusion, many things are said by men who believe otherwise. Hence it will be well to dwell further on this; and according to my ability I shall endeavor to defend not only my own opinion but Archimedes too, who seems to have been of the same opinion.[114]

Continuing the proof of his fourth proposition he therefore started to address extensively the erroneous claim of his adversaries and their alleged proofs that a balance removed from the horizontal position will actually not remain indifferent to this displacement, but rather return to its original position.

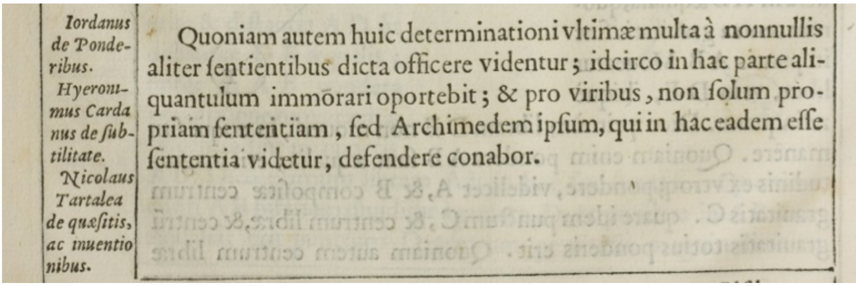
[112] DelMonte (1577, 1r), Renn and Damerow (2010, 57). Translation in Drake and Drabkin (1969, 259). In the second part Guidobaldo clearly referred to the Latin translation *aequeponderare* of Archimedes' expression for being in equilibrium. The translation of Drake and Drabkin has been corrected here accordingly.

[113] For a comprehensive study of Galileo's use of the term and its historical context, see Galluzzi (1979).

[114] DelMonte (1577, 5v), Renn and Damerow (2010, 68). Translation in Drake and Drabkin (1969, 262).

3.8.1 Overview of Guidobaldo's criticism of the claims of his adversaries

The long, somewhat chaotic supplement of some fifty pages[115] to Guidobaldo's fourth proposition of the chapter on the balance starts with a marginal note. This explicitly lists the treatises that were the subject of Guidobaldo's critique (see figure 3.16). In the first place he referred to Jordanus' *De ponderibus*, known to him from the commented edition of Apianus which he owned.[116] In the second place he referred to Cardano's *De subtilitate*,[117] and finally to Tartaglia's *Quesiti*.[118] As we have discussed in the preceding sections, these treatises in fact all contain the claim that a balance in equilibrium, if it is moved out of the horizontal position, will spontaneously move back into the horizontal.[119]



Iordanus de Ponderibus. Hyeronimus Cardanus de ſubtilitate. Nicolaus Tartalea de quæſitis, ac inuentionibus.

Quoniam autem huic determinationi vltimæ multa à nonnullis aliter ſentientibus dicta officere videntur; idcirco in hac parte aliquantulum immorari oportebit; & pro viribus, non ſolum propriam ſententiam, ſed Archimedem ipſum, qui in hac eadem eſſe ſententia videtur, defendere conabor.

Figure 3.16: Marginal note listing the treatises Guidobaldo is criticizing.

Guidobaldo proceeded, over many pages, to derive assertions within the framework of Jordanus, Tartaglia, and Cardano, including his own opinion about the indifferent equilibrium of a balance, initially derived

[115] DelMonte (1577, 5v–30r), see Renn and Damerow (2010, 68–117).

[116] de Nemore (1533), see chapter 5 for the discussion of Guidobaldo's marginal notes in his copy of the treatise. It seems that this version of Jordanus' doctrine was the only one to which he explicitly referred, although he could have also seen the edition of *De ratione ponderis* by Tartaglia (de Nemore, 1565). On the manuscript tradition and an English translation, see Moody and Clagett (1960, 169–227).

[117] Cardano (1550, 16–20) and the corresponding part contained in *Opera omnia* (Cardano, 1966, vol. 3, 369–371).

[118] Tartaglia (1546).

[119] de Nemore (1533, *propositio secunda*), page 305 in the present edition; Cardano (1550, 17, emphasized by a marginal note); Tartaglia (1546, 88v, *libro ottavo, quesito XXXII, propositione V*); see also the corresponding proposition in Tartaglia's edition of Jordanus' *De ratione ponderis* (de Nemore, 1565, 3v, *quaestio secunda*).

with the help of the concept of center of gravity. His motives are explained in the already mentioned comment inserted by Pigafetta (see page 87ff.):

> ed affine che questa nova opinion sua, dimostrata a pieno nella predetta quarta propositione, resti totalmente chiara, non si è già contentato egli d'haverla dimostrata con vive ragioni, e certe solamente, ma come buon filosofo, procedente con via di reale dottrina, e di fondata scienza, (imitando Aristotele, il qual ne' principii de suoi libri, investigando dottrina migliore, ha datto contra la opinione de gli antichi, solvendo le ragioni addotte da loro:) ha ben voluto, essendo la verità una sola, proporre le opinioni de' suoi predecessori, ed esaminare le loro ragioni, le quali sembrano provar il contrario, e solverle, la loro fallenza dimostrando co'l presente discorso [...]
>
> Anzi di più per confermatione della verità soggiunge, che questi tali non hanno saputo fare le loro demostrationi; poi che co'l proprio modo di speculare usato da loro, e con le loro medesime ragioni prova la sua intentione, e sentenza essere verissima, appoggiando si alla dottrina di Aristotele sempre, e facendo toccar con mano, che egli con esso lui è d'accordo nelle questioni mechaniche.
>
> And to the end that this new opinion of his, fully demonstrated in the aforesaid fourth proposition [about the indifferent equilibrium], should be completely clear, he has not been content to demonstrate it with vivid and certain reasoning alone, but, like a good philosopher, proceeding by the path of true doctrine and well-founded science (imitating Aristotle, who at the beginning of his books, in quest of the best doctrine, has given the contrary opinions of the ancients, analyzing the reasons which they accepted), he has wished, because there is but one truth, to propound the opinions of his predecessors and examine the reasons by which they prove the contrary, and to resolve these, showing their fallacy in the present argument [...]
>
> Moreover, as a confirmation of the truth, he adds that they did not know how to construct their proofs; for by their own mode of theorizing and their very own reasons, he proves his opinions to be most true, supporting them always on the doctrine of

> Aristotle and making it clear that he is in accord with him in the mechanical questions.[120]

Mentioning Aristotle here is significant. It shows that Guidobaldo could not easily dismiss a concept as closely related to Aristotelian dynamics as is *positional heaviness*. His faithfulness to Aristotle was evidently a major motive for his engagement in the equilibrium controversy, as also his marginal comments to the works of Jordanus and Benedetti show.

Guidobaldo's discussion of arguments in the treatises of his adversaries, which are related to his allegedly fundamental "discovery" of the indifferent equilibrium of a balance suspended from its center of gravity, is composed of several sections. Typically these sections begin with a paraphrase or reference to an argument of one of these treatises which Guidobaldo considered worthy of substantial rejection. He then presented opposing arguments or derived implicit consequences that justified his criticism. The result is a flow of meandering arguments and counterarguments. They extend the proof of his fourth proposition to an irritating web of inferences that cannot easily be disentangled. Given that the arguments of Guidobaldo's adversaries are themselves partly hybrids of various threads of implications, an adequate understanding of the disproportionate "proof" of his fourth proposition requires taking into account the context of the arguments in their treatises. A brief overview of the different sections of Guidobaldo's continuation of his proof may help to follow his arguments.

1. Guidobaldo first developed a general argument against the proposition that a balance in equilibrium if moved into an oblique position will return to the horizontal. He showed that the implications of this proposition are incompatible with the concept of the *center of gravity* upon which Archimedes' theory of equilibrium is based.[121]

2. Guidobaldo then paraphrased and extended a counterargument of Tartaglia's fictitious interlocutor Mendoza against his claim that the balance will return to the horizontal.[122] This counterargument is based on the idea that it must be possible to compensate the alleged

[120] DelMonte (1581, 29r). Translation in Drake and Drabkin (1969, 295–296), with slight modifications.

[121] DelMonte (1577, 6r–6v), Renn and Damerow (2010, 69–70), for an English translation see Drake and Drabkin (1969, 262–263).

[122] Tartaglia (1546, 90v–91r), for an English translation see Drake and Drabkin (1969, 128–129).

greater positional heaviness of the upper end of the balance in an oblique position by adding a weight to the lower end.[123]

3. Tartaglia rejected this counterargument by referring to an idea of Jordanus concerning the possibility of infinitely small differences between "mixed" angles which are composed of straight and curved legs.[124] Guidobaldo paraphrased Tartaglia's rejection of Mendoza's counterargument, followed by an extensive argument against this rejection.[125] At the end of this argument Guidobaldo introduced the condition that the directions to the center of the world for both ends of the beam of the balance are not parallel, attributing this assumption also to his adversaries although they did not make explicit use of it in their arguments. This condition later became a major concern in his reception of Benedetti's work. Contrary to the claim of Guidobaldo's adversaries, it follows that the lower end becomes positionally heavier than the upper end, a conclusion to which he returned in the sequel.

4. Next, Guidobaldo paraphrased three different arguments for dealing with the changing positional heaviness in dependence of the position of the beam of the balance. They differ in the measure of the positional heaviness, the first determining the positional heaviness by the horizontal distance of the weight from the vertical axis of support, the second by the vertical component of the actual trajectory, the third by the angle between the beam of the balance and the direction toward the center of the world. They have in common that they seem to determine somehow the effect of the obliquity by which the weights are hindered in descending directly to the center of the world.

   Guidobaldo first reported Cardano's argument that the weight of a body attached to the beam of the balance is heavier the more distant it is from the support of the balance (see section 3.7).[126] This

[123]DelMonte (1577, 6v), Renn and Damerow (2010, 70), for an English translation see Drake and Drabkin (1969, 263).

[124]Tartaglia (1546, Book 8, sixth proposition, 91r–92r), for an English translation see Drake and Drabkin (1969, 129–131). Tartaglia had evidently taken this argument from the manuscript he used in his later edition of Jordanus' *De ratione ponderis*, de Nemore (1565, 4r), for an English translation see Moody and Clagett (1960, 179).

[125]DelMonte (1577, 6v–8r), Renn and Damerow (2010, 71–73), for an English translation see Drake and Drabkin (1969, 263–265).

[126]DelMonte (1577, 8v), Renn and Damerow (2010, 74), for an English translation see Drake and Drabkin (1969, 265–266). See also Cardano (1550, 16–20) and (Cardano, 1966, vol. 3, 369–371) which is discussed above.

distance is measured, as we have seen, by the horizontal distance of the suspended weight from the vertical through the center of the balance.[127] Accordingly, a body suspended from a balance arm would be heaviest in the horizontal position of that arm, the position in which its motion would also be swifter than in any other position. This conceptualization of *positional heaviness* is close to the modern concept of *torque*, suggesting that the positional heaviness at the upper and the lower position must be the same (see section 1.4).

The second argument is based on taking the vertical distances of descent as a measure of the positional heaviness.[128] Guidobaldo knew this argument from his copy of Apianus' edition of Jordanus' treatise *Liber de ponderibus*, where the argument forms the basis of the fourth and fifth postulates and is further elaborated in Apianus' commentary to Jordanus' second proposition.[129] The argument also appears in Jordanus' treatise *De ratione ponderis* and is thus also contained in Tartaglia's edition (see section 3.5).[130] As discussed above, an extensive version of the argument is furthermore contained in Tartaglia's *Quesiti* (see section 3.6).[131] The argument is also appended by Cardano to his virtually correct conceptualization of the positional heaviness as representing – in modern terms – the torque. It seemed to imply the same conclusion as his first argument about the changing positional heaviness in dependence of the obliquity of the beam of the balance. This argument, however, provided a strong reason in favor of the erroneous conclusion that the balance must return to the horizontal position.

The third argument, which Guidobaldo again explicitly attributed to Cardano,[132] involves the concept of *meta* and is based on taking as a measure the angle between the beam of the balance and the direction

[127] DelMonte (1577, 8v) added the marginal note: *Cardanus primo de subtilitate*. The note concerns Cardano (1550, 16–17). Drake and Drabkin (1969, 266, footnote 20) attribute misleadingly also Guidobaldo's following references to Jordanus and Tartaglia to this proof.

[128] DelMonte (1577, 8v–9r), Renn and Damerow (2010, 74–75), for an English translation see Drake and Drabkin (1969, 266–267).

[129] de Nemore (1533, B ii recto–B ii verso), pages 305–306 in the present edition.

[130] de Nemore (1565, 3v), for an English translation see Moody and Clagett (1960, 176–179).

[131] Tartaglia (1546, 89r–89v), for an English version see Drake and Drabkin (1969, 125–127).

[132] Guidobaldo referred to Cardano (1550, 17–18).

toward the center of the world.[133] From the fact that the angle, thus defined, is greater in the upper than in the lower position, Cardano concluded that also the positional heaviness must be greater in the upper position.

5. Before detailing a refutation of his adversaries, Guidobaldo countered both Cardano's and Tartaglia's arguments in a general way by stressing that from the observation that a weight is swifter or its motion straighter, it does not follow that it will therefore also be heavier:

    > Neque enim intellectus quiescit, nisi alia huius ostendatur causa; cum potius signum, quam vera causa esse videatur.
    >
    > Now the intellect is not satisfied unless this can be demonstrated from some other cause, for this appears to be merely a sign rather than a cause.[134]

    He also noticed that all arguments referring to the swiftness of motion do not actually infer the positional heaviness of a body from its position, but only from its departure from that position.

    Guidobaldo started his detailed account of the arguments of his adversaries with a discussion of Cardano's claim that the positional heaviness is determined by the distance from the vertical through the center of the lever, that is, the first argument he had introduced earlier.[135] In the sequel he gave an account of the changing effect of the obliquity of the beam of a balance from his own perspective.[136] His account included the cosmological context, in contrast to the tacit assumption of Cardano that the lines drawn from different points of the balance to the center of the world are parallel and thus cannot meet at this point. Guidobaldo's arguments are mainly based on physical reasons for his own claim that a weight (*pondus*) attached to the end of the beam of a balance is more or less heavy (*gravius*) according to the amount of support the beam gets from the center of the balance in dependence on the obliqueness of its position.

[133] DelMonte (1577, 9r), Renn and Damerow (2010, 75), for an English translation see Drake and Drabkin (1969, 267).
[134] DelMonte (1577, 9v), Renn and Damerow (2010, 76). Translation in Drake and Drabkin (1969, 267–268).
[135] DelMonte (1577, 9r–9v), Renn and Damerow (2010, 75–76), for an English translation see Drake and Drabkin (1969, 267–268).
[136] DelMonte (1577, 9v–15r), Renn and Damerow (2010, 76–87), for an English translation see Drake and Drabkin (1969, 268–275).

6. Guidobaldo continued with a critique of the second claim of his adversaries, in this case, as mentioned above, maintained by Jordanus, Tartaglia, and Cardano as well.[137] All of them considered the vertical descent of a weight attached to the end of the beam of a balance, that is, the projection of the circular path of the end of the beam on the vertical direction to the center of the world, to be a measure of the obliquity of its path and thus of the weight's changing positional heaviness. Guidobaldo's refutation made use of two arguments. He first referred to the cosmological fact that the directions from different points of the circular path of the end of the beam toward the center of the world cannot be parallel and thus only approximately represent the positional heaviness. From the failure of his adversaries to take this fact into account, he concluded that all their demonstrations are false. His second argument conceded that the difference of the directions toward the center of the world is so small as to be imperceptible, and that their assumption that the straight descents of the weights are parallel was feasible. On this basis he then showed that their definition of positional heaviness was ambiguous and leads to contradictory results.

7. After demonstrating that the arguments of his three adversaries may lead to untenable conclusions, Guidobaldo added a sophisticated geometrical proof that on the basis of their assumptions about the relation between the vertical descent, the obliqueness of descent, and the positional heaviness, he could actually infer the opposite of their claim, namely that the positional heaviness at a position closer to the vertical is greater than the positional heaviness at a position more distant to the vertical, which is counter-intuitive.[138]

8. What follows is a short commentary on the origin of the errors of his adversaries. Guidobaldo argued that in general, inferences from false assumptions are false.[139]

9. After this general commentary, Guidobaldo once again returned to the dependence of the positional heaviness on the geometrical constellation of the inclined balance, now paying attention to the fact

[137] DelMonte (1577, 15r–17r), Renn and Damerow (2010, 87–91), for an English translation see Drake and Drabkin (1969, 275–277).

[138] DelMonte (1577, 17r–17v), Renn and Damerow (2010, 91–92), for an English translation see Drake and Drabkin (1969, 277–278).

[139] DelMonte (1577, 17v–18r), Renn and Damerow (2010, 92–93), for an English translation see Drake and Drabkin (1969, 278–279).

that both weights should be considered as being connected by the beam so that their motions cannot be considered independently.[140] He tried to show that, even if the assumptions of his adversaries were accepted, that is, if the directions from the weights attached to the ends of the beam toward the center of the world were assumed to be parallel, and that the positional heaviness depended on how straight (*rectus*) their descent is, it does not necessarily follow from their arguments that the beam returns to the horizontal.
At this point Guidobaldo introduced the idea that one has to consider both weights not separately, but connected by the beam of the balance. From this perspective he reconsidered the arguments of his adversaries. He first discussed the dependence of the positional heaviness on the vertical descent of weights attached to the beam, as claimed by Jordanus, Tartaglia, and Cardano. He then discussed the dependence on the horizontal distance to the vertical through the point of suspension of the balance, as claimed by Cardano. In both cases Guidobaldo argued that the positional heaviness must be equal. In the first case he drew attention to the fact that one must not compare two descents, but rather a descent on one side with a rise on the other. In the second case the positional heaviness is equal by definition so that both measures lead to the same conclusion, in agreement with Guidobaldo's claim that the balance is in indifferent equilibrium. As an act of virtuosity Guidobaldo added the argument that if one compares ascents rather than descents, the balance will move, according to the logic of his adversaries, into a vertical and not a horizontal position.

10. Guidobaldo continued with an investigation of the situation in which the directions meet at the center of the world, instead of being parallel, and with a discussion of the meaning of obliqueness and straightness as criteria for the positional heaviness, offering his own reinterpretation of this concept.[141] He concluded that, contrary to the opinion of his adversaries, the positional heaviness in the lower position must be greater and not smaller than the weight in the upper position.

[140] DelMonte (1577, 18r–19r), Renn and Damerow (2010, 93–95), for an English translation see Drake and Drabkin (1969, 279–281).
[141] DelMonte (1577, 19r–19v), Renn and Damerow (2010, 95–96), for an English translation see Drake and Drabkin (1969, 281).

11. This conclusion contradicts, of course, not only the claims of Guidobaldo's adversaries, but also his own claim that an equilibrated balance suspended at its center of gravity, if moved to an oblique position, will remain there. Therefore he now made use of his main argument, which he had introduced earlier, namely that the two weights on the balance have to be considered in conjunction.[142] He criticized his adversaries for not taking this circumstance into account. His argument is implicitly based on the modern idea that the horizontal components of the directions toward the center of the world cancel each other out so that the remaining directions of gravity are parallel and the straightness of descent is the same for both weights.

12. So far Guidobaldo had extensively discussed and refuted all the inferences his adversaries had drawn from taking horizontal and vertical measures as defining the magnitude of the positional heaviness. At this point he moved on to the last claim of his initial overview.[143] Cardano had argued that the angle between a beam supported from above and the *meta*, the direction toward the center of the world, determines the positional heaviness of a body attached to the end of the beam. He concluded that in this case the positional heaviness of the upper body exceeds that of the lower body. He further claimed that if the balance is supported from below, the positional heaviness of the lower body will exceed that of the upper body. Guidobaldo replied that there was no reason whatsoever for this assertion. Moreover, he argued that this assertion would lead to a contradiction if it were taken into account that a balance can be supported, at the same time, from above and below.

13. Guidobaldo finally discussed extensively the issue of a balance supported from above or from below[144] as it had been treated in the Aristotelian *Mechanical Problems*, that is, with regard to a material beam in which the point of suspension of the balance does not necessarily lie on the line connecting the centers of gravity of the two weights. Cardano had indeed ascribed to Aristotle his assertion that the angle between the beam and the *meta* determines the heaviness of

[142] DelMonte (1577, 19v–20v), Renn and Damerow (2010, 96–98), for an English translation see Drake and Drabkin (1969, 281–283).
[143] DelMonte (1577, 20v–21v), Renn and Damerow (2010, 98–100), for an English translation see Drake and Drabkin (1969, 283–284).
[144] DelMonte (1577, 21v–30r), Renn and Damerow (2010, 100–117), for an English translation see Drake and Drabkin (1969, 284–294).

its weight (*gravius pondus reddere*).[145] Cardano had further maintained, as discussed above, that Jordanus neither proved nor even understood this relation. In addition he had claimed that experience (*experimentum*) supports his assertion.[146] Guidobaldo first discussed Cardano's misunderstanding of Aristotle's proposition. He conceded the difficulty in constructing a balance that is supported exactly at its center of gravity, but maintained that this is not a principal difficulty but only a question of practical precision. He then continued with his own interpretation of Aristotle's arguments, adding detailed proofs of his own of all possible constellations including equilibrated balances with unequal weights attached to them, compensated by corresponding unequal arms, and finally balances with a bent beam. The gist of this discussion is the support of his own claim from the Aristotelian treatment of the balance.

In summary, this long and somewhat chaotic supplement to Guidobaldo's fourth proposition of the chapter on the balance is concerned with a number of basic ideas intimately related to the concept of *positional heaviness*. In the following, some of these ideas will be addressed in more detail, also in order to demonstrate how carefully Guidobaldo studied the contemporary literature and how he worked the fruits of these readings into his own line of reasoning. Our later discussion of the marginalia to Benedetti's work vividly illustrates how this process of reception actually worked.

3.8.2 Exploiting the concept of *center of gravity*

Guidobaldo began his discussion[147] by generally refuting his adversaries' claim that a balance with equal weights attached at equal distances from its beam will, if the beam is moved from the horizontal position, not be indifferent to this displacement, but rather return to its original horizontal position. He pointed out a specific consequence of this claim:

> Hanc eorum sententiam nullo modo consistere posse ostendam. Non enim, sed si quod aiunt, evenerit, vel ideo erit, quia pondus D pondere E gravius fuerit, vel si pondera sunt aequalia, distantiae, quibus sunt posita, non erunt aequales.

[145] The discussion concerns the second problem of the Aristotelian *Mechanical Problems*, see Aristotle (1980, 346–351).
[146] See our discussion in section 3.7.
[147] See point 1 of the preceding overview, page 95.

> For if what they say is true, this result will occur because either the [upper] weight D is heavier than the [lower] weight E or the weights are equal but the distances at which they are placed are not equal.[148]

Guidobaldo argued that, since the weights are equal, the return to the horizontal position would involve a shift of the common center of gravity of the two weights at the arms of the balance. This, however, would be in conflict with the third proposition of Archimedes' *On the Equilibrium of Planes*[149] and with Pappus' definition of the center of gravity he used in his own proof of the contrary statement:

> Cum pondera eandem inter se se servent distantiam. Unius cuiusque enim corporis centrum gravitatis in eodem semper est situ respectu sui corporis.
>
> For the weights remain the same distance apart, and the center of gravity of any body stays always in the same place with respect to that body.[150]

In the following part[151] Guidobaldo turned to the other possibility, that is, to the claim that the balance might return to the horizontal position due to an increase of the weight[152] of the ascending side of the balance. He used an indirect proof working with the counterargument of Tartaglia's interlocutor Mendoza that, if the two bodies on the arms of the balance should attain different weights, that difference could be compensated by placing an additional weight on the side that has become lighter. The latter conclusion then would lead to a contradiction.[153] While on the one hand the center of gravity of the balance now in equilibrium must still be at the center of the balance, according to the law of the lever the addition of a weight would move the center of gravity out from the center of the balance. Guidobaldo argued that the existence of two centers of

---

[148] DelMonte (1577, 6r), Renn and Damerow (2010, 69). Translation in Drake and Drabkin (1969, 262).
[149] Archimedes (1953, 190), see also the first proposition in Tartaglia's edition (Archimedes, 1543b, 5).
[150] DelMonte (1577, 6r), Renn and Damerow (2010, 69). Translation in Drake and Drabkin (1969, 263).
[151] DelMonte (1577, 6v), Renn and Damerow (2010, 70), see Drake and Drabkin (1969, 263).
[152] Guidobaldo avoided here as in other places the term *positional heaviness* used by his adversaries.
[153] Point 2 of the overview, page 95.

gravity contradicts Archimedes' theory, indirectly demonstrating that in the oblique position the balance would remain stationary (*manebunt*).

3.8.3 The intricacies of the concept of *curvilinear angles*

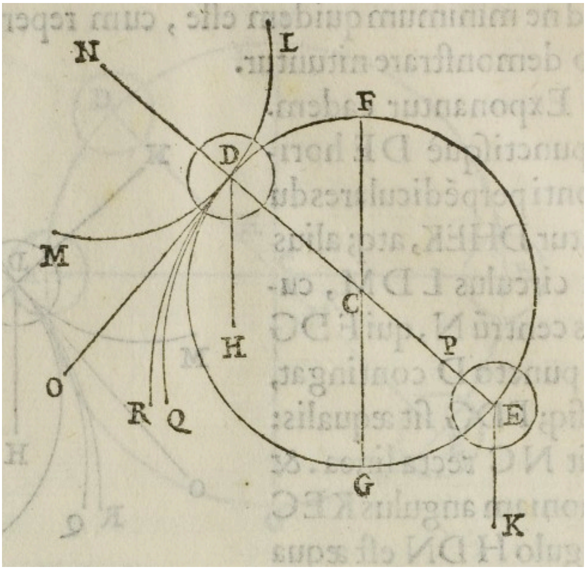


Figure 3.17: Guidobaldo refuted the argument that the *curvilinear angle MDG*, which from a modern point of view is zero, is the smallest possible angle by inserting the curves *QD* and *RD*. According to the line of reasoning of his adversaries the angles *MDQ* and *MDR* must be smaller than *MDG*.

The objection put forward by Tartaglia's interlocutor Mendoza that a greater positional heaviness of the upper part of an inclined balance might be compensated by an additional weight on the lower part, had been refuted by Tartaglia using an argument going back to Jordanus.[154] Tartaglia had argued that the difference in positional heaviness may just be infinitesimally small (*not just minimal, but still less*) so that it cannot be compensated by any finite weight. In his book Guidobaldo summarized this argument as follows:

[154] See point 3 of the overview, page 96.

> Excessum enim ponderis *D* supra pondus *E*, cum quantitatis rationem habeat, non solum minimum esse, verum in infinitum dividi posse immaginabamur, quod quidem ipsi, non solum minimum, sed ne minimum quidem esse, cum reperiri non possit, hoc modo demonstrare nituntur.
>
> For, the excess of weight *D* over weight *E* having some ratio and quantitative part, we imagined it to be not only minimal but also capable of infinite division. They seek in the following manner to prove that no such weight can be found, since it is not just minimal, but still less.[155]

Indeed, Tartaglia had argued that this is exactly what happens because the ratio of the mixed angles (*angulus mixtus*), as Guidobaldo called them, included between circumference and perpendicular at the two sides of the balance, that is, between the path of the weight and the direction to the center of the world, is supposedly smaller than any other ratio that exists between greater and smaller quantities.[156] The term *mixed angle* designates angles with a curved leg. Guidobaldo reduced them to the special case of circles and a tangent touching each other. From a modern point of view the degree of such angles is zero, independent of the curvature of their legs. Guidobaldo, however, shared with Tartaglia the opinion that such angles differ from each other, although the difference is infinitely small.

In his response Guidobaldo first argued that it is easily possible, by considering circles of larger diameters, to construct situations in which the ratio between the two angles is even smaller so that the claim that the ratio is the smallest possible one is refuted (see figure 3.17). He then pointed to the fact that the lines connecting the weights to the center of the world are not parallel but must converge at that center. On this basis he argued that the lower weight actually becomes positionally heavier than the weight that has been raised,[157] because the small but finite angle between perpendiculars and the directions to the center of the world outweighs any effect of infinitely small "angles" (see figure 3.18).

[155] DelMonte (1577, 6v–7r), Renn and Damerow (2010, 70–71). Translation in Drake and Drabkin (1969, 263–264).

[156] Tartaglia (1546, 130– 131), thirty-third question, sixth proposition; for an English translation see Drake and Drabkin (1969, 128–132). See also the discussion on page 73.

[157] DelMonte (1577, 7r–8r), for an English translation see Drake and Drabkin (1969, 264–265).

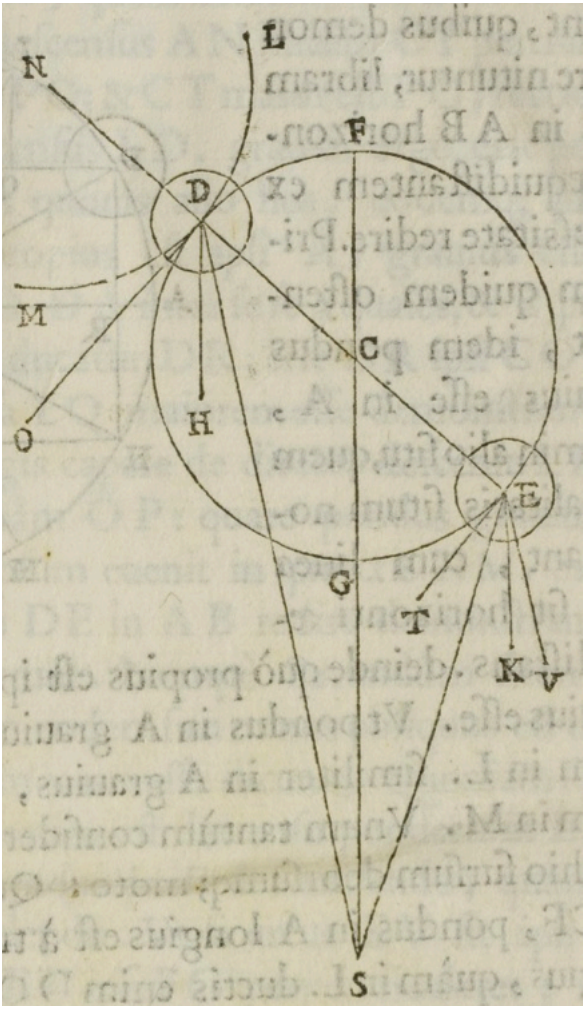


Figure 3.18: According to Guidobaldo, if S represents the center of the world the *mixed angle SEG* between the circular path of the weight at E and the direction to the center of the world is less than the *mixed angle SDG*. Thus, contrary to what his adversaries claim, by their own suppositions the weight placed at E must be heavier than that at D.

As we shall discuss below (see section 3.9), Benedetti shared this conclusion but based this argument on entirely different assumptions.[158] In contrast to Benedetti, Guidobaldo stayed entirely within Tartaglia's conceptual framework, comparing curvilinear angles as indicators of positional heaviness. It is thus also clear that Guidobaldo's insistence on the convergence of perpendiculars at the center of the world was not exaggerated precision, but rather a valid argument in a context in which infinitesimally small angles are being considered.[159]

3.8.4 Guidobaldo's reaction to Cardano's first argument

In responding to the arguments of his adversaries,[160] Guidobaldo began with a general remark on their failure to offer physical reasons for the changing effect of the obliquity of the beam of a balance. He then adressed Cardano's claim that the closer a weight is to the vertical of the beam the less it weighs, offering his own account for this claim (see figure 3.19).

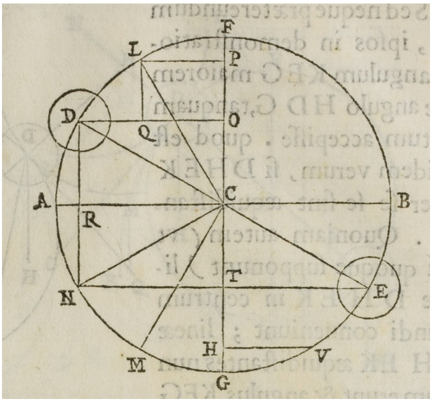


Figure 3.19: Discussing Cardano's first measure of positional heaviness by the horizontal distances *LP*, *DO*, *AC* Guidobaldo explained the changing effect of the weight by the different extent to which the weight presses on the circumference of the circle traced by the balance.

[158] Benedetti (1585, 148– 149), pages 336– 337 in the present edition, for an English translation see Drake and Drabkin (1969, 175–176).
[159] See the discussion in van Dyck (2006a).
[160] DelMonte (1577, 9r–15r), Renn and Damerow (2010, 75–87), for an English translation see Drake and Drabkin (1969, 267–275). See point 5 of the overview, page 98.

In dealing with the varying distance of a weight from the vertical position of the beam, Guidobaldo reconstructed the whole line of reasoning by which the weight of a body on a balance arm has different effects according to the position of the arm from his own perspective, governed by attention to the relation between weight, support and center of the world. He summarized his own account stressing the physical reasons for the changing effect of a weight in different positions on the circle described by the beam of the balance:

> Idem ergo pondus propter situum diversitatem gravius, leviusque erit. Non autem quia ratione situs interdum maiorem re vera acquirat gravitatem, interdum vero amittat, cum eiusdem sit semper gravitatis, ubicunque reperiatur; sed quia magis, minusque in circumferentia gravitat [...]
>
> Therefore the same weight, by diversity of position, will be heavier or lighter, and this not because by reason of its place it sometimes truly acquires greater heaviness and sometimes loses it, being always of the same heaviness wherever it is, but because it presses [*grava*] more or less on the circumference [...][161]

The proximity of the descent of a weight moving in constrained motion, on the one hand, and the natural motion of a weight to the center of the world, on the other, is determined by the angle of contact between the circular path of constrained descent and the straight line of direct descent to the center of the world. In this way Guidobaldo concluded, in particular, that it is not in the horizontal position of the balance arm that a body weighs most but at that point where a straight line drawn from the center of the world touches, as a tangent, the circle described by the balance arm. In the following we shall call that point for ease of reference the *extreme point*. At this point the lever arm forms a right angle with the path of direct descent to the center of the world. Accordingly, at this point the constrained descent of the weight along the circle will be closest to its natural descent along a straight line. It is also at the extreme point where the balance arm sustains the weight less than if it were at any other place on the circumference. The position of the balance arm at this point will be parallel to the horizontal, though not at the fulcrum of the balance but at the position of the center of gravity of the suspended body.

[161] DelMonte (1577, 10v), Renn and Damerow (2010, 78). Translation in Drake and Drabkin (1969, 269).

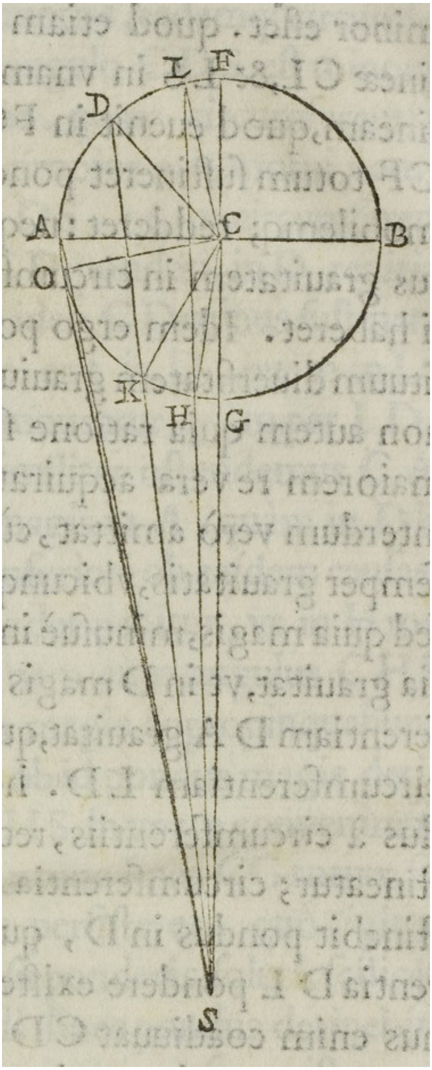


Figure 3.20: Contrary to Cardano, Guidobaldo took into account that the directions from the two weights at the end of a balance cannot be parallel and that therefore the extreme point at which a weight is heaviest differs from point *A*.

Evidently, if the center of the world were infinitely distant and all lines of direction converging at it were perpendiculars and parallel to each other, then the extreme point would mark the horizontal position of the balance arm, also at the fulcrum. For a finite distance of the center of the world, the point where the weight is heaviest lies instead slightly below the horizontal through the fulcrum (see figure 3.20).

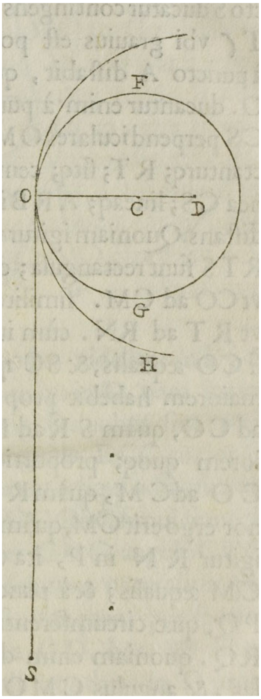


Figure 3.21: Guidobaldo argued that the weight placed at *O* will be heavier along the arm *DO* than along the arm *CO* because the curvilinear "angle" *SOH* is less than the "angle" *SOG*.

Guidobaldo then showed that the same line of reasoning also allowed him to conclude that the weight at the extreme point is heavier the longer the balance arm, a crucial feature of any acceptable concept of *positional heaviness* (see figure 3.21). His argument is based on comparing curvilinear angles. In fact, the larger circle marked by the larger balance arm will make the smaller "angle" with the line of straight and natural descent to the center of the world. He then continued to consider the balance from a *cosmological perspective*, taking into account its finite distance from the center of the world (see figure 3.20). As we have discussed, this perspective was suggested to him by the attempt to set mechanical devices and processes into the context of an Aristotelian cosmos, a characteristic feature of preclassical mechanics. Guidobaldo demonstrated that the closer the balance is to the center of the world the farther the extreme point (where the weight is heaviest) will lie from the horizontal position of the balance arm (as seen from the fulcrum). He even proceeded to study cases in which the balance is located so that the center of the world lies either on or within the circle described by the balance arm. In his analysis Guidobaldo stuck to the principle that the positional heaviness (which he avoided to designate in this way) is determined by the closeness between constrained and direct descent to the center of the world, as given by the angle of contact between these two descents.

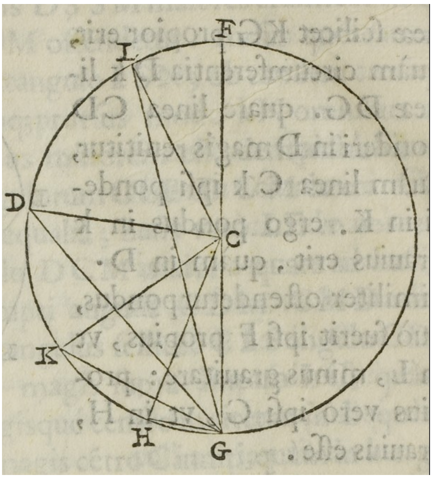


Figure 3.22: Guidobaldo considered the extreme case in which the center of the world lies at the bottom of the circle described by the balance.

Particularly interesting is the case in which the balance is located so that the center of the world lies at the bottom of the circle described by the balance arm (see figure 3.22). For this case Guidobaldo showed that the closer the weight is to the bottom the heavier it becomes since, in any other position, it receives support from the balance arm and thus does not attain its full effect. The drawing accompanying the argument shows a circle with various chords connecting points on the circumference with the bottom of the circle; these points are also connected by radii to its center. Guidobaldo compared the constrained motion along the circumference with the direct motion along the corresponding chord to the center of the world located at the bottom of the circle. The exact same constellation of motions would later play a crucial role in Galileo's theory of motion, albeit with a different interpretation (see figure 3.23). It seems that Galileo simply transposed Guidobaldo's cosmological model to a terrestrial situation. The center of the world located at the bottom of the circle then simply becomes again the lowest point of the motion of the beam of a balance, while the various radii represent positions of the beam at different angles. But what about the chords? In a terrestrial setting they can only be interpreted as inclined planes connecting various points along the circumference with the bottom of the circle. Alternatively, the circle itself could also be conceived as representing the cross-section of a sphere or a cylinder constraining the motion. In any case, the motion of a weight left to itself along the circle, whether constrained by the beam of a balance or the surface of a sphere, would then be the motion of a pendulum.

Guidobaldo's cosmological model of a balance touching the center of the world could thus serve as the blueprint for a cornerstone of Galileo's new science of motion, the comparison of the motion of a pendulum along a certain arc with the motion along an inclined plane representing the chord of that arc. In 1602 the motion of a pendulum as well as the motion of fall along inclined planes representing the chords of a circle became the subject of a famous letter by Galileo to his patron Guidobaldo, who had been skeptical with regard to Galileo's claim about the isochronism of these motions.[162] Indeed, Guidobaldo's analysis does not suggest any such isochronism. Its demonstration required a recognition of the laws of motion along differently inclined planes. This Galileo attained on a conceptual basis that was closer to Benedetti's mechanics than to Guidobaldo's theory (see section 3.10). Galileo's famous insights into the relation between the motion of a pendulum and the motion along inclined planes thus ultimately derived from integrating elements of both Guidobaldo's and Benedetti's work.

[162] Favaro (1968, vol. 10, 97–100). See also the discussion in Renn et al. (2001) and in Büttner (2009).

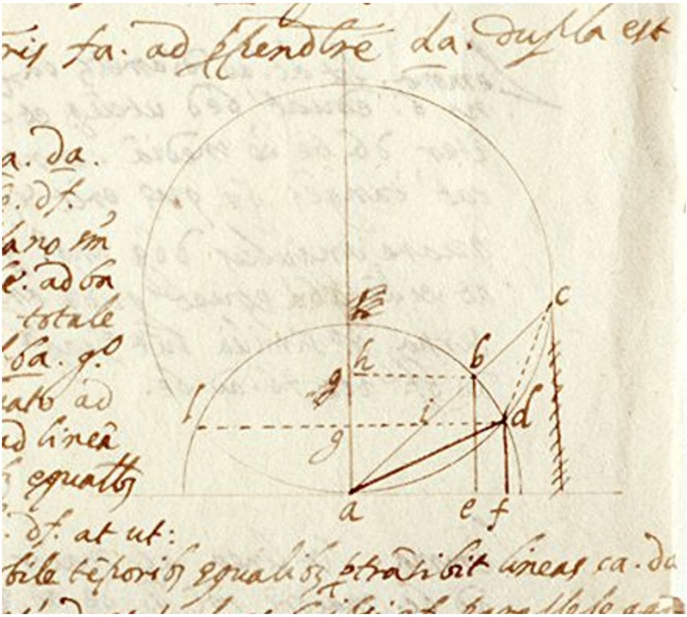


Figure 3.23: Part of a page of Galileo's manuscript MS 72 (Galilei, 1602, fol. 172r) related to theorem 6 of Galilei (1638) in (Favaro, 1968, vol. 8), the so-called "Theorem of Chords." According to this theorem a falling body will require equal times to traverse the distances *CA* and *DA*.

Guidobaldo finished his excursion on the question of how a balance behaves in the vicinity of the center of the world with a remark on the material beam of the balance which, of course, has weight itself. With the help of the concept of center of gravity he was able to quickly settle the issue. All that needed to be done was to find the center of gravity of the entire constellation of the arm of the balance and the weight attached to it. This constellation could then be treated as before as an idealized beam with a weight attached to it. Before going further, Guidobaldo summarized what he had identified so far as being the false assumptions of his adversaries, in particular, that a weight is heaviest in the horizontal position of the beam, which cannot be the case if the finite distance to the center of the world is taken into account.

3.8.5 Guidobaldo's reaction to the main argument of his adversaries

Having exhausted the issue of the distance of the balance beam from the vertical, Guidobaldo, as he had announced before, next entered a series of arguments shared by Jordanus, Tartaglia, and Cardano.[163] These arguments concerned the determination of the positional heaviness by straightness and obliqueness in the sense of the amount to which a given descent partakes more or less in the direct descent to the center of the world. His adversaries took the horizontal projections of equal parts of the circular trajectory to the vertical as a measure of this partaking (see figure 3.24). Comparing the different descents along equal arcs of the upper and the lower weight, for geometrical reasons they thus came, as we have discussed, to the conclusion that the beam of the balance must return to the horizontal because the descent of the upper weight exceeds that of the lower.

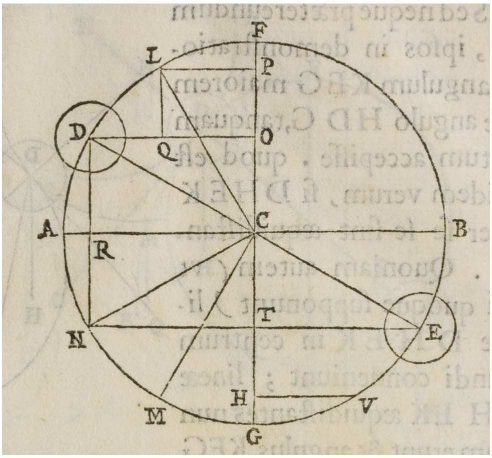


Figure 3.24: According to Jordanus, Tartaglia, and Cardano, the vertical lines *PO* and *OC* represent the vertical descents of the starting points of the displacements along the equal arcs *LD* and *DA*. Therefore, the positional heaviness at point *L* must be smaller than at point *D*. Consequently, the balance must move spontaneously into the horizontal position.

[163] See point 6 of the overview, page 99.

Since Guidobaldo had earlier emphasized that the lines connecting the weights with the center of the world cannot be parallel, he could now argue that the alleged proofs of his adversaries were altogether fallacious:

> Ex quibus non solum suppositio illa, qua libram *DE* in *AB* redire demonstrant, verum etiam omnes fere ipsorum demonstrationes ruunt.
>
> Thus they ruin not only the assumption from which they demonstrate that the balance *DE* returns to *AB*, but also almost all their demonstrations.[164]

Guidobaldo's response to his adversaries, however, left room for the counterargument that the deviations from the parallelism of the lines to the center of the world is negligibly small and that their arguments would work at least approximately. Therefore, Guidobaldo conceded this parallelism and proceeded to demonstrate that measuring positional heaviness by vertical projections leads to inconsistencies, even under these conditions.[165] Jordanus, Tartaglia, and Cardano attempted to determine the magnitude of the positional heaviness by the vertical components of their further descent. They thus fell into the trap that they had to determine an attribute of a weight at a particular point by the length of a line, neglecting that this implicit definition did not sufficiently clarify how and in which direction the endpoint of this line had to be placed.

This is the tacit background of Guidobaldo's construction of counterexamples to their claim. He compared subsequent descents along equal arcs such as *LA* and *AM* corresponding to the equal vertical descents *PC* and *CH*. According to the implicit definition of his adversaries, the positional heaviness at points *L* and *A* should thus be equal (see figure 3.24). However, these subsequent descents in fact bring the beam into two different inclinations in which the endpoints of the beam are obviously of different positional heaviness.

Guidobaldo extended this discussion with further arguments, claiming in particular that when comparing the positional heaviness of descents on the two sides of the balance, their vertical components with regard to the horizontal position of the beam must be taken into account because otherwise further difficulties arise. He finally concluded that this definition of the magnitude of positional heaviness by vertical descents is inconsistent:

[164] DelMonte (1577, 15v), Renn and Damerow (2010, 88).

[165] See point 7 of the overview, page 99.

> Ergo ex diversitate tantum modi considerandi, idem pondus, et gravius, et levius esse continget. Non autem ex ipsa natura rei. Insuper ipsorum suppositio non asserit, pondus secundum situm gravius esse, quanto in eodem situ minus obliquum est principium ipsius descensus. Suppositio igitur superius allata, hoc est, secundum situm pondus gravius esse, quanto in eodem situ minus obliquus est descensus; non solum ex his, quaedix-imus, ullo modo concedi potest; sed quoniam huius oppositum ostendere quoque non est difficile: scilicet idem pondus inaequalibus circumferentiis, quo minus obliquus est descensus, ibi minus gravitare.
>
> Thus from a mere diversity in manner of consideration, and not from the nature of the thing, it would come about that the same weight was heavier or lighter. Moreover, their assumption does not affirm that the positional heaviness will be greater when at the same place the commencement of the descent is less oblique. Hence the postulate [they] adopted above, that is, that the weight is positionally heavier according as the descent from the same place is less oblique, is not to be conceded at all, for the reasons we have given; and not only that, but it is not difficult to show the exact opposite; that is, that the less oblique the descent of the same weight along equal arcs, the less it weighs.[166]

Guidobaldo proceeded to consider equal arcs as before but now shifted the lower arc AM to result into the arc OP. By a rather involved geometrical proof he was now able to show that by applying the definition of his adversaries, the positional heaviness in the upper position L must be greater than the positional heaviness in the almost horizontal position O, which is absurd.

Guidobaldo concluded with a general methodological reflection:[167]

> Non igitur ex rectiori, et obliquiori motu ita accepto colligi potest, secundum situm pondus gravius esse, quanto in eodem situ minus obliquus est descensus. atque hinc oritur omnis ferme ipsorum error in hac re, atque deceptio: nam quamuis per accidens interdum ex falsis sequatur verum, per se tamen ex

[166] DelMonte (1577, 16v–17r, page number 16, misprinted as 14), Renn and Damerow (2010, 90–91). Translation in Drake and Drabkin (1969, 277).

[167] See point 8 of the overview, page 99.

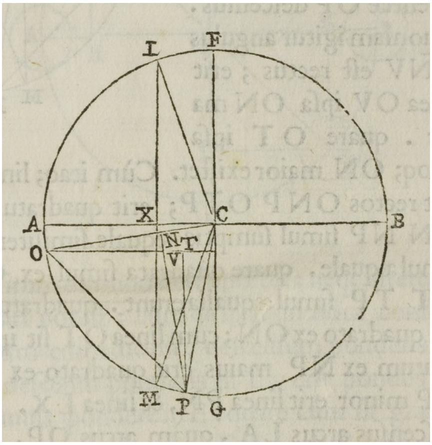


Figure 3.25: Guidobaldo argued that according to the definition of his adversaries and contrary to their claims the positional heaviness in point *L* must be greater than in point *O*, since the vertical descent *LX* is longer than the descent *TP*.

falsis falsum sequitur, quemadmodum ex veris semper verum, nil idcirco mirum, si dum falsa accipiunt; illisque tanquam verissimis innituntur; falsissima omnino colligunt, atque concludunt. Decipiuntur quinetiam, dum librae contemplationem mathematice simpliciter assummunt; cum eius consideratio sit prorsus mechanica: nec ullo modo absque vero motu, ac ponderibus (entibus omnino naturalibus) de ipsa sermo haberi possit: sine quibus eorum, quae librae accidunt, verae caulae reperiri nullo modo possint.

Therefore it is not possible to deduce from the degree of straightness or bending of the motion (taken in their sense) that the weight is positionally heavier [*secundum situm pondus gravius esse*] according as, at a given place, the fall is less bent. And from this arises most of their error and delusion in this matter. And though at times the truth may accidentally follow from false assumptions, nevertheless it is the nature of things that from the false the false generally follows, just as from true things the truth always follows. So it is no wonder that, when they assume false things as true and use these as a basis, they

> deduce and conclude things that are quite false. These men are, moreover, deceived when they undertake to investigate the balance in a purely mathematical way, its theory being actually mechanical; nor can they reason successfully without the true movement of the balance and without its weights, these being completely physical things, neglecting which they simply cannot arrive at the true cause of events that take place with regard to the balance.[168]

3.8.6 The necessity of considering the weights in conjunction

Finally, Guidobaldo approached his goal of demonstrating with the help of a modified but still ambiguous concept of *positional heaviness* the indifferent equilibrium of a balance, assuming that the weight itself is changing.[169] This rather tedious procedure is in stark contrast to his straightforward earlier proof which made use of the concept of the *center of gravity.* Now, at the beginning of the home stretch, he introduced the key idea that distinguishes his own approach from that of his adversaries, namely that the weights on a balance cannot be considered in isolation "as if now one and now the other were placed in the balance, but never both of them together."[170]

Neglecting the cosmological context for the time being, Guidobaldo first showed that the procedure of determining positional heaviness by the amount to which a given descent partakes more or less in the direct descent, a procedure he had just refuted with a *reductio ad absurdum*, allowed for the conclusion that a balance, when removed from the horizontal, would stay in its oblique position and not return to its original place (see figure 3.26). In his argument he used the key idea of the connection of the two weights, stressing that one should not compare the descents of the weights but rather the descent of one (from position D) with the simultaneous rise of the other weight (from position E):

> Erit itaque descensus ponderis in D ascensui ponderis in E aequalis, et qualis erit propensio unius ad motum deorsum, talis etiam erit resistentia alterius ad motum sursum. Resistentia scilicet violentiae ponderis in E in ascensu naturali potentiae

[168] DelMonte (1577, 17v–18r), Renn and Damerow (2010, 92–93). Translation in Drake and Drabkin (1969, 278–279).
[169] See point 9 of the overview, page 99.
[170] Drake and Drabkin (1969, 279). Translated from DelMonte (1577, 18r), see Renn and Damerow (2010, 93).

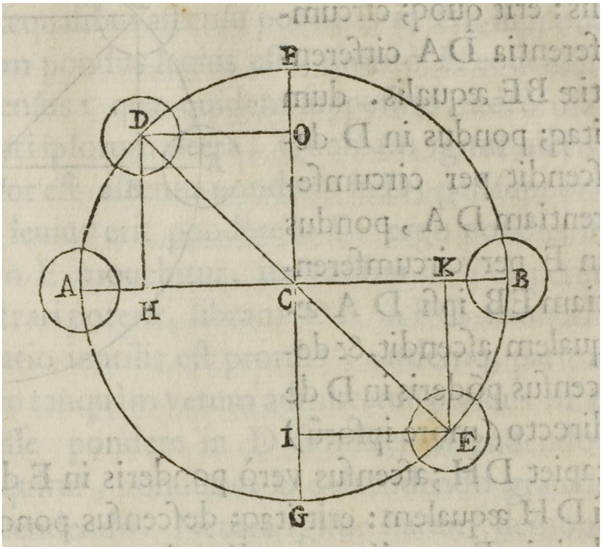


Figure 3.26: Guidobaldo's drawing on page 18r related to his proof of the indifferent equilibrium of a balance in an oblique position.

ponderis in *D* in descensu contra nitendo apponitur; cum sit ipsi aequalis. Quo enim pondus in *D* naturali potentia deorsum velocius descendit, eo tardius pondus in *E* violenter ascendit. Quare neutrum ipsorum alteri praeponderabit, cum ab aequali non proveniat actio. Non igitur pondus in *D* pondus in *E* sursum movebit. Si enim moveret; necesse esset, pondus in *D* maiorem habere virtutem descendendo, quam pondus in *E* ascendendo; sed haec sunt aequalia: ergo pondera manebunt. Et gravitas ponderis in *D* gravitati ponderis in *E* aequalis erit.

Therefore, the descent of the weight placed at *D* will be equal to the rise of the weight placed at *E*, and whatever the inclination of the one is to downward movement [*propensio [...] ad motum deorsum*], such will also be the resistance of the other to upward motion [*resistentia ad motum sursum*]. That is, the resistance to the force of the weight [*violentia ponderis*] placed at *E* in its ascent opposes itself to the natural power [*naturalis potentia ponderis*] of the weight placed at *D*, because of their

> equality, so that by however much the weight placed at D goes with its natural power more swiftly downward, by so much the weight placed at E is more slowly forced upward. So that neither of the two will weigh more [*praeponderare*] than the other; there being no action that proceeds from equality, the weight [*pondus*] placed at D will not move the weight placed at E upward, because, if it did, it would be necessary that the weight placed at D should have stronger force [*maior virtus*] in descending than should the weight placed at E in rising. But these things are equal; therefore the weights [*pondus*] will remain at rest and the weighing down of the weight [*gravitas ponderis*] placed at D will be equal to the weighing down of the weight placed at E.[171]

Not leaving any doubt among his readers that he nevertheless considered the procedure applied to be worthless, Guidobaldo inserted another virtuoso-like *reductio ad absurdum.* He used the same procedure of determining positional heaviness by the amount to which a descent partakes in the vertical, to conclude that the balance would, when removed from its original horizontal position, ultimately assume a vertical position. The argument only works when the connection of the two weights on the balance is once again ignored. The trick of Guidobaldo's *reductio ad absurdum* is not to compare descents, as Jordanus and Tartaglia had done in order to show that the balance returns to its horizontal position, but to compare ascents instead. When previously the descent of the upper weight was straighter than that of the lower weight so that it acquired a greater positional heaviness, its rise is now more oblique than that of the lower weight so that it acquires a smaller positional heaviness. As a consequence, the lower weight which acquires a greater positional heaviness sinks to the bottom and the balance attains a vertical position.

> Quae quidem suppositio, adeo manifesta esse videtur, veluti ipsorum altera.
>
> This assumption seems as evident as theirs. [...][172]

Thus Guidobaldo commented and concluded that neither of these demonstrations is true.

[171]DelMonte (1577, 18v), Renn and Damerow (2010, 94). Translation in Drake and Drabkin (1969, 279–280).

[172]DelMonte (1577, 19r), Renn and Damerow (2010, 95). Translation in Drake and Drabkin (1969, 280).

3.8.7 Guidobaldo's reinterpretation of the concept of *positional heaviness*

It is at this point that Guidobaldo introduced his own interpretation of the changing effect of a weight attached to the end of a beam in dependence of its inclination.[173] Following up on his earlier discussion,[174] this interpretation takes the cosmological context into account, i.e., the finite distance to the center of the world. Furthermore, this interpretation determines obliquity in terms of the deviation of the actual path of the displacement from the closest route to this center. This deviation is measured by the curvilinear angle between both paths at the initial point of the displacement. In order to discriminate the weight *pondus* from its effect Guidobaldo qualified the way in which a weight acts under different circumstances using the term *gravitare*. He formulated his reinterpretation in the following way:

> Praeterea si ipsorum suppositionem, eorumque verborum vim recte perpendamus; alium certe habere sensum conspiciemus. nam cum semper spatium, per quod naturaliter pondus movetur, a centro gravitatis ipsius ponderis ad centrum mundi, instar rectae lineae a centro gravitatis ad centrum mundi productae, sit sumendum; tanto huiusmodi ponderis descensus, magis, minusque obliquus dicetur; quanto secundum spatium instar praedictae lineae designatum, magis, aut minus (naturalem tamen locum petens, semperque magis ipsi appropinquans) movebitur; ita ut tanto obliquior descensus dicatur, quanto recedit ab eiusmodi spatio: rectior vero, quanto ad idem accedit. et in hoc sensu suppositio illa nemini difficultatem parere debet, adeo enim veritas eius conspicua est; rationique consentanea: ut nulla prosus manifestatione egere videatur.
>
> In addition to this, if we shall examine their assumption, and the force of their argument, we shall certainly see that these have a different meaning. For since the space through which the weight moves naturally must be from the center of gravity of this weight toward the center of the world, along a straight line drawn from the center of gravity to the center of the world, it will be said that a descent of the weight made in this way will be more or less oblique according to the space designated, and that it will move more or less along the said line, always

[173] See point 10 of the overview, page 100.
[174] See point 3 of the overview, page 96, and chapter 3.8.3.

> going to seek its natural place by the closest route. Thus the descent is said to be more oblique, the more it departs from that space, and straighter the more it approaches it. Now in this sense the assumption need not give rise to difficulty on the part of anyone, because this is so clear in its truth and its agreement with reason that it does not appear to need to be made evident in any way.[175]

Guidobaldo thus explained his understanding of the concept of *positional heaviness*, an understanding that he evidently no longer saw as being in conflict with his own line of reasoning, which emphasized the concept of center of gravity and its relation to the center of the balance and the center of the world. The concept of *positional heaviness* was indeed an underdetermined concept, susceptible to various interpretations that were not necessarily in agreement with each other.[176] These interpretations, moreover, could vary in conciseness. A more concise interpretation typically allows for more powerful conclusions, but is also more liable to conflict with other branches of the conceptual network in which it is employed. Guidobaldo's vaguer version of the concept avoided conflicts with his elaborate framework: it allowed for comparisons between weights of larger and smaller positional heaviness, it avoided the ambiguities that result from making the positional heaviness dependent on finitely extended descents, but it did not comprise a quantification as will be encountered in Benedetti's case. In other words, there was no quantitative measure for the positional heaviness of a given body under given circumstances.

Nevertheless, Guidobaldo's concept opened up a wide range of conclusions, in particular with regard to the *cosmological* behavior of balances and weights closer or more distant from the center of the world. With this Guidobaldo implicitly also left a challenge to his successors employing his achievements in their own work: For instance, how stable are these conclusions or what modifications do they require when reconsidered from the perspective of a modified interpretation of the concept of *positional heaviness* such as that of Benedetti or Galileo? As we have mentioned above, Galileo for instance turned Guidobaldo's statements about weights on a balance being closer or more distant from the center of the world into statements about weights on lesser or more steeply inclined planes.

[175] DelMonte (1577, 19r), Renn and Damerow (2010, 95). Translation in Drake and Drabkin (1969, 281).

[176] In the words of Yehuda Elkana, it was a "concept in flux," (Elkana, 1970).

3.8.8 From positional heaviness to indifferent equilibrium

After Guidobaldo had reinterpreted the notion of *positional heaviness*, he now approached the task of demonstrating the indifferent equilibrium of a balance. He had just shown that this reinterpretation would apparently even lead to the conclusion that the lower weight tends more to the center of the world than the upper weight. As a consequence, it seemed to follow that the balance would spontaneously move into a vertical position as soon its horizontal position is slightly disturbed, which is indeed a correct consequence from a modern point of view which Guidobaldo, however, could not accept. In order to show that, with the help of his reinterpretation of the positional effect of a weight, he could nevertheless derive the indifferent equilibrium of a balance, Guidobaldo now made crucial use of the fact that the two weights are connected. He thus complemented his reinterpretation with the idea to consider the balance as a system with an interaction of the two weights (see figure 3.27).[177]

His crucial argument was that if the two weights are joined together, one has to consider the descent not of each single weight, but of their center of gravity toward the center of the world. As a consequence, the natural motions of the two weights fixed to the balance will not be along straight lines converging at the center of the world, but along parallels to the straight line that connects their center of gravity with the center of the world. He argued that the connection of the two weights by the balance forces the weights into these parallel downward tendencies.

> Si vero pondera in *ED* sibi invicem connexa, quatenusque sunt connexa consideraverimus; erit ponderis in *E* naturalis propensio per lineam *MEK*: gravitas enim alterius ponderis in *D* efficit, ne pondus in *E* per lineam *ES* gravitet, sed per *EK*.
>
> But if the weights at *E* and *D* are joined together and we consider them with respect to their conjunction, the natural inclination of the weight placed at *E* will be along the line *MEK*, because the weighing down of the other weight [*gravitas alterius ponderis*] at *D* has the effect that the weight [*pondus*] placed at *E* must weigh down [*gravitet*] not along the line *ES*, but along *EK*.[178]

[177] See point 11 of the overview, page 100.
[178] DelMonte (1577, 20r), Renn and Damerow (2010, 97). Translation in Drake and Drabkin (1969, 282).

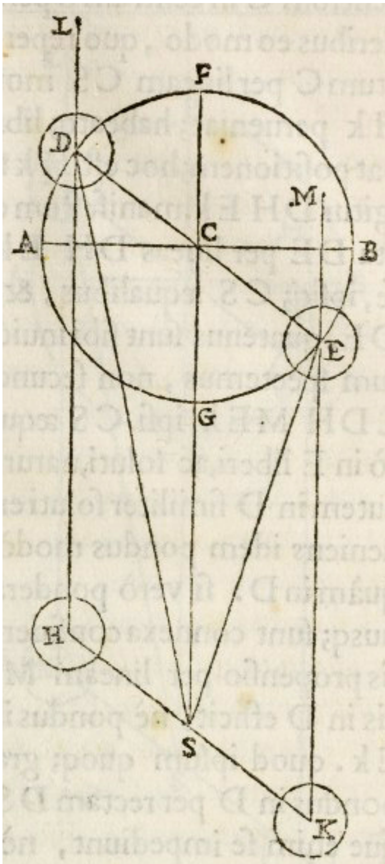


Figure 3.27: Guidobaldo considered the weights in conjunction and argued that the balance resulting tendencies of the weights downward are parallel.

Having thus established the legitimacy of considering the natural descents of the two weights as being parallel, Guidobaldo could now use the same consideration as he had done before: comparing the rise of one weight with the decline of the other. The downward tendencies of the two weights were then identified by means of the closeness between constrained and natural descent (or rise) as measured by the angles of contact between these two paths:

> Cum autem suppositio illa, quae ait, secundum situm pondus gravius esse, quanto in eodem situ minus obliquus est descensus; tanquam clara, atque conspicua admittatur; proculdubio haec quoque accipienda erit; nempe, secundum situm pondus gravius esse, quanto in eodem situ minus obliquus est ascensus. Cum non minus manifesta, rationique sit consentanea. Aequalis igitur erit descensus ponderis in *E* ascensui ponderis in *D*. eandem enim obliquitatem habet descensus ponderis in *E*, quam habet ascensus ponderis in *D*; et qualis erit propensio unius ad motum deorsum, talis quoque erit resistentia alterius ad motum sursum.
>
> Now assuming that the weight is positionally heavier to the degree that its descent from a given place is less oblique, one will also admit without doubt that the weight will be positionally heavier according as its rise at a given point will be less oblique, since this is no less evident or agreeable to reason. Therefore the descent of the weight at *E* will be equal to the rise of the weight at *D*, because the descent of the weight at *E* has as much of the oblique as does the rise of the weight at *D*; and whatever may be the inclination of the one to downward movement, this likewise will be the resistance of the other to upward movement.[179]

Guidobaldo finally concluded:

> Ex quibus sequitur pondera in *D E*, quatenus sunt sibi invicem connexa, aeque gravia esse.
>
> From which it follows that the weights at *D* and *E*, considered in conjunction, are equally heavy.[180]

3.8.9 Guidobaldo's interpretation of Aristotle's balances

In agreement with his plan Guidobaldo next addressed the last argument in favor of the balance returning to the horizontal position, involving the idea of the *meta* of the balance.[181] This *meta* is represented by the lower

[179] DelMonte (1577, 20r–20v), Renn and Damerow (2010, 97–98). Translation in Drake and Drabkin (1969, 282–283).

[180] DelMonte (1577, 20v), Renn and Damerow (2010, 98). Translation in Drake and Drabkin (1969, 283).

[181] See point 12 of the overview, page 101, as well as section 3.7.

half of the perpendicular line through the fulcrum when the support is from above. He was quite aware of the obscure character of argument:

> [...] nihil meo iudicio concludit. Figmentumque hoc de trutina, et meta potius omittendum, ac silentio praetereundum esset, quam verbum ullum in eius confutatione sumendum; cum sit prorsus voluntarium.
>
> [...] in my opinion this [i.e. the return to the horizontal] does not follow, and this fiction about the support and the *meta* should just be left out and passed over in silence; for to say anything about it only confuses the issue, the whole thing being arbitrary.[182]

Nevertheless, Guidobaldo went to some length to reveal the illusionary character of the argument, apparently because the argument seemed to be supported by the Aristotelian *Mechanical Problems* as well as by empirical evidence.[183] As to the role of empirical evidence, Guidobaldo stressed that the case in which the balance is supported at its center is particularly difficult to realize in practice, in contrast to the cases in which the support is either from above or from below:

> Quocirca si centrum in ipsa libra esse consideraverimus, ad sensum confugiendum non est: cum artificilia ad summum illud perfectionis gradum ab artifice deduci minime possint.
>
> Hence if we consider the center to be in the balance, one cannot have recourse to the senses, for artificial devices cannot be brought to such a degree of perfection.[184]

This consideration may have provided a challenge and the starting point for Guidobaldo's efforts to actually produce such an indifferent balance, as it is described in Pigafetta's inserted letter quoted above (see page 87ff.).

The remainder of Guidobaldo's discussion of the Aristotelian *Mechanical Problems* deals mainly with the other two cases discussed by Aristotle, i.e., the cases when the support is either from above or from below (see figure 3.28). He had actually already dealt with these cases in the deductive part of his book, in propositions 2 and 3, on the basis of the concept

[182] DelMonte (1577, 20v–21r), Renn and Damerow (2010, 98–99). Translation in Drake and Drabkin (1969, 283).

[183] See point 13 of the overview, page 101.

[184] DelMonte (1577, 22r), Renn and Damerow (2010, 101). Translation in Drake and Drabkin (1969, 285), modified by the authors.

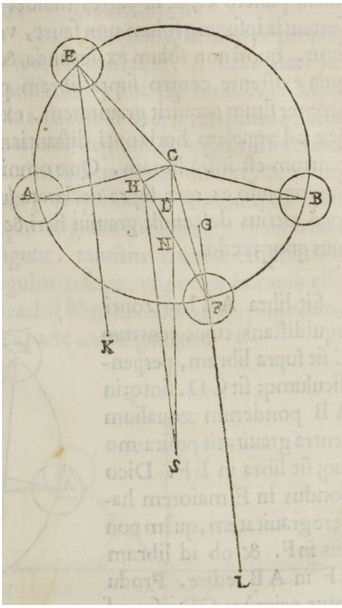


Figure 3.28: In agreement with Aristotle, Guidobaldo argued that a balance supported from above when displaced from the horizontal position would return to the horizontal. His proof differs, however, from the proof of Aristotle. Guidobaldo argued that the upper weight *E* requires a greater heaviness than the lower weight *F* because the descent of the upper weight toward the center of the world *S* is less oblique than the rise of the lower weight.

of center of gravity. Now Guidobaldo re-expressed these conclusions with the help of his concept of the dependence of the heaviness (*gravitas*) on the positional circumstances:

> Quod non solum ex secunda, et tertia huius liquet; verum quia existente centro supra libram pondus elevatum maiorem propter situm acquirit gravitatem. Ex quo contingit redditus librae ad aequalem horizonti distantiam. E contra vero, quando centrum est infra libram. Quae omnia hoc modo ostendentur; supponendo ea, quae supra declarata sunt. Scilicet

> pondus ex quo loco rectius descendit, gravius fieri. et ex quo rectius ascendit, gravius quoque reddi.
>
> This [i.e. Aristotle's opinion that a balance supported from above will return to the horizontal, while a balance supported from below will move toward the lower side] is clear not only from the second and the third propositions of the present book, but also because, if the center is above the balance, the higher weight acquires a greater positional heaviness, considering the return of the balance to the position parallel to the horizon. The contrary happens when the center is below the balance. These things are demonstrated in the following manner, what has been said above being assumed: that is, that the weight will be heavier in that place from which its descent is straighter, and is likewise heavier at the place from which its rise would be straighter.[185]

Throughout the arguments that follow, Guidobaldo made use of defining that straightness in terms of angles of contact as he had done before. But he also occasionally employed Tartaglia's other definition of straightness, in terms of the descent partaking more or less in the vertical. The use of the latter definition, however, is prefaced by a note of caution on the intrinsic falsity of this approach:

> Ex ipsorum quinetiam rationibus, ac falsis supositionibus iam declaratos librae effectus, ac motus deducere, ac manifestare libet; ut quanta sit veritatis efficacia appareat, quippe ex falsis etiam elucescere contendit.
>
> Besides, we may use their logic and their false assumptions to produce the effects and motions of the balance already explained, so that from this one may see the power of truth and how it forces itself to shine forth even from false things.[186]

The excursion on Aristotle's balances ends with an extension of the previous arguments to the case in which an additional weight is placed on one arm of the balance, to the case in which the arms of the balance are of different lengths, and to the cases in which the arms are curved or form an angle.

[185] DelMonte (1577, 23r), Renn and Damerow (2010, 103). Translation in Drake and Drabkin (1969, 286).

[186] DelMonte (1577, 25v), Renn and Damerow (2010, 108). Translation in Drake and Drabkin (1969, 289).

3.8.10 Arguments in further propositions

This lengthy digression is followed in the Italian version by the comment of the editor Pigafetta (page 87ff.). Guidobaldo then resumed the deductive part of his treatise with a continuation of the systematic treatment of the balance. What will be discussed in the following is the remainder of Guidobaldo's book, giving exclusive attention to the question of weights and forces acting in a direction other than the perpendicular or toward the center of the world and, of course, to the issue of positional heaviness. It is in the subsequent book on the lever that these topics again play a significant role.

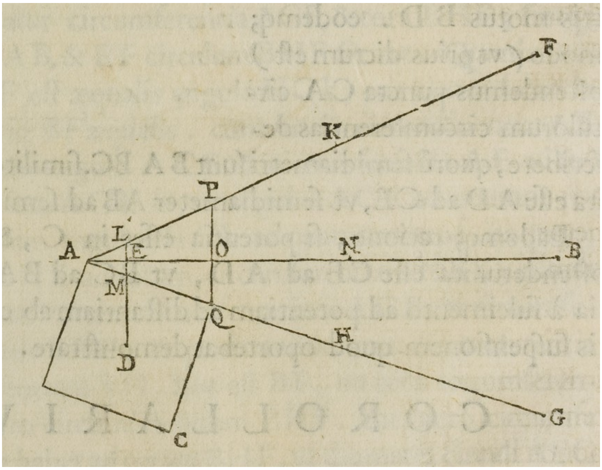


Figure 3.29: Guidobaldo extended the vertical through the center of gravity of the weight to the beam of the lever in order to determine its effective heaviness, a conclusion that is not compatible with classical mechanics.

The crucial theorem is proposition 5 (see figure 3.29):

> Potentia quomodocunque vecte pondus sustinens ad ipsum pondus eandem habebit proportionem, quam distantia a fulcimento ad punctum, ubi a centro gravitatis ponderis horizonti ducta perpendicularis vectem secat, intercepta, ad distantiam inter fulcimentum, et potentiam.

> The power that sustains the weight in any way by means of the lever will have the same proportion to the weight as that of the distance from the fulcrum to the point on the lever, vertical to the center of gravity of the weight, to the distance between fulcrum and the power.[187]

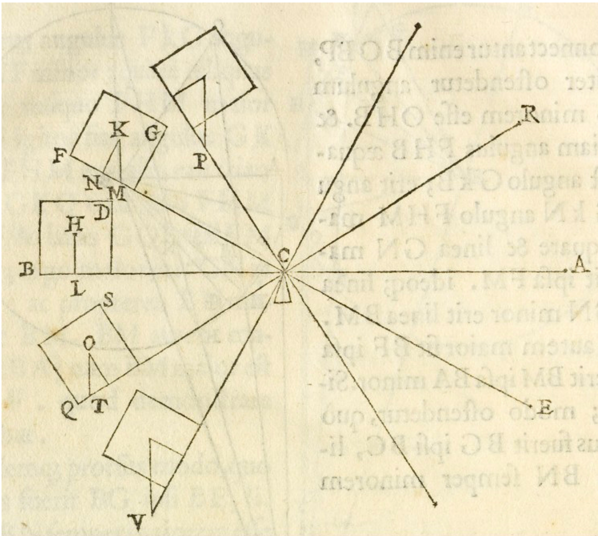


Figure 3.30: Guidobaldo considered the lever in various positions, always determining the effective lever arm by vertically projecting the position of the center of gravity of the sustained weight upon the lever arm.

In order to fully appreciate the significance of what Guidobaldo had in mind, it is necessary to consider the ensuing applications of this theorem as well, for instance proposition 8 (see figure 3.30):

> Potentia pondus sustinens centrum gravitatis supra vectem horizonti aequidistantem habens, quo magis pondus ab hoc situ vecte elevabitur; minori semper, ut sustineatur, egebit potentia: si vero deprimetur, maiori.

[187] DelMonte (1577, 43v), Renn and Damerow (2010, 144). Translation in Drake and Drabkin (1969, 300).

> If the power sustaining the weight which has its center of gravity above a horizontal lever is given, then, the more the weight is raised from this position by means of the lever, the smaller the power required to sustain it. But if it shall be lowered, greater power is required.[188]

In other words, Guidobaldo attempted to address the variation of the force required to lift a weight placed from above on a lever, when that lever is either raised or lowered. Now while it does seem that in this analysis Guidobaldo made use of the projection on the horizontal, measuring the effective length of the lever arm of a weight appended or sustained at an angle, this is actually not the case. What he considered is merely the projection of the center of gravity of the sustained weight along the vertical upon the lever arm in an oblique or horizontal position. In the case of a weight placed from above on a lever arm, the base point of this projection on the lever is then taken as determining the effective length of the lever arm. This is different from the way in which the effective weight (or rather the torque) would have to be determined according to classical physics. From a modern perspective, it would be the point where the perpendicular through the center of gravity of the weight placed on the lever arm crosses the horizontal (and not the arm of the lever as Guidobaldo had it) that determines the effective length of the lever arm, namely the distance of this point from the fulcrum. The two procedures only coincide in the trivial case in which the lever is in a horizontal position. In summary, although Guidobaldo's approach is vexingly close to that of Benedetti and Galileo, it does not actually yield the same correct results.

Another instance in which Guidobaldo needed to take into account something like positional heaviness can be found in his book on the wheel and the axle (see figure 3.31). Proposition 1 of this book states that

> Potentia pondus sustinens axe in peritrochio ad pondus eandem habet proportionem, quam semidiameter axis ad semidiametrum tympani una cum scytala.
>
> The power sustaining the weight by means of the wheel and axle is in the same ratio to the weight as the radius of the axle to the radius of the wheel including the handle.[189]

[188] DelMonte (1577, 49r), Renn and Damerow (2010, 155). Translation in Drake and Drabkin (1969, 301–302), with slight modifications.

[189] DelMonte (1577, 107r), Renn and Damerow (2010, 271). Translation in Drake and Drabkin (1969, 318).

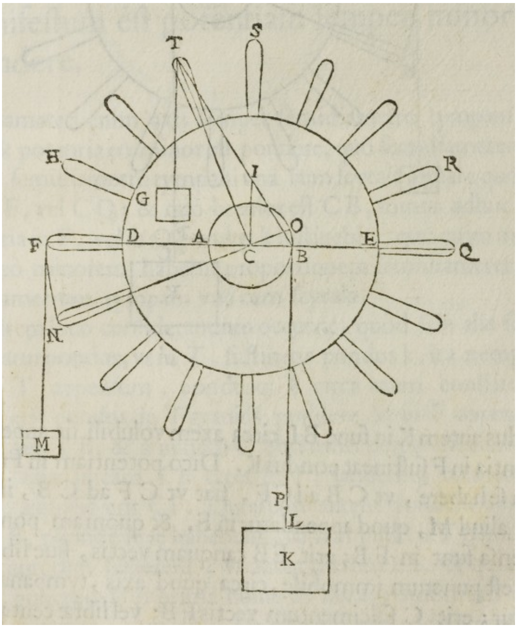


Figure 3.31: Guidobaldo reduced the wheel to the case of a balance in which the power may act in a non-vertical direction. He did not offer a general procedure for determining the effective lever arm.

In the context of explaining this proposition Guidobaldo also considered the case in which the power is applied to a handle not in a horizontal but in an oblique position. He treats this case according to the model of a balance at the end points of which the weight to be moved and the moving weight (representing the power) are attached. This balance is in an oblique position, its fulcrum supposed to be at the center of gravity of the two weights (and not at the center of the wheel). The relation between those two weights, in case of equilibrium, can hence be derived from the law of the lever. Remarkably, Guidobaldo did not make use of the treatment of the bent lever according to the procedure of determining effective lever arms by horizontal projection, a procedure that would, in this case, have yielded the same result. His approach is indeed not a generic one at all, but depends entirely on the adequate identification of the lever problem at hand.

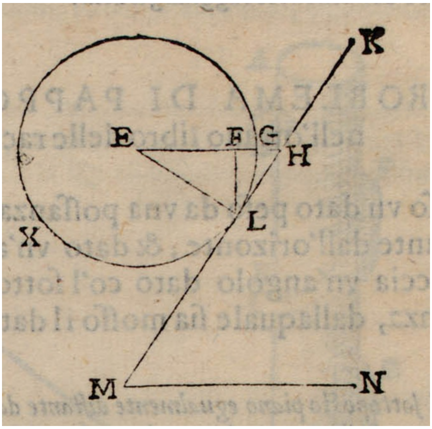


Figure 3.32: Pigafetta, the translator of Guidobaldo's book, added Pappus' erroneous proof of the law of the inclined plane.

The risky character of this identification may be well illustrated by a third instance in which something like positional heaviness plays a role. In his book on the screw, Guidobaldo dealt with its reduction to the inclined plane which in turn is reduced to the balance. For the latter reduction he referred to Pappus. Pappus' analysis of the inclined plane has therefore at this point been inserted into the Italian edition by Pigafetta (see figure 3.32). It considers a spherical body placed on an inclined plane and imagines that the force needed to move its weight along the plane can be determined by imagining the whole situation being equivalent to that of an appropriately positioned balance. The procedure is indeed quite similar to the one we have just encountered in the discussion of wheel and axle, albeit much more problematical. A balance with unequal arms in horizontal position is erected from the point at which the sphere touches the inclined plane, its fulcrum being positioned vertically above it. At one side of the balance the body to be moved is attached, and on the other side, a weight, whose magnitude is to be determined, that balances the body to be moved. The length of this other side is given by the point at which the horizontal balance arm touches the plane. In other words, from the geometrical constellation the lengths of both lever arms are known so that the magnitude of the counter-balancing weight can be obtained, and from it, and some dynamical assumptions, the power needed to move the given weight along the inclined plane. Although the procedure reproduces correctly some qualitative features of the problem, such as the increase of power with steepness of the plane, it turns out to be incompatible with classical physics, implying that an infinitely large power would be required to lift a weight vertically.

In his 1588 book on Archimedes,[190] Guidobaldo briefly returned to the issue of the distances at which weights act to form a center of gravity. But instead of entering a detailed discussion of the positional effect of a weight, he limited himself to emphasizing that one should always consider weights arranged in a straight line (see figure 3.33):

> Quare cum Archimedes tam in hoc postulato, quam in sequentibus, supponit pondera in distantiis esse collocata, intelligendum est distantias ex utraque parte in eadem recta linea existere. Nam si (ut in secunda figura) distantia *AB* fuerit aequalis distantiae *BC*, quae non in directum iaceant, sed angulum constituant; tunc pondera *AB*, quamvis sint aequalia, non aeque ponderabunt. nisi quando (ut in tertia figura) iuncta *AC*, bifariamque divisa in *D*, ductaque *BD*, fuerit haec horizonti perpendicularis, ut in eodem tractatu nostro exposuimus. Distantias igitur in eadem recta linea semper existere intelligendum est. ut ex demonstrationibus Archimedis perspicuum est.
>
> For this reason, since Archimedes assumes in this postulate as well as in the following ones that the weights are placed at certain distances, it is to be understood that these distances exist on both sides in the same straight line. For if (as in the second figure) the distance *AB* were equal to the distance *BC*, which do not lie along a straight line, but constitute an angle, the weights *AB*, although they are equal, do not weight equally [non aeque ponderabunt], other than when (as in the third figure) *AC* is connected and divided in half at *D* and *BD* being drawn, the latter would be perpendicular to the horizon, as we have discussed in our treatise. Therefore the distances are always understood to be along the same straight line, as is evident from the demonstrations of Archimedes.[191]

[190]DelMonte (1588). For extensive historical discussion, see Frank (2007).
[191]DelMonte (1588, 25).

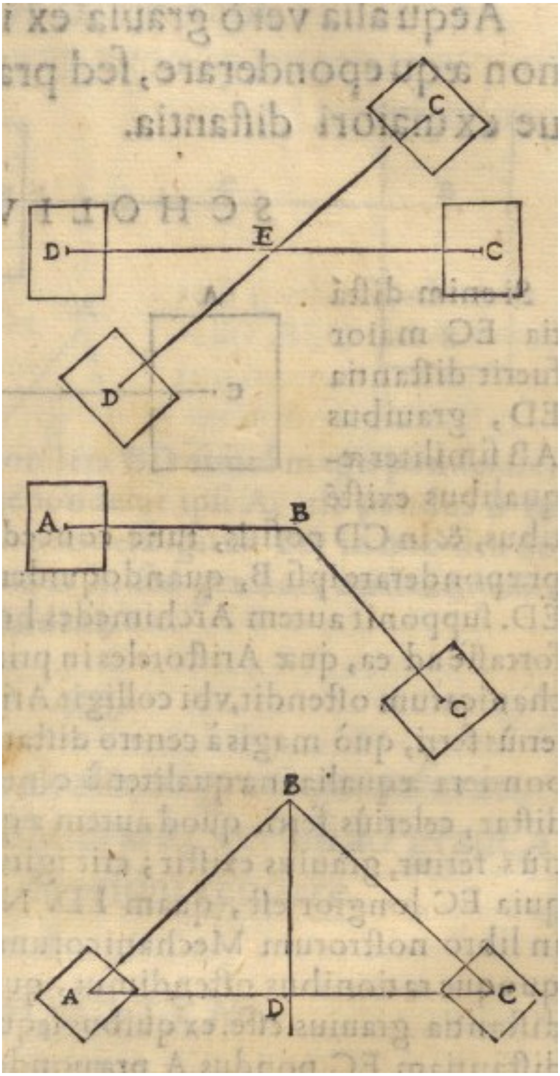


Figure 3.33: In his 1588 book on Archimedes, Guidobaldo stressed that the distances of weights have to be considered always along the same straight line, without giving a prescription for the case of a bent lever.

3.8.11 The flaws of Guidobaldo's understanding of the positional effect of heaviness

After this lengthy discussion, following Guidobaldo's meandering arguments, let us summarize his views on positional heaviness. He abandoned the framework of Jordanus and Tartaglia as he considered this to be incompatible with his own treatment based on the concept of center of gravity. He considered their exclusive focus on the straightness of descent as resulting from a purely mathematical perspective, neglecting the fact that the descent of a body is related, on the one hand, to the mechanical constraints of the motion of weights suspended from a balance, and, on the other, to the direction of the descent toward the center of the world.

In view of his extensive discussion of the concept as well as the arguments of his adversaries, which were based on it, one may ask if Guidobaldo himself believed in a concept of *positional heaviness* that may be freed from the contradictions he had revealed, or whether his use of that concept was exclusively polemical in character. At first glance one may be inclined to answer this question in favor of the merely polemical use, as indeed most interpreters have done.

On closer inspection, however, it has turned out that Guidobaldo did not reject the concept of *positional heaviness* altogether. He reinterpreted it using a conceptual differentiation between the weight (*pondus*) and its heaviness (*gravitas*) as a result of the positional circumstances of its descent. In his own treatment, Guidobaldo avoided the definition of a quantitative measure of the positional effect of a weight in a constrained situation so as not to run into the same ambiguities he himself had criticized. Instead, he limited himself to a consideration of the tendency of the center of gravity to join the center of the world along the most direct path possible, without having the mathematical means at his disposal to make such a procedure concise. He used ad-hoc suppositions to obtain the results he believed to be true. In this way, however, he exposed himself to the same kind of criticism that he had raised against his adversaries.

In summary, neither in the polemical nor in the deductive part of Guidobaldo's treatise on mechanics, a generic procedure for quantitatively determining positional or effective heaviness by a projection of the lever arm onto the horizontal can be found that corresponds to the modern procedure for determining the torque of a weight on a bent lever. Similarly, no treatment of this problem of forces not acting along the vertical was offered by Guidobaldo. Against this background, the question of how he might react to an analysis in which such procedures are employed becomes relevant as it must have constituted a foundational challenge for his mechanics. Pondering this question would have remained a matter of mere speculation were we not in the fortunate position of having Guidobaldo's extensive marginalia on Benedetti's book at our disposal.

3.9 Benedetti's approach to positional heaviness

For Benedetti's mechanics a specific way of determining positional heaviness, measured by the distance from the fulcrum to the line of inclination of a weight (*linea inclinationis*, the line connecting the center of gravity of the weight to the center of the elements) played a crucial and unambiguous role.[192] Benedetti's situation was, in fact, somewhat different from that of his predecessors. Nothing speaks against the assumption that he was familiar with the preceding literature, in particular, with the works discussed above. He thus also knew the pitfalls and contradictions into which the discussion of the positional effect could lead. In the beginning of his treatise on mechanics he introduced the positional aspect of weight, without, however, explicitly introducing *positional heaviness* as a technical term.

Benedetti then carefully analyzed how a weight changes its effect in dependence on the position of a moveable arm of a balance.[193] Following the tradition of his predecessors, he explained this change of effective weight in physical terms by claiming that a weight is impeded by the balance arm in following its straight path to the center of the region of the elements, as he expressed himself. As a consequence, it exerts a certain pressure on the arm which varies according to its inclination. This pressure will be greater the nearer the arm is to the vertical, while it would vanish in the horizontal position. Hence the weight will be positionally heavier when the moveable arm of the balance is along the horizontal than in any other position.

More specifically, Benedetti proposed a quantitative measure of a given weight or of a motive force according to its position. In contrast to Tartaglia's measure of positional heaviness, he determined the (positional) weight using a projection by means of perpendiculars drawn from the center of the balance to the line of inclination, corresponding to the effective length of the lever arm. This argument resembles Cardano's first argument, that is, to take the horizontal component of the distance to the

[192] Benedetti (1585, 141-142), pages 329–330 in the present edition. See also sections 6.1 and7.1.
[193] Benedetti (1585, 142-143), pages 330–331 in the present edition. See also sections 6.1 and 7.2.

support as a measure of the positional heaviness. However, going beyond Cardano who only considered the case of parallel directions to the center of the world, Benedetti also considered lines of directions not directed along the vertical. From the perspective of classical physics Tartaglia's (and Jordanus') concept of *positional heaviness* is closer to issues of energy conservation and the work principle, while that of Benedetti (and later of Galileo) is closer to the modern concept of torque (see section 1.4).

Benedetti may have taken his lead from Cardano's argument or even from its criticism by Guidobaldo. But he may also have turned a result that he could have found in Jordanus' *De ratione ponderis*, edited by Tartaglia,[194] into a key principle of his mechanics. As we have discussed, proposition 8 of *De ratione ponderis* states in fact that when the arms of a balance form an angle, then if their ends are equidistant from the vertical line passing through the axis of support, equal weights suspended from them will be of equal heaviness (see figure 3.15).[195] Jordanus' proof is a rather complex indirect proof that is based on showing that if such a balance were not in equilibrium, a weight by descending through a certain vertical distance would be able to lift an equal weight by a larger vertical distance, which is assumed to be impossible. If Jordanus was indeed Benedetti's starting point, which seems likely as he referred to de Nemore (1565), then he must have dropped any consideration of such vertical descents and rather took the end result of Jordanus' analysis, namely the distance of a weight from the line of suspension, as a general criterion for its positional effect.[196]

Benedetti also made use of his method to determine the effectiveness of a force according to its position by treating a balance with a weight on one arm and a force acting on its other arm at an angle other than 90 degrees.[197] The line of inclination is hence, in this case, not a perpendicular but given by the direction in which the force acts. Nevertheless, Benedetti's procedure is general enough to cover this case as well. Accordingly, a perpendicular is drawn from the center of the balance to the oblique line of inclination, and it is the length of that perpendicular which determines the effective lever arm of the force. This treatment happens to be in agreement with the way in which the torque of the force would be determined according to classical physics (see section 1.4).

[194] de Nemore (1565).
[195] Moody and Clagett (1960, 185–187). See also the discussion in sections 2.1 and 3.5.
[196] We have observed such a process of reinterpretation also in many other instances of conceptual development, see e.g. Damerow et al. (2004), and referred to it as a *Copernicus process*.
[197] Benedetti (1585, 143), page 331 in the present edition. See also sections 6.1 and 7.3.

Benedetti also dealt with the fact that the beam of a balance is not a mathematical line but a material body which he imagined as having a rectangular cross-section.[198] Furthermore, he treated balances supported at one of their ends. His idea was to use his treatment of forces not acting along the vertical in order to reduce such a material balance to the simple case of a bent lever. For this purpose he first imagined a balance supported from below with two bodies attached to its arms that are conceived as exerting their weights from the top of the rectangular beam of the balance. Connecting the fulcrum of the balance with the places of the weights Benedetti thus obtained a triangle positioned on its top. He then argued that what determines the effectiveness of the weights are the distances of their lines of inclination from the center, i.e. the horizontal projections of the sides of that triangle. Next he generalized the treatment to the cases of balances supported at their center or at another point. Benedetti claimed that such an analysis of the material beam had never been achieved before in the literature.[199]

Benedetti further addressed alleged errors in Tartaglia's *Quesiti*.[200] His main target, however, was Tartaglia's understanding of the concept of *positional heaviness*. He did agree with Tartaglia's general claim which has been quoted above (see section 3.7).

> Dalle cose dette, et dimostrate di sopra, se manifesta qualmente un corpo grave in qual si voglia parte, che lui se parta, over removi dal sito della equalità lui si fa più leve, over leggiero secondo il sito, over luoco, et tanto più quanto più sara remosso da tal sito [...]
>
> From the things said and demonstrated above, it is manifest how a heavy body, whenever parted or removed from the position of equality, is made positionally lighter, and the more so, the more it is removed from that position.[201]

But he disagreed with the cause assigned by Jordanus and Tartaglia to this effect. According to Benedetti,

> quia vera causa per se ab eo oritur, quod a centro librae dependeat ut primo cap. huius tractatus ostendi.

[198] Benedetti (1585, 144-146), pages 332–334 in the present edition. See also sections 6.1, 7.4 and 7.5.
[199] Drake and Drabkin (1969, 171).
[200] Benedetti (1585, 148-151), pages 336–339 in the present edition. See also sections 6.2, 7.6 and 7.7.
[201] Tartaglia (1546, 90r). Translation in Drake and Drabkin (1969, 127).

> the true cause emerges by itself from the fact that the weight hangs down [in part] from the fulcrum of the balance, as I showed in the first chapter of this treatise.[202]

In other words, Benedetti stressed that by taking into account the distance from the fulcrum to the line of inclination his approach to the positional effect of a weight is distinct from and superior to Tartaglia's consideration in the Jordanus tradition of straightness of descent.

More specifically, Benedetti refuted several of Tartaglia's claims. In particular, he disputed the central claim in the equilibrium controversy that when a balance is moved from its horizontal position it will return to this position because the body that had been moved upward will attain greater positional heaviness than the body which had been moved downward.[203] As we have seen above, Jordanus' and Tartaglia's argument was based on comparing the descents of the two weights. In other words, the balance would thus have to break in the middle in order to visualize these descents. Benedetti now pointed to the simple fact, already emphasized by Guidobaldo, that, when one weight descends, the other must ascend, and that the corresponding arcs will always be similar to each other and placed in the same way. He concluded that no positional difference in heaviness can be produced in the way that Tartaglia argued.[204]

Nevertheless, Benedetti did not believe in an indifferent equilibrium of such a balance when considered in a cosmological context. In the continuation of his argument, he rather came to the from a modern viewpoint correct conclusion that, when such a balance in equilibrium is displaced from its original horizontal position, the weight that has been lowered will actually assume a greater positional heaviness than the one that has been lifted up:

> Pondus igitur ipsius *A* in huiusmodi situ, pondere ipsius *B* gravius erit.
>
> Therefore the weight of *A* in this [lower] position will be heavier than the weight of *B*.[205]

He reached this conclusion by taking into account that the lines of inclination of the two weights are not parallel to each other but must

[202] Benedetti (1585, 148), page 336 in the present edition. Translation in Drake and Drabkin (1969, 175).
[203] Thirty-second question, fifth proposition; see Drake and Drabkin (1969, 124–127).
[204] Drake and Drabkin (1969, 175).
[205] Benedetti (1585, 148), page 336 in the present edition. Translation in Drake and Drabkin (1969, 176).

converge at the center of the elements. The effective lever arms of the two weights must hence be determined by perpendicular lines drawn from the center of the balance to these lines of inclination. It now turned out that the perpendicular line, corresponding to the weight that had been lowered, is larger than the line corresponding to the weight that had been lifted. Consequently, the lower weight had become heavier positionally so that one would expect the balance to tilt into a vertical position (see figure 1.4).

Benedetti added some more critical remarks on Tartaglia's way of considering positional heaviness. Tartaglia had argued in *Quesiti*, as we have seen, that the upper weight attains a greater positional heaviness than the lower one, but that this difference is arbitrarily small and can therefore not be compensated by any finite weight.[206] This conclusion was reached by comparing curvilinear *angles of contact* on each side of the balance. In his analysis of this argument Benedetti again took into account that the lines of inclination are not parallel to each other but must converge toward the center of the elements, just has Guidobaldo had done before him. Clearly, since Tartaglia's argument hinges on angles of contact, which are infinitesimally small compared to ordinary angles, even that small deviation from being parallel must matter in this case. Taking this into account, Benedetti was able to construct a contradiction, thus refuting Tartaglia's argument. He concluded:

> Omnis autem error in quem Tartalea, Iordanusque lapsi fuerunt ab eo, quod lineas inclinationum pro parallelis vicissim sumpserunt, emanuit.
>
> Now the whole error into which Tartaglia and Jordanus fell arose from the fact that they took the lines of inclination as parallel to each other.[207]

Other propositions of Tartaglia, such as his demonstration using the concept of *positional heaviness* of basic properties of a balance with unequal arms in propositions 7 and 8 of the *Quesiti*, were dismissed as lacking in rigor when compared to the corresponding demonstrations of Archimedes. Concerning the inclined plane treated in Tartaglia's proposition 14, Benedetti interpreted it by reducing it to a balance in a position parallel to the plane. He did not consider the vertical displacements that

[206] Thirty-third question, sixth proposition; see Drake and Drabkin (1969, 130–131). See the discussion in section 3.6.

[207] Benedetti (1585, 150), page 338 in the present edition. Translation in Drake and Drabkin (1969, 177).

are crucial to the analysis given by Jordanus and Tartaglia. Instead, he rather argued that Tartaglia's geometrical analysis was flawed by the neglect of the convergence of the lines of inclination at the center of the elements and that the same body which Tartaglia considered at different positions on the same inclined plane cannot have the same weight in these positions since, in Benedetti's interpretation, they correspond to places at different lengths along the arm of a balance.

In summary, Benedetti had introduced a way to determine the positional effect of a weight or a force that essentially gives, in the cases he considered, the same results as the application of the modern concept of *torque*. In particular, Benedetti had managed to go beyond the consideration of weights tending downward to include forces acting in an arbitrary direction. In this way, he was also able to take into account the fact that, on a spherical earth, the lines of inclination of weights on a balance are not parallel. He did not manage, however, to successfully apply his measure of positional heaviness to challenging objects such as the inclined plane.

3.10 Benedetti, Guidobaldo, and Galileo

Historians of science have always regarded Benedetti's *Diversarum speculationum […] liber*[208] as remarkable because of the close similarity of some of its parts with Galileo's early writings. The range of themes and methods common to both authors is indeed astonishing. Both Benedetti and Galileo proposed a theory of the motion of fall based on Archimedean hydrostatics, both considered the acceleration of this motion and its causes, both formulated what in hindsight appear as proto-inertial principles, both dealt with the bent lever in a similar fashion, both analyzed the relation between vibrating strings and musical tones, both formulated similar views on the irradiation of surfaces, both expressed similar views on thermal and hydrostatic phenomena, and, last but not least, both embraced the Copernican world system.[209] Many of these themes and ideas belonged to the shared knowledge of preclassical mechanics. Yet, in some respects the agreement of their approaches is so striking that one may wonder whether it is due to mere coincidence.

In the introduction to *Mechanics in Sixteenth-Century Italy* Stillman Drake writes:

[208] Benedetti (1585).
[209] For an overview, see Bordiga (1985).

> The question of Benedetti's influence, particularly on the young Galileo, is one of great interest and importance in the history of mechanics in the sixteenth century.[210]

Still, in spite of many pages dedicated to the issue over more than one century, the question of Benedetti's direct impact on Galileo has remained unclear, in particular as Benedetti was never mentioned by him. Did Galileo ever consider Benedetti's mechanics, his theory of motion, and his cosmology in any detail? Did Benedetti's work shape Galilei's own views during his formative years, and, if so, why did he never refer to it?

There are a number of possible connections that have been considered in the past. For instance, Galileo's Pisan colleague Jacopo Mazzoni mentioned Benedetti in *In universam Platonis et Aristotelis philosophiam praeludia* from 1597.[211] He also received a letter from Galileo, written on May 30, 1597 arguing for the Copernican world view.[212] In his book Mazzoni referred to Benedetti's discussion of the possibility that motion along a straight line can be continuous,[213] a theme that was later taken up by Galileo in chapter 20 of *De Motu*, which also contains an explicit reference to Copernicus.[214] Another potential intermediary was Galileo's friend Paolo Sarpi who discussed Benedetti's theory of fall in *Pensieri naturali e metafisici*.[215] While earlier historians of science such as Caverni, Duhem, Wohlwill, and Mach stress Benedetti's role for the history of mechanics and his pivotal role for Galileo's subsequent achievements, the more recent historical literature tends to deny the possibility of an influence of Benedetti on Galileo.[216]

[210] Drake and Drabkin (1969, 36).

[211] Mazzoni (1597).

[212] Favaro (1968, vol. 2, 194–202).

[213] See Benedetti (1585, 183– 184). For a historical discussion of the context of this argument in contemporary technology, see Freudenthal (2005).

[214] Mazzoni (1597, 193) and Galilei (1960b, 326). It is conceivable that such issues had been discussed, inspired by Benedetti's work, between Galileo, Mazzoni, and Guidobaldo during the latter's stay in Tuscany in 1589. We would like to thank Pier Daniele Napolitani for drawing our attention to this possibility and to the above-mentioned passages.

[215] Cozzi and Sosio (1996). For an overview of such potential connections, see the discussion in Bordiga (1985, 732–736) who also mentions Mersenne, Clavius, and Cardinal Michelangelo Ricci as possible intermediaries.

[216] See the discussion by Ventrice in Bordiga (1985, 732– 736) who mentions Drake, Drabkin, Fredette, and Galluzzi among those who are skeptical about a concrete influence of Benedetti on Galileo. A notable exception are the commentaries by Carugo and Geymonat to their edition of Galileo's *Discorsi* (Carugo and Geymonat, 1958). Bertoloni Meli even discusses the possibility of Guidobaldo and Galileo discussing Benedetti, but nevertheless rejects any substantial influence by the latter on

The present edition of the parts of Benedetti's book that deal with mechanics together with Guidobaldo's critical notes may help to provide a somewhat surprising answer to this question. It was most probably Guidobaldo, Benedetti's fervent opponent in matters of mechanics, who served as a conduit to Galileo and, at the same time, made it virtually impossible for Galileo to openly admit to this influence if did he not want to risk the protection of the most important patron of his early career.

Among the most striking and consequential similarities between the work of the young Galileo and that of Benedetti is the latter's theory of the motion of fall,[217] with the treatment of the motion of fall as motion in a medium according to Archimedean principles, just as it was presented in Galileo's early work *De motu*.[218] After carefully examining the issue, Drabkin arrived at the following conclusion:

> While there may be some instances of the influence, direct or indirect, of the earlier author on the later, in many cases the similarities seem to be the reflection of a common heritage and tradition, the use of the same sources, or the concurrence of independent minds.[219]

Drake surmised, referring to a period extending at least up to 1596:

> But of course it may be simply that Galileo had not yet heard of Benedetti.[220]

In line with this assessment, even recent, more careful studies of the emergence of Galileo's science have basically ignored a possible impact of Benedetti on its formative period.

There are, however, other perhaps less striking similarities between Galileo and Benedetti. One of them is the prominence of the bent lever in their writings on mechanics, which contrasts with the rather minor role it

Galileo's thinking because that influence would have supposedly arrived too late, see Bertoloni Meli (2006, 61–65).

[217] This topic was the first subject in the letter of dedication in *Resolutio* (Benedetti, 1553). It was taken up again with different arguments in the two editions of *Demonstratio* (Benedetti, 1554, 1555). Finally it became the subject of a chapter on the Aristotelian theory of motion in *Diversarum speculationum [...] liber* (Benedetti, 1585, 168–197) which remained without any marginal notes by Guidobaldo.

[218] See Galilei (1909a, 1960b).

[219] Drabkin (1964, 630).

[220] Drake and Drabkin (1969, 37).

played for Guidobaldo.[221] A related similarity is the analysis of the bent lever in terms of special concepts expressing the varying effect of a weight according to its position.

In his mechanical writings, most likely postdating his move to Padua, Galileo used such a term for expressing the varying effect of a weight, his famous concept of *momento* which he defined as follows:[222]

> Momento è la propensione di andare al basso, cagionata non tanto dalla gravità del mobile, quanto dalla disposizione che abbino tra di loro diversi corpi gravi.
>
> Moment is the tendency to move downward caused not so much by the heaviness of the movable body as by the arrangement which different bodies have among themselves.[223]

Galileo's concept of *momento*[224] and his analysis of the bent lever, crucial to both his mechanics and his theory of motion, evidently emerged from the midst of the controversy about the concept of *positional heaviness*. In this controversy Galileo took a position much closer to Benedetti than to Guidobaldo. Instead of *positional heaviness* Galileo used the concept of *momento* or *momentum* that Guidobaldo had introduced in his book by quoting Commandino's definition of the center of gravity. But while Guidobaldo made no further use of it in his mechanics, Galileo took this concept with respectable lineage in the Urbino school, gave it a new meaning that was taken over from Benedetti and made it a pillar of his own framework, including, following Commandino, the definition of the center of gravity:

> Centro della gravità si diffinisce essere in ogni corpo grave quel punto, intorno al quale consistono parti di eguali momenti.
>
> Center of gravity is defined to be that point in every heavy body around which parts of equal moments are arranged.[225]

The evidence for our claim concerning Benedetti's legacy in Galileo's work comes from Guidobaldo's marginal notes on Benedetti's book, as

[221] Benedetti's achievements in this regard have been recognized in the history of mechanics since Mersenne's French edition, Galilei (1634); see the discussion in Bordiga (1985, 184–187). For a more recent assessment, see Benvenuto (1987).

[222] Remarkably, the concept of *momento* is also used in Stigliola's treatise from 1597, see Stelliola (1597) and, for historical discussion, Gatto (2006).

[223] Favaro (1968, vol. 2, 159). Translation in Galilei (1960a, 151). See also Galilei (2002).

[224] See the extensive discussion in Galluzzi (1979).

[225] Favaro (1968, vol. 2, 159). Translation in Galilei (1960a, 151). See also Galilei (2002).

well as from his entries in a research notebook, known under the title of *Meditatiunculae*,[226] which contain traces of Galileo's intervention in this controversy.

According to Benedetti and Galileo (and in contrast to Tartaglia and Guidobaldo) the effective length of the lever arm, obtained by drawing a perpendicular from the fulcrum of the balance to the line of inclination determines the effectiveness of a weight or a mechanical constellation. As we have seen, the measure of *positional heaviness* used by Benedetti had already played a role in Cardano's treatment of the balance in *De subtilitate* where it occurs together with two other measures.[227] While these measures in a similar way qualitatively determine the changing effect of a weight attached to a balance in dependence on the obliquity of the beam, they result in quantitatively different values. Cardano was either unaware of these differences, or he simply did not pay attention to the possibility of defining a quantitative measure of the magnitude of *positional heaviness*.

As we have also discussed, in contrast to Cardano, Benedetti unambiguously introduced a quantitive measure for the magnitude of a given weight or force in dependence of the positional circumstances (see section 3.9). His prescription is strikingly similar to that of Galileo:

> Quod quantitas cuiuslibet ponderis, aut virtus movens respectu alterius quantitatis cognoscatur beneficio perpendicularium ductarum a centro librae ad lineam inclinationis.
>
> That the magnitude of one given weight or the magnitude of one motive force in comparison with another can be found by means of perpendiculars drawn from the center of the balance to the line of inclination.[228]

Similarly Galileo wrote in his *Mechanics*:

> Ma qui è d'avvertire, come tali distanze si devono misurare con linee perpendiculari, le quali dal punto della suspensione caschino sopra le linee rette, che dai centri della gravità delli due pesi si tirano al centro commune delle cose gravi.
>
> But here it must be noted that such distances must be measured with perpendicular lines dropped from the point of sus-

[226] DelMonte (1587).
[227] Cardano (1550, 16–20), see section 3.7.
[228] Benedetti (1585, 143), page 331 in the present edition. Translation in Drake and Drabkin (1969, 169).

> pension upon the straight lines drawn from the centers of gravity of the two weights to the common center of heavy bodies.[229]

In his *Mechanics*, Galileo later stressed once more how important it is to carefully define the effective distances of weights from their support:

> Un'altra cosa, prima che più oltre si proceda, bisogna che sia considerata; e questa è intorno alle distanze, nelle quali i gravi vengono appesi: per ciò che molto importa il sapere come s'intendano distanze eguali e diseguali, ed in somma in qual maniera devono misurarsi.
>
> There is one thing that must be considered before proceeding further, and this concerns the distances at which heavy bodies come to be weighed; for it is very important to know the sense in which equal and unequal distances are to be understood, and in what manner they must be measured.[230]

He also made it clear in his analysis of the inclined plane by means of the bent lever that this procedure is critical in determining the *momento* of a given weight.[231] As discussed earlier, in his *Diversarum speculationum [...] liber* Benedetti convincingly demonstrated the effectiveness of this method in determining the magnitude of a force or weight according to its position.

Another remarkable similarity between Benedetti's and Galileo's mechanics is the proof of the law of the lever. Both started by considering a uniform weight supported from its center of gravity which is then broken into unequal pieces sustained by strings from a support above the uniform weight. The proof then argues about the possibility to rearrange these strings without disturbing the equilibrium. Benedetti's somewhat obscure proof was critically annotated by Guidobaldo (see section 7.15). A similar proof is then much more clearly expressed in Galileo's mechanics.[232] Finally, Benedetti's discussion of projectile motion may have stimulated Galileo and Guidobaldo's joint experiment using an inclined plane to trace the trajectory of a projectile (see page 199).

The very existence of Guidobaldo's marginal notes on Benedetti's *Diversarum speculationum [...] liber* provides one definitive answer to the question of who had actually read this book. The fact that Guidobaldo

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[229] Favaro (1968, vol. 2, 160). Translation in Galilei (1960a, 152).
[230] Favaro (1968, vol. 2, 164). Translation in Galilei (1960a, 156–157).
[231] Favaro (1968, vol. 2, 181) and Galilei (1960a, 173).
[232] Favaro (1968, vol. 2, 161–163) and Galilei (1960a, 153–154).

del Monte had read it, however, is not entirely new.[233] Guidobaldo's notebook, the famous *Meditatiunculae*,[234] contains a few entries referring to Benedetti's book which furthermore suggest that Guidobaldo must have studied this work at about the same time he began to correspond with Galileo.[235] It has also been shown that other entries in the *Meditatiunculae* document common research and discussions by Guidobaldo and Galileo.[236]

This coincidence strongly suggests that Guidobaldo served as a conduit to his younger colleague Galileo, transmitting knowledge about some of Benedetti's methods that he himself rejected. A closer inspection of the *Meditatiunculae* reveals striking evidence for the likelihood of this scenario. Remarkably, the page of Guidobaldo's notebook facing the page with his most detailed critique of Benedetti shows Galileo's famous construction relating the bent lever to the inclined plane (see figure 3.34).[237]

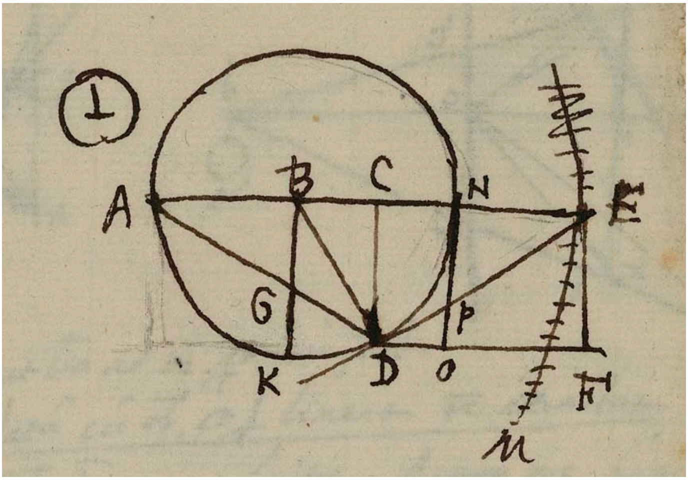

Figure 3.34: Galileo's construction relating the bent lever to the inclined plane in Guidobaldo's notebook.

[233]See Renn et al. (2001, 74).
[234]DelMonte (1587). For a historical discussion and a transcription, see Tassora (2001).
[235]See, in particular, DelMonte (1587, 145–146), extensively discussed in sections 5.2 and 7.3 below.
[236]See Renn et al. (2001).
[237]DelMonte (1587, 145bis), this page may have been inserted.

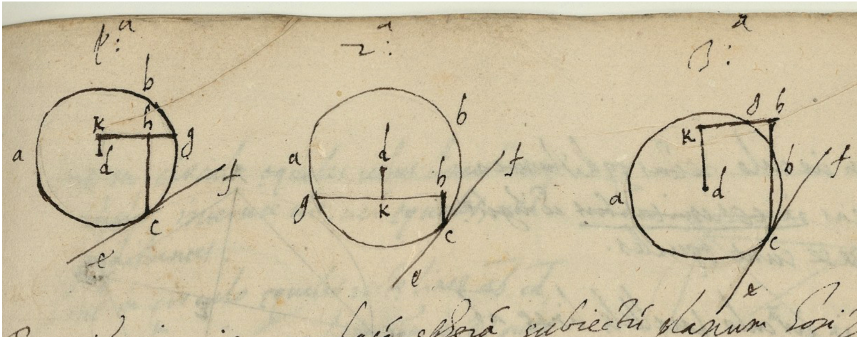

Figure 3.35: Guidobaldo's construction related to the inclined plane in his notebook. The construction was adapted from Pappus' erroneous solution.

This circumstance is all the more noteworthy as Guidobaldo's notebook also contains, on an earlier page, his own problematic adoption of Pappus' analysis of the inclined plane (see figure 3.35).[238] In his writings Galileo had criticized this analysis,[239] substituting it with his own solution of the problem which makes use of the bent lever conceptualized in the same way as Benedetti did. Guidobaldo therefore must have learned about this proof from Galileo, and he must also have seen the connection to Benedetti's methods. It is in any case most unlikely that the two scientists did not discuss this connection and it is quite plausible that Galileo became familiar with Benedetti's work through Guidobaldo. Galileo started corresponding with Guidobaldo in 1588, three years after the publication of Benedetti's book and shortly before he embarked on what later became known as his writings *De Motu*.[240] First Galileo wrote a dialogue version and then an essay in twenty-three chapters. Only this second version contains his proof of the law of the inclined plane, the argument about continuity of motion along a straight line, and the mention of Copernicus. Most probably this version was written after Galileo became familiar with Benedetti's work. His treatise on mechanics, which for the first time explicitly discussed the problem of the effective lever arm, was only written much later, certainly after he had visited Guidobaldo in 1592 on his way

[238] DelMonte (1587, 64).
[239] Galilei (1960a, 172).
[240] Galilei (1960b). For a thorough discussion of the chronology of these writings, see Giusti (1998).

to Padua (see section 2.3). It therefore seems most probable that Galileo was familiar with key ideas of Benedetti when he wrote these works.

A more detailed examination of Guidobaldo's argument in his notebook makes it evident that the discussion of Benedetti's work was not only a turning point for Galileo, but also for Guidobaldo himself. In fact, this discussion must have convinced him, at least to some extent, both of the legitimacy and the fertility of Benedetti's procedure for determining the positional effect of a weight. In fact, he was able to justify this procedure with the help of the concept of *center of gravity* central to his own mechanics.

In the approach to the bent lever problem that he took in his notebook, Guidobaldo considered an arrangement familiar from Benedetti's book, a balance with one deflected arm and with two weights that are in inverse proportion to the projections of their lever arms on the horizontal (see figure 3.36). He assumed this balance to be in equilibrium and determined the center of gravity of the two weights according to the principles of his own book on mechanics. He found that their center of gravity lies at the crossing point between a vertical line through the support and the line connecting the two weights, thus justifying the assumption that it is in equilibrium.

For geometrical reasons it is now clear that the distances of the weights from the center of gravity are in the same proportion as the projections on the horizontal, and inversely as the weights themselves. Guidobaldo had thus managed to relate Benedetti's procedure for determining positional heaviness to his own use of the concept of *center of gravity*, a typical example of how concepts become related by being applied to the same problem. At first he concluded that this is what was to be shown, but then continued the argument to include Galileo's consideration of inclined planes (see figure 3.36):

> Libra *ABD* habeat *AB* horizonti aequidistans. Ponderaque in *AD* maneant. Primum ducta *DC* ad *AB* perpendiculari, dico pondus *D* ad pondus *A* esse, ut *AB* ad *BC*. Sit *BK* ipsi *AB* perpendicularis, et in centrum mundi tendat, iungaturque *AGD*. Et quod pondera manent, erit ex meis mechanicis punctum *G* centrum gravitatis, et ut pondus *D* ad pondus *A*, ita *AG* ad *GD*. Et quod *BG DC* sunt parallelae erit *AB* ad *BC* ut *AG* ad *GD*, hoc est ut pondus *D* ad *A*. Quod demonstrare oportebat.

> Let the balance *ABD* have [the part] *AB* equidistant from the horizon. And let the weights in *AD* be at rest. If first *DC*

is drawn perpendicularly to AB, I say that the weight D is to the weight A, as [is] AB to BC. Let BK be perpendicular to AB and tend toward the center of the world, and let AGD be joined. And since the weights are at rest, according to my [book on] mechanics the point G will be the center of gravity, and as the weight D is to the weight A, so is AG to GD. And since BG DC are parallel, AB will be to BC as AG to GD, that is, as the weight D to [the weight] A. Which was to be demonstrated.[241]

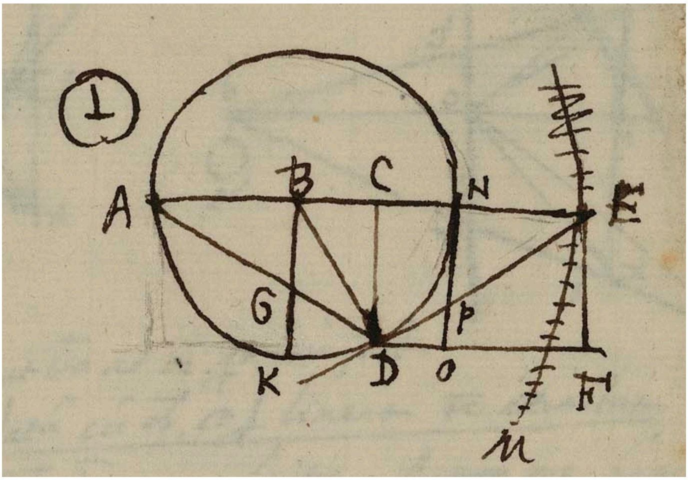

Figure 3.36: Guidobaldo's analysis of Galileo's construction relating inclined plane and bent lever.

Before the claim that this is what was to be demonstrated, Guidobaldo inserted a passage in which he prepared the argument concerning inclined planes by reformulating what he had found in terms of positional heaviness without actually using the term. What he nevertheless needed was a way to describe the changing effect of a weight with position in order to then argue about the powers capable of supporting weights on differently inclined planes. So he attempted to reformulate the equilibrium of two different weights on a bent lever in terms of the different effects of one and the same weight, which he introduced as a reference weight, on the two sides of

[241] DelMonte (1587, 145bis).

the bent lever. Expressing himself in a rather cumbersome and confusing way, he argued, from what he had shown before, that these effects are proportional to the projections of the lever arms on the horizontal:

> Ex hoc patet aequalia pondera in *A D* [...] esse pondus *A* ad pondus *D* ut *AG* ad *GD*. Sit n[empe] ob evitandam confusionem pondus *L*, quod intelligatur in *A* aequale existens ipsi ponderi in *D*. Quod n[empe] pondus in *D* ad pondus in *A* cui aequeponderat est ut *AG* ad *GD*. Pondus vero in *D* eandem habet gravitatem ut pondus in *A*. Ergo pondus *L* ad pondus *A* et ad pondus in *D* est ut *AG* ad *GD*. et per consequens ut *AB* ad *BC*. Levius ergo est pondus in *D* quam pondus in *B*, quanto minor est *BC* quam *BA*.
>
> From this it is evident that, equal weights [being] in *A D* [...], that the weight *A* is to the weight *D* as is *AG* to *GD*. Let now, in order to avoid confusion, the weight *L*, which is understood to exist in *A*, be equal to the same weight at *D*. Because in fact the weight at *D* is to the weight at *A*, with which it is in equilibrium, as is *AG* to *GD*. But the weight at *D* has the same heaviness as the weight at *A*. Therefore the weight *L* is to the weight *A* and to the weight at *D* as is *AG* to *GD*, and in consequence as *AB* to *BC*. Hence the weight at *D* is lighter than the weight at *B* by as much as *BC* is smaller than *BA*.[242]

After this preparation, following Galileo's argument, Guidobaldo related the bent lever to the inclined plane, comparing the power supporting a weight in the vertical direction with the power supporting the same weight along the plane. He formulated the law of the inclined plane by stating that these powers are inversely as the length of the plane is to its height. He then proceeded to relate the inclined plane as well as the vertical to the balance or rather to the bent lever with different positions of its arm, by claiming that a weight on the plane is as if it were on an arm that is vertical to the plane. He could then apply the result previously derived for the bent lever to the inclined plane. For geometrical reasons his argument refers to an inclined plane formed by the intersection of the line representing the original inclined plane and the vertical line tangent to the circle formed by the possible positions of the deflected lever arm (see figure 3.36):

[242] DelMonte (1587, 145bis).

> Sit deinde planum *DE* horizonti inclinatum, et per *EF* horizonti recta sitque *DF* horizonti aequidistans. Dico potentiam pondus sustinentem in *EF* ad potentiam idem pondus sustinentem super *DE*, ita esse, ut *DE* ad *EF*. Intelligatur idem pondus in *N*. Quoniam nempe pondus in *N* super *NO* est, ac si esset libra *ABN* essetque pondus in brachio *BN*, cum sit *BNP* angulus rectus. Similiter ob eandem causam pondus in *D* super *DE* est ac si esset in brachio *BD*, cum sit *BDE* quoque rectus. Hoc nempe modo pondera tangunt plana quod nempe similiter pondus in *N* super planum *NPO* est ac si esset in brachio *BN* pondus vero in *N* est aequegrave ut in *A* erit pondus in *N* ad pondus *D* ut *AB* hoc est *BD* ad *BC*. Et quod triangula *CDE* *EDF* *PDO* sunt similia, et *CDE* simile est ipsi *BDC* erit *BD* ad *BC* ut *DE* ad *EF*, hoc est ut *DP* ad *PO*.
>
> Let then *DE* be a plane inclined to the horizon, and through *EF*, perpendicular to the horizon, let *DF* be equidistant to the horizon. I say that the power supporting the weight at *EF* is to the power supporting the weight over *DE* as is *DE* to *EF*. It is to be understood that the same weight is in *N*. Now evidently the weight at *N* is over *NO*, as if the balance were *ABN* and the weight on the arm *BN*, because *BNP* is a right angle. Similarly for the same reason the weight at *D* over *DE* is as if it were on the arm *BD*, because *BDE* is also a right [angle]. In this way namely the weights touch the planes since indeed the weight at *N* over the plane *NPO* is similarly as if it were on the arm *BN*. But the weight at *N* is equally heavy as at *A*. The weight at *N* will be to the weight *D* as *AB*, that is *BD*, is to *BC*. And since the triangles *CDE* *EDF* *PDO* are similar, and *CDE* is similar to *BDC*, *BD* will be to *BC* as *DE* is to *EF*, that is, as *DP* to *PO*.[243]

Guidobaldo returned to his introduction of a reference weight representing the power of supporting a weight in the vertical. This weight corresponds to the weight at the deflected arm of the balance and is to the weight on the horizontal arm as is the horizontal arm to the projection of the deflected arm. He could now conclude that the same proportion also holds for the powers supporting weights along the vertical and the inclined plane, respectively:

---

[243] DelMonte (1587, 145bis).

> Pondus atque in *N* sustinetur a pondere *L*. Pondus vero *D* sustinetur a pondere in *A*, pondus vero in *L* ad ipsum in *A*, est ut *AB* hoc est *BD* ad *BC*. Ergo potentia sustinens pondus super *NO* ad potentiam pondus sustinens super *DPE* est ut *BD* ad *BC*.
>
> And the weight at *N* is supported by the weight *L*. But the weight *D* is supported by the weight at *A*, but the weight at *L* is to the same in *A*, as is *AB*, that is BD, is to *BC*. Therefore the power supporting a weight over *NO* is to the power supporting the weight over *DPE* as *BD* is to *BC*.[244]

In a final step of his proof Guidobaldo then transferred the proportions he had found for the small triangle generated by his geometrical construction to the originally considered inclined plane:

> Eodem atque modo sustinetur pondus super *DP*, veluti super *DE*, et super *PO*, ut super *EF*. Ergo potentia sustinens pondus super *DE* ad eam, quae sustinet pondus super *EF* est ut *DE* ad *EF*. Quod demonstrare oportebat.
>
> And in the same way the weight is supported over *DP*, or over *DE*, and over *PO*, as over *EF*. Therefore the power supporting the weight over *DE* is to that which supports the weight over *EF* as is *DE* to *EF*. Which was to be demonstrated.[245]

Guidobaldo had thus recapitulated Galileo's proof of the inclined plane based on relating it to the bent lever. Since he had previously shown that Benedetti's treatment of the bent lever could be justified using the concept of *center of gravity*, he had reached a new understanding of the problem of the inclined plane within the framework of his own mechanics.

Let us summarize the situation with regard to the interaction between Guidobaldo, Galileo, and Benedetti: At the beginning of 1588, Galileo and Guidobaldo exchanged their views on the technicalities of proofs in the Archimedean tradition. A more intensive and regular scientific exchange then developed, as we may conclude from the few surviving letters. Thus Guidobaldo had sent Galileo his commentary on Archimedes for commentary and criticism, as we know from a letter of May 28, 1588.[246] From another letter to Galileo, dated December 8, 1590,[247] we may conclude

[244] DelMonte (1587, 145bis).
[245] DelMonte (1587, 145bis).
[246] Favaro (1968, vol. 10, 33–34).
[247] Favaro (1968, vol. 10, 45).

that Guidobaldo used to receive letters from Galileo on an almost daily basis and that Galileo sent his findings likewise to his mentor Guidobaldo. As mentioned above, recent research into Guidobaldo's biography has revealed evidence that Guidobaldo and Galileo must have met as early as 1589 in Tuscany.[248] They might even have met jointly with Galileo's teacher Mazzoni who, as we have also seen, cited Benedetti in his work. Thus Guidobaldo, Mazzoni and Galileo may have discussed Benedetti's *Diversarum speculationum …liber* with the consequence that Galileo reconsidered his work in progress on motion and, in particular, his treatment of motion along inclined planes. This treatment essentially relies on Benedetti's theory of the bent lever and was included in Guidobaldo's notebook. But Benedetti's impact on Galileo probably went even further than that. Galileo may now have taken the Copernican hypothesis much more seriously than before, discussing this as well as other subjects with Mazzoni. In the above-mentioned letter of 1597 Galileo praised Mazzoni for his *Praeludia* and reminded him of the controversial issues on which they meanwhile had reached an agreement, trying now to also press him on the Copernican hypothesis (see page 143).[249]

It is in any case difficult to imagine that Guidobaldo did not discuss his views on Benedetti's mechanics with Galileo, views that he considered at the same time misguided as well as profoundly challenging, as his marginal notes make evident.

3.11 Theoretical excursus: mental models in the transmission of knowledge

3.11.1 The basic mental models of early mechanical knowledge

An analysis of the long-term development of mechanical knowledge requires an appropriate description of the architecture of this knowledge. As pointed out earlier (see section 1.2), one must take into account, in addition to the theoretical knowledge usually considered in the history of science, two further types of knowledge, intuitive physics and practical mechanical knowledge. In order to describe the interaction between these different layers, it has turned out to be useful to adopt the concept of *mental model* from cognitive science and to adapt it to the needs of his-

[248] Menchetti (2012).

[249] This scenario was developed in a joint discussion with Pier Daniele Napolitani.

torical analysis.[250] Mental models are cognitive instruments for drawing conclusions from experiences in the context of given knowledge.

More specifically, mental models are knowledge representation structures which allow inferences to be drawn from prior experiences about complex objects and processes, even when only incomplete information on them is available. Furthermore, conclusions based on mental models can be corrected in light of new information. The concept of mental model is thus particularly suited to explain the long-term continuity of certain aspects of physical thinking. Mental models are also capable of mediating between existing theories and experiences which may or may not be difficult to subsume under these theories. They thus constitute a particularly important theoretical instrument for understanding what happened in the early modern period. It was indeed the period of preclassical mechanics that was characterized, as we have discussed, by the encounter between traditional theories and novel *challenging objects* that were difficult to subsume under the existing theories so that these objects became a stimulus for further developments (see section 3.3).

A mental model consists of a relatively stable network of possible inferences relating inputs that are variable. Cognitive science often uses the term *slots* to indicate the nodes in the structure which have to be filled with inputs satisfying specific constraints. Applying a mental model presupposes the assimilation of specific knowledge to its structure, that is, input information compatible with the constraints of the slots is mapped into them. Filling the slots is the crucial process that decides on the appropriateness and applicability of a mental model for a specific object or process. Once the mapping is successful – if the input information satisfies the constraints of the slots – the reasoning about the object or process is, to a large extent, determined by the mental model.

An example of fundamental importance of the history of mechanics is the *motion-implies-force model* which, when involved in the interpretation of a process of motion, yields the conclusion that the moved object is moved by a force exerted upon it by some mover. While this conclusion is incorrect from the perspective of classical physics, contradicting as it does Newton's principle of inertia, it is in agreement with Aristotelian dynamics (see section 3.4.1). What is more important in our context, the *motion-implies-force model* represents elementary human experiences. In fact, when observing some moving object, one usually presumes that there is some mover at work which drives the object by its force, even when the

[250] On the concept of *mental model*, see Renn and Damerow (2007). In the following we also make use of Renn et al. (2003).

mover itself and its force cannot be directly observed. The missing information about the mover is simply added by the *default setting* of the model based on prior experiences. If, however, additional empirical information eventually becomes available, e.g. specifying the kind of mover, then this information replaces the original default setting without, however, challenging the model itself.

Mental models relevant to the history of mechanics either belong to generally *shared knowledge* or to the *shared knowledge* of specific groups. Accordingly, they can be related to the three types of knowledge mentioned above. First, there are the basic models of intuitive physics, such as the motion-implies-force model just described. Another group of mental models is part of the professional knowledge of more or less specialized practitioners. Their historical transmission is related to the transmission of the real instruments that embody them. And, finally, there are the mental models that belong to theoretical knowledge and that are communicated by an explicit description of their structure and of the conditions of their applications.

A foundational experience of practitioners' knowledge since ancient times has been the equivalence of the weight of a body and the force required to lift it up. This equivalence is prototypically embodied in a real model, namely that of the balance with equal arms. In fact, the force that keeps the balance in equilibrium is equal to the weight in the scale pan. Hence we call this model of compensation between force and weight the *equilibrium model*. However, the practical knowledge of the technicians and engineers of antiquity also involved other basic experiences, and, in particular, the experience of how one can free oneself from the constraint of the equivalence between weight and force. In fact, the art of the mechanician consisted precisely in overcoming the natural course of things with the help of instruments such as the lever. According to this understanding, a mechanical instrument serves to achieve, with a given force, an *unnatural* effect that could not have been achieved without the instrument. We therefore call the model underlying this understanding the *mechanae model* – according to the Greek word μηχανή which means both mechanical instrument and trick, and which is at the origin of the word mechanics.

A key mental model of theoretical knowledge resulted from an integration of the mechanae model with the equilibrium model in the context of a theoretical reflection on the practical knowledge related to the balance with unequal arms that occured in the context of Aristotelian physics, and in particular in the Aristotelian *Mechanical Problems*. We call this model the *balance-lever model.* It can be understood as a generalization of the equilibrium model, associated with the ordinary balance with equal arms. In the case of an equal-arms balance, weight differences are balanced by weights; in the case of an unequal-arms balance, they are balanced by changing the position of the counterweight along the scale or by fixing the counterweight at the end of the beam and changing the position of the suspension point. This necessarily generalized the equilibrium model: weights can be compensated not only by weights but also by distances. It was this practical knowledge related to balances with unequal arms, invented in Greek antiquity some time before Aristotle, which provided the empirical basis for the formulation of the law of the lever.[251]

Another key mental model of mechanical knowledge is the *center of gravity model.* It can be applied to any given heavy body, allowing one to mentally replace it by its total weight and its center of gravity. Its slots are therefore the heavy body itself, its total weight, and the center of gravity. The structure of the model is determined by noting that any axis through the center of gravity turns the body into a lever in equilibrium. In other words, the center of gravity model allows any body to be conceived as a generalized balance with a fulcrum and a distribution of weights around it in equilibrium. In contrast to the fulcrum, however, the center of gravity no longer has to be a physically distinguished point that can be identified by visual cues, but its identification is rather the result of the application of the model to a heavy body. In fact, the center of gravity model can be applied to any body, whether it physically resembles a balance or not. This is the step taken by Archimedes in his work on the equilibrium of plane figures.[252]

To what kind of knowledge does the center of gravity model belong? It is clearly rooted in practical knowledge dealing with balances as it is embodied in the equilibrium model and also in observations on the stability of bodies. On the other hand, understanding the center of gravity model actually requires an explicit or implicit description of its properties. In other words, neither the emergence nor the transmission of this mental model is conceivable without its representation by written language. The very fact that the model is applicable to all heavy bodies suggests that it could hardly have emerged in the context of practitioners' knowledge dealing with specialized domains, but that the model rather belongs to theoretical knowledge.

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[251] See the discussion in Damerow et al. (2002) and Renn and Damerow (2007).
[252] Archimedes (1953).

The center-of-gravity model resulted from a reflection on the applicability of the equilibrium model to all bodies. Indeed, the application of a mental model to different objects and processes and the outcome of such applications may themselves become the object of reasoning that produces new knowledge, provided that such knowledge is appropriately represented – in our case by written language. Knowledge about knowledge structures may then in turn change these knowledge structures. Thus, the application of a mental model may lead to changes – in our case to a generalization – of that model by a deliberate reorganization of its structure as the result of the accumulated meta-knowledge obtained by reflection. As an example for such a reorganization, take the transformation of the concept of fulcrum into that of the center of gravity. While in the equilibrium model the fulcrum is primarily characterized by its physical properties as the turning point of a balance, and only then by the functions it takes on as a consequence of the application of the model, in the more developed model these secondary properties now become the primary properties of the center of gravity.

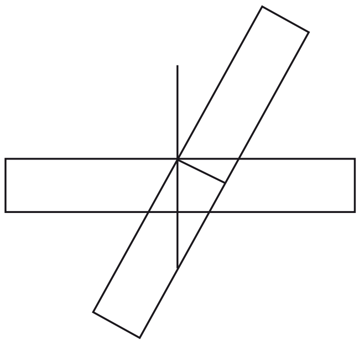

Figure 3.37: If a material beam of a balance is supported from above, and deflected from its horizontal position, it returns to this position according to the Aristotelian *Mechanical Problems* because the upper part of the balance, to the right of the plane indicated by the perpendicular line through the suspension point, is heavier than the lower part to the left of this plane.

A starting point for the development of the center-of-gravity model is found in the Aristotelian *Mechanical Problems* discussed above in section 3.4. There the Aristotelian author considered, as we have seen, an equal-arms balance with an extended, material beam and distinguished between the case in which the balance is suspended from above and the case in which it is supported from below. In these two cases the balance displays different behaviours when its equilibrium is disturbed, for instance by adding or removing a weight. The solution to the problem of why it rises again when supported from above is based on a consideration of the perpendicular line across the point of suspension, which represents a plane dividing the balance into two parts (see figure 3.37). The relation between the weights of these two parts of the balance now decides whether or not the balance rises again. In this way, the equilibrium model is generalized to apply to the suspended beam itself, without the weights usually attached to a balance. The criterion for whether it moves or remains at rest is now no longer the relation between such weights, but that between the two parts divided by the perpendicular plane across the point of suspension.

Although applied to the special case of the material beam of a balance either suspended from above or supported from below, this model works quite generally for all bodies and, if elaborated systematically, naturally singles out the case in which the two parts are always of equal weight. Indeed, if the suspension point across the beam is appropriately moved, a point is reached where the downwardly displaced side of the beam is neither greater nor lesser than the other side. For the material beam this happens if it is suspended from the middle rather than from above or below, in other words, if it is suspended from its center of gravity – a conclusion that the Aristotelian author does not actually draw. But a reflection on the Aristotelian argument could yield a first characterization of the center of gravity as the point from which a suspended body will remain at rest and preserve its position. In this way, the center-of-gravity model eventually resulted from a reflective abstraction of the equilibrium model rooted in practical knowledge, made possible because of the representation of this knowledge in terms of written language, in this case in the Aristotelian text.

The center-of-gravity model provided the backbone for Archimedes' proof of the law of the lever based on redistributing weights under the constraint that the equilibrium is maintained (see figure 3.38). The legitimacy of this argument, however, has often been disputed, in particular vividly and powerfully by the historian and philosopher of science Ernst Mach

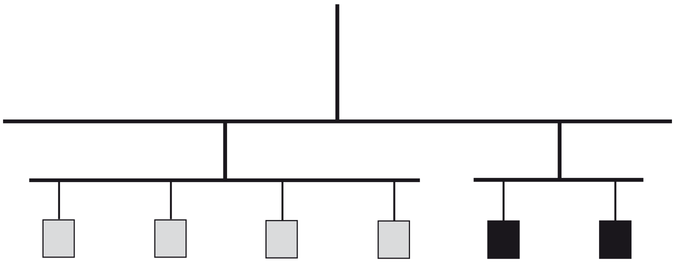

Figure 3.38: Archimedes' proof of the law of the lever is based on a redistribution of weights justified with the help of the concept of center of gravity. A constellation of unequal weights at unequal distances from the fulcrum may thus be replaced, as shown in the figure, by a set of equal weights evenly distributed with regard to the fulcrum. The two constellations are equivalent because the redistribution of weights leave the center of gravity invariant. The redistribution may be visualized by replacing the unequal weights by equilibrated balances, appended from the same suspension points and carrying evenly distributed weights. This illustrates how the concept of center of gravity makes it possible to conceive any given body as a balance and how this concept can be iteratively applied to justify such redistributions of weights.

around the turn of the last century.[253] He argued that Archimedes' proof actually presupposes what has to be shown, the law of the lever. In fact, he argued that the proof involves the assumption that equal displacements of a weight placed on a beam from and towards the point of support cancel each other. This assumes that the effect of a weight placed on a beam is a linear function of distance, a presupposition essentially equivalent to the law of the lever. A closer look at Archimedes' proof reveals, however, that he did not actually talk about such displacements of weights at all. This objection to Mach's analysis has been raised by several historians and has been masterfully elaborated in Dijksterhuis's book on Archimedes.[254] In his analysis Dijksterhuis correctly emphasized that, in the critical step of his proof, Archimedes made use of the concept of center of gravity in

[253]Mach (1988, 10–24). Mach also reports other criticisms.
[254]Dijksterhuis (1956, chap. 9).

order to justify that the original weights keep the system in equilibrium. Indeed, Archimedes argued that these weights maintain the equilibrium because they are placed at the respective centers of gravity of the two groups of equally spaced weights which correspond to them and which, taken together, keep the beam in equilibrium because their overall center of gravity coincides with the point of support of the beam.

Archimedes' proof essentially presupposes three properties of the center of gravity: First, the center of gravity of a symmetric configuration as used in the proof will be at the middle point of the configuration. Second, if a body is supported at its center of gravity, it will be in equilibrium. And third, bodies of equal weight may be substituted for each other without changing the state of equilibrium as long as their centers of gravity coincide. The latter point is crucial for Archimedes' argument: Because of the new abstract quality which the concept of fulcrum assumes when generalized to the concept of center of gravity, it could now be applied iteratively, allowing, in particular, the point of suspension of a weight on a balance to be conceived as the fulcrum of another balance (see figure 3.38). The properties listed above are also used in the proofs of Benedetti and Galileo (see section 7.15). The iterative application of the concept of *center of gravity* is in fact the critical feature of these proofs, working with different constellations of weights on a balance, considered to be equivalent with regard to their center of gravity.

Against this background, it now becomes possible to characterize the concept of *positional heaviness* as resulting from a reflective abstraction following a route alternative to that which led to the center-of-gravity model. As our historical analysis has shown, the concept of *positional heaviness* emerged in a situation where the concept of *center of gravity* was not available and thus left room for this alternative conceptualization (see section 3.4).

3.11.2 The positional-heaviness model

The basic idea of the concept of *positional heaviness* was to express the equilibrium of two different weights on a balance with unequal arms, not as a statement of proportionality – equilibrium results when the lengths of the lever arms are inversely proportional to the weights – but as resulting from an equality. Equilibrium then results when the two different weights are nevertheless equal in positional heaviness, given that a weight acquires a larger or smaller positional heaviness the longer or shorter the lever arm is on which it acts. As we have seen, the concept of *positional heaviness* proved to be effective also in treating devices such as the bent lever and the inclined plane because it lent itself to dealing with the varying effect of a weight in dependence on the angle of inclination.

From a theoretical point of view the concept of *positional heaviness* is based on a special *mental model*. Like the center-of-gravity model, this mental model also resulted from a reflective abstraction based on generalizing the equilibrium model to all balances, including balances with unequal arms. But in contrast to the center-of-gravity model, the *positional heaviness model* did not lead to a generalization of the concept of *fulcrum*, but rather to a generalization of the concept of *weight* attached to a mechanical constellation such as a balance. The original *positional heaviness model* allowed for a differentiation between a weight and its effect without establishing any precise relation between them – other than requiring that equilibrium is associated with equal positional weights. The model thus paved the way for a number of theoretical attempts to specify such a relation under the condition that they are conformal with general properties of the model such as the monotonous relation between the magnitude of the observable effect and the magnitude of the *positional heaviness* which describes this effect. As we have discussed, several different and partly incompatible measures of positional heaviness were triggered in the framework of its underlying mental model which provided a coherence of qualitative knowledge, in spite of such alternatives.

Mental models are usually context-specific and not universally valid. Thus, the concept of *positional heaviness* made sense only against the background of the specific context of knowledge about mechanical devices used in medieval and early modern technology and of the body of available contemporary theories available to organize this knowledge. In particular, when the concept of positional heaviness was first coined by Jordanus, it was shaped by a context in which the Archimedean concept of center of gravity was not available, in which the perplexing differentiation between a weight and its effect under certain conditions could make reference to Aristotle's theory of fallacies, and in which a deductive organization of knowledge on the Euclidean model served as an epistemic ideal. The reflection on such theories and their function constituted a specific *image of knowledge*[255] and thus another context-specific condition of the concept.

In the early modern period, the context for using, rejecting, or elaborating the positional-heaviness model changed. New technological devices and a broader program of mechanical explanation entered the scene. The possibility to reduce an effect such as the power-saving potential of a pul-

[255] See Elkana (1978).

ley or a complex machine to the paradigm of the balance from which the concept of *positional heaviness* had been derived now became a critical, context-specific condition of the applicability of the model. Another context-specific condition was the cosmological assumption that weights always tend to move toward the center of the world. This was an almost universally accepted Aristotelian premise of preclassical mechanics. Furthermore, the adherence to the Archimedean kind of deductive theories as a model for structuring the body of mechanical knowledge provided an image of knowledge characteristic for the Renaissance and early modern context. This explains the common mathematical framework of the attempts to render the meaning of the concept of *positional heaviness* in precise mathematical terms. And finally, the availability of Archimedean writings made it necessary to confront the consequences of interpreting a mechanical problem against the background of the positional-heaviness model with the implications of the center-of-gravity model.

Mental models can be adapted to new experiences attained, in particular, in the process of studying challenging objects. The example of an equilibrated balance deflected into an oblique position of its beam shows that this adaptation may be a multifaceted process. The analysis of the different approaches of Guidobaldo and his adversaries demonstrates, on the one hand, that these scholars and engineers shared a basic understanding of the difference between a weight and its effect. This common understanding was based on a shared mental model which reflected general experiences obtained from the practical handling of balances. We have shown, on the other hand, that the specific ways in which Guidobaldo and his adversaries applied the shared mental model to particular experiences such as those concerning the behavior of a deflected balance and to results of experiments dedicated to studying this behavior in detail challenged the application of the model and determined in various ways its more or less successful adaptation.

These adaptations were, however, limited by some common context-specific historical conditions. In particular, no concept of mathematical function existed that would have allowed experiences involving several physical dimensions such as space, time, and weight to be integrated into a single composite physical magnitude such as torque or work. In the case of the challenging object of a deflected balance, it was thus impossible to integrate the effect of geometrical properties of the deflected balance and the effect of physical properties of the attached weights into a composite magnitude representing all of these effects. The changing effect of a weight in dependence on the obliquity of the beam of a deflected balance could either be explained as resulting from circumstantial conditions that could not be assimilated to one theoretical framework, or the effect could be represented by a new concept of weight that would be determined by specific geometrical or mechanical aspects of the experiences with deflected balances. Realizing the latter possibility led to the specific problems of defining a precise concept of *positional heaviness*. The deficiency in adequately representing the interplay between various physical magnitudes was fundamental, to the effect that not even the equilibrium of a balance could be conceived of as an equality of values of a composite physical magnitude. The introduction of generalized composite mechanical properties such as the product of lengths and weights would have been, however, the precondition for a conceptual development leading to the concept of torque used in classical mechanics to define the equilibrium of a balance.

The historical development we have reviewed nevertheless did eventually lead to a certain reconciliation of the original alternative conceptualizations in terms of center of gravity and positional heaviness. This must be understood as the result of the extensive elaboration and controversial evaluation of the consequences of the application of the underlying shared mental models to an ever increasing array of challenging objects. It is thus no accident that this relative and preliminary stabilization of the conceptual structures of preclassical mechanics went along with a growing network of arguments connecting these mechanical problems and hence the mental models underlying their conceptualization. Galileo's proof of the inclined plane theorem with the help of the bent lever, analyzed with the help of his concept of *momento*, itself an adaptation of the concept of *positional heaviness*, may stand as a representative for this process of knowledge integration.

Mental models link past with present experiences and thus allow conclusions to be drawn from incomplete information. In the present case the knowledge about a challenging object such as the bent lever, the inclined plane or the equilibrated balance deflected by a force acting in an arbitrary direction, was too incomplete to determine a general theory covering all of this knowledge. Only mental models such as the *positional-heaviness model* made it possible to link this knowledge to prior experiences and to the tradition of theoretical mechanics based on the law of the lever, as well as to Aristotelian cosmology and the medieval science of weights. Among the relevant prior experiences was, in particular, the familiarity with balances in equilibrium which were normally conceived of as being in horizontal position, a *default assumption* which was even reflected in the designation of the equilibrium as the position in which the beam of the balance is parallel to the horizon.[256]

Mental models bridge various levels of knowledge that represent the same object in various forms. Early modern technology was based largely on the intuitive knowledge of practitioners and, in particular, on rules of thumb derived by engineers who organized their work. Early modern practitioners had ample experience with the dependency of forces and weights on geometrical and mechanical constellations. However, such knowledge was not usually written down and transmitted only by participation in its application in practical contexts. It was mental models such as the *positional-heaviness model* that allowed this practical knowledge to be combined with knowledge at the level of theoretical reflection based on theories of mechanics available at the time.

If one tries to confront the historical concept of *positional heaviness* with potential counterparts in classical physics, it becomes particularly evident that the effectiveness of the historical concept requires an explanation along the lines just sketched, rather than conceiving it in some sense as an ancestor of modern concepts (see section 1.4). For the cases of the equilibrium of a balance with unequal arms and that of a bent lever, the equality of positional heaviness corresponds, in modern terms, to the equality not of weights or forces but of torques. This translation, however, does not work in the case of equilibrium on an inclined plane. In this case the equality of positional heaviness for different weights placed on differently inclined planes has to be interpreted in terms of the changing component of the force of gravitation in the direction of the inclined plane and as a consequence of the principle of work. The equilibrium of two bodies of different weights placed on differently inclined planes and connected with each other by a weightless rope can then be explained by the compensation of weight and distance in the sense that the larger weight travels a shorter vertical distance along the less steeply inclined plane than the smaller weight hanging down along the more steeply inclined plane, when both are being moved in accordance with their constraints. In other words, the compensation between lengths and weights receives different explanations in the two cases, excluding the possibility of translating positional heaviness with a single modern term such as torque, vector component of force, or work.

[256]See, for instance, the term *parallelum epipedo orizontis* in the *Liber de canonio*, the expression *equatur linea super equidistantiam orizontis* in the *Liber karatonis*, and the expression *situm equalitatis esse equidistantiam super superficiei orizontis* in the *Liber de ponderibus*; for reference see Moody and Clagett (1960, passim).

In summary, both the concept of center of gravity and the concept of positional heaviness are reflective abstractions resulting from mental models rooted in practical experience, in particular, the equilibrium model and the balance-lever model. The representation of these models in the medium of written language constituted not only the basis for using these models far beyond the original extension of their range of applicability but also the presupposition for reflecting on the properties of the model as they are revealed by applying it to particular challenging objects. Whereas mental models such as the motion-implies-force model emerged from a reflection on the operations directly performed with a real object, secondary abstractions such as that yielding the concepts of center of gravity or of positional heaviness resulted from a reflection on mental operations represented by language and performed in order to explore the properties of the model and its application. Without the representation in terms of language, such concepts would have hardly emerged.

Against this background, we recognize that the proofs encountered in our historical sources presuppose not only the practical knowledge about balances with equal and unequal arms, which gave rise to the equilibrium and the balance-lever model, but also concepts of theoretical knowledge such as center of gravity and positional heaviness, which resulted as reflective abstractions from these models. These proofs involve, however, not only these abstract concepts but also, just like the proofs of Euclidean geometry after which they are often modelled, complex constructions corresponding to physical arrangements and the operations performed on them. While in Euclidean geometry the physical operations reflected in the written text are constructions performed with compass and ruler, here they typically correspond to operations with balances. In this way, the practical knowledge about balances continued to provide the empirical grounding that made these proofs convincing.

Chapter 4
Jordanus' Treatise *De ponderibus* Edited by Petrus Apianus

In the following we shall first briefly summarize the structure and the contents of the book annotated by Guidobaldo and then present his marginal notes in their context, with detailed explanations and quotations of the passages on which he chose to comment.

The *Liber Jordani de ponderibus* is based on fourteenth-century manuscripts edited by Petrus Apianus in 1533 at Nuremberg.[1] It comprises a prologue, seven postulates, and thirteen theorems (see also sections 2.1 and 3.5). After each theorem, the Apianus edition presents a short commentary as well as a generally much more extended supplementary commentary, either by the editor himself or compiled from other manuscript sources, as Moody and Clagett contend.[2]

4.1 The prologue

The prologue introduces the science of weights as being subject to both philosophy and geometry. The author then emphasizes that the arm of a balance describes a circle. The effect of a weight in different positions of the arm of the balance is derived from the properties of motion along a circle, conceived in the manner of the Aristotelian *Mechanical Problems.* An arc is compared to its chord in order to assess its curvature. A longer arc of the same circle is thus considered to be more curved than a shorter one, and an arc of a given length is more curved in a smaller circle than in a larger one. The author furthermore assumed that the more curved a motion is, the more contrary it is to a straight line and the more violence it contains. But the more violence or more impediment the motion of a body acquires, the more its heaviness is diminished. This consideration constitutes the

[1] de Nemore (1533).

[2] The following description is based on Moody and Clagett (1960, 145– 149). For a discussion of the extensive commentary and its manuscript basis, see also Moody and Clagett (1960, 293–305).

basis for the statement that a body becomes lighter the more the arm of a balance descends and ultimately for defining positional heaviness. After explaining how such considerations of contrariety of motions relate to a situation in which a heavy body is at rest, the author claimed to have prepared the postulates that are stated next. They are characterized as being in no need of proof and as constituting assumptions of the science of weights.

4.2 The postulates

The first two of the seven postulates (see page 63) are rooted in Aristotelian natural philosophy, stating that the movement of every body is toward the center of the world and that the heavier a body is, the faster it descends. The next three postulates deal with the more or less straight or oblique character of a body's descent. The third postulate introduces the notion of "heavier in descending," defined by the directness of the motion to the center. The fourth postulate defines positional heaviness by stating that a body is positionally heavier if its descent is less oblique. Obliqueness is, in turn, defined in the fifth postulate. A descent is called more oblique if it partakes less of the vertical. The sixth postulate indirectly characterizes a body as having less positional heaviness by the fact that it moves upward as a consequence of the descent of the other body. The seventh postulate finally characterizes the position of equality by equidistance to the plane of the horizon.

4.3 The first theorem on the proportion of descents and ascents of heavy bodies

The first theorem specifies the second postulate. It states that the velocities of descent are in direct proportion to the weights of heavy bodies, while the contrary motions of descents and ascents are in inverse proportion to each other. This theorem lays the ground for a derivation of the law of the lever by starting from Aristotelian dynamics as it is then pursued in theorem 8. The direct proportionality between weight and velocity of descent is a common conclusion from Aristotelian dynamics (see section 3.4.1). The inverse proportionality characteristic of the law of the lever may then be derived from the contrariety of descent and ascent of the two arms of a lever. The first short comment makes this implication explicit, while the second, more detailed and technical comment discusses the conclusion in a scholastic style, refuting arguments of a possible adversary but also referring to propositions of Euclid and Archimedes.

4.4 The second theorem on the equilibrium position of a balance

The second theorem states that a balance with equal arms will not leave the horizontal position when two equal weights are attached. It also states that when the balance is brought into a different position it will return to the horizontal. The first commentary just refers to the fourth postulate for a justification of the latter statement, which is essentially to the definition of positional heaviness. This is hardly understandable without knowledge of the full argument in favor of this claim as it is familiar from other writings. The longer second commentary then provides the missing explanation showing how the fourth postulate is to be applied in order to arrive at the desired conclusions. The argument it provides is essentially identical to that of the *Elementa*, while the accompanying figure is somewhat different.

4.5 The third theorem on the irrelevance of the lengths of the pendants

The third theorem argues that the lengths of the supporting chords of the weights attached to the balance are inconsequential for the equilibrium. The first short comment sketches an indirect proof, arguing that if one body descends, the other side would have to be less heavy. This contradicts the premise that the weights on the two sides of the balance are equal. As a justification it refers, somewhat surprisingly, to the second postulate. The second longer comment essentially follows the reasoning of the *Elementa* and is based on the notion of *positional heaviness*. Remarkable is a concluding reference to the fact that the different distances of the suspension chords from the center of the world have been rightly neglected.

4.6 The fourth theorem on the decrease of positional heaviness

The fourth theorem claims that whenever a weight attached to the beam of a balance descends it becomes positionally lighter. The first commentary simply refers to the fourth postulate which states that a body becomes positionally heavier if its descent is less oblique. The second commentary then argues in more detail, again following the same logic as the *Elementa*, that when a weight is displaced from the horizontal position it will move through arcs that capture less of the vertical so that in fact the weight becomes, by the fourth postulate, less heavy positionally. This theorem is omitted from the *De ratione ponderis*.

4.7 The fifth theorem on the descent of the longer arm

The fifth theorem states that when the arms of a balance are unequal, but the weights attached are equal, the balance will descend on the side of the longer arm. The first commentary notes that the motion of the longer arm describes a larger circle and then refers to the third postulate which states that a body is heavier in descending if its movement toward the center is more direct. The extensive second commentary elaborates in great detail the mathematical relation between circle and arc and refers to Euclid, in particular to the edition of the *Elements* by Johannes Campanus,[3] as well as to Ptolemy and Archimedes. The reference to Ptolemy's *Almagest*, with which Apianus was intimately familiar through his work on cosmography,[4] suggests that Apianus, who around this time must have been working also on his famous sine tables,[5] may have contributed his own thoughts to this commentary or even be its author, an issue which, however, remains controversial.

4.8 The sixth theorem on the bent lever

The sixth theorem is one of the problematic statements about the bent lever dropped from the *De ratione ponderis*. It states that when equal weights are suspended so that one weight is attached from the shorter arm in horizontal position, while the other weight is attached to the longer arm which is, however, bent so that its end is at the same distance from the vertical as is the shorter arm, then the weight on the longer arm will become positionally lighter. According to classical physics, as well as according to theorem 8 of the *De ratione ponderis* and Benedetti's rule, the two weights should, however, be in equilibrium. In the *Elementa* this erroneous conclusion is reached by arguing that the descent of the weight on the longer arm is more oblique than the descent of the weight on the shorter arm, comparing arcs that capture equal amounts of the vertical. The first comment to the theorem is again rather vague and

[3] Johannes Campanus, 1220–1296.
[4] Apianus (1524).
[5] Apianus (1541).

hardly indicates an approach to demonstrating this conclusion. The more explicit second comment again follows the logic of the demonstration in the *Elementa*.

4.9 The seventh theorem on the freely swinging pendant

The seventh theorem states that when equal weights are suspended from equal arms, one by a freely moving chord of suspension and the other by a rigidly fixed rod at a right angle to the arm of the balance, the weight that is freely swinging will be positionally heavier. The theorem thus amounts to the surprising claim that the equilibrium of a balance depends on the mobility of its arms in such a way that the weight on the mobile arm has supposedly a greater effect than the weight on the arm that is fixed. What actually happens in this case according to classical physics is that such a balance in a horizontal position is in stable equilibrium, while a balance with two freely moving chords of suspension would be in indifferent equilibrium.

The first commentary is exceptionally long while still not being very helpful to the non-initiated. It begins as usual with an explication of the terms involved, followed by an indication of how the proof is to be carried out. The hint it gives, however, is limited to the enigmatic statement that the arm that can swing freely describes a greater circle in its descent. The demonstration found in the *Elementa* reduces this case to the bent lever considered in the preceding theorem. The arm of the balance with a fixed rod is mentally replaced by a bent lever along the hypothenuse of the triangle formed by the arm and the rod. The weight at the end of this bent lever has thus the same distance from the vertical line through the point of suspension as has the weight on the other arm of the balance. All that now remains is to compare the obliquity of the descents of these weights as it was considered in the previous theorem. The first comment proceeds by referring to the confusion that may arise when neglecting the difference between mobile and fixed arms, in particular when trying to establish the second theorem. As we have seen, the second theorem in fact claims that the balance always returns to the horizontal position, or in modern terms, that its equilibrium is stable. This, however, is certainly not the case when it has two freely moving chords of suspension. It is therefore remarkable that the first commentary explicitly states that theorem 7 was invented in the course of an experiment aimed at verifying theorem 2. It refers to the possibility that the claim of theorem 2 may seemingly be refuted by such an experiment if one considers a balance with freely moving chords.

The second comment then provides the same argument as the proof of the *Elementa*.

4.10 The eighth theorem on the law of the lever

The eighth theorem states the law of the lever in terms of positional heaviness. The first commentary merely rephrases the claim and refers back to the first theorem. The proof of the *Elementa* is based on reducing the compensation of weights and lengths in a balance with unequal arms to a compensation of weights and heights to which they are lifted according to Aristotelian dynamics, using the preparation provided by theorem 1. The argument is in fact based on the statement inferred from theorem 1 that what suffices to lift a certain weight to a given height will also suffice to lift another weight to a different height if these weights and distances are inversely proportional to each other. The second commentary elaborates this idea in great detail, adding references to the relevant theorems of Euclid.

4.11 The ninth theorem on the equal positional heaviness of bodies in different positions

The ninth theorem states that two oblong bodies of equal weight and shape, one suspended in a vertical position, the other at its midpoint in a horizontal position, have the same positional heaviness. The first commentary first rephrases the theorem and then vaguely indicates that it can be proved by remarking that the semicircles described by these weights are equal. The proof of the *Elementa* decomposes the weight in horizontal position into two equal weights hung at equal distances from the midpoint of the original weight. It then shows that each of these weights balances half of the weight on the other side of the balance. The second commentary develops this idea and remarks in conclusion that this theorem constitutes, according to some, the end of Euclid's book on the balance.

4.12 The four theorems of the *De canonio*

The four remaining theorems are taken from the *Liber de canonio* (see also section 2.1). They deal with the weight of a material beam and its role for the equilibrium of a balance. The structure of the text remains the same. After the theorem a first commentary explains the terms, rephrases the theorem, and hints at a proof. The second commentary then provides technical details including a figure and typically referring to theorems of Euclid's *Elements*.

Chapter 5
Guidobaldo's Marginal Notes in Jordanus' Book

Guidobaldo evidently owned a copy of Apianus' edition of Jordanus.[1] This copy survived and is annotated in Guidobaldo's hand. These marginal notes are presented here not with the aim to offer an exhaustive philological analysis, but in order to gain insight into the equilibrium controversy, in particular with regard to the concept of *positional heaviness.* The presentation of the notes comprises a short characterization of the relevant passages in Jordanus' text, including quotations of the passages to which Guidobaldo referred – if these passages could be identified, then the marginalia themselves are presented and interpreted.

5.1 Second theorem: rejecting Jordanus' stance in the equilibrium controversy

Guidobaldo left several notes on this theorem and its commentaries, some of them very minor. The second theorem deals, as mentioned above, with the equilibrium position of a balance, claiming that a balance always returns to its horizontal position when it is displaced from it. Guidobaldo in contrast was, as we have also discussed extensively, convinced that the balance remains in whatever position it has and does not return to the horizontal.

Guidobaldo's first note refers to Jordanus' statement of the theorem:

> Cum fuerit aequilibris positio aequalis, aequis ponderibus appensis, ab aequalitate non discedet, etsi ab aequidistantia separetur, ad aequalitatis situm revertetur.
>
> If an equilibrated [balance] is in horizontal position [*positio aequalis*], with equal weights suspended, it will not leave the horizontal position [*aequalitate*]; and if it is removed from the

[1] de Nemore (1533).

> horizontal position [*aequidistantia*], it will return to the horizontal position [*aequalitatis situm*].[2]

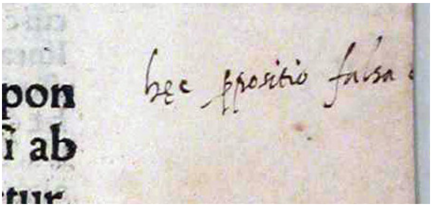

Figure 5.1: First marginal note to the second proposition.

In his first short marginal note Guidobaldo simply rejects this claim. He noted:

> haec propositio falsa e[st]
>
> this proposition is false

Guidobaldo's second note, on the next page, refers to the detailed argument in the second commentary, in particular to the following statement:

> Non enim ulterius descendet B, eo quod descensus eius versus D magis obliquus est, quam ascensus C ad aequalitatem, B enim et C iam equaliter distant a situ aequalitatis [...]
>
> In fact B does not descend further because its descent toward D is more oblique than the ascent of C to the horizontal position since B and C have the same distance from the horizontal position [...][3]

The argument compares, as usual, the obliquity of descents in order to establish which weight is positionally heavier. In the present case it is argued that the descent of the [lower] weight B is more oblique than that of [upper] weight C so that weight B is positionally lighter than weight C, with the effect that the balance returns to its horizontal position.

[2]de Nemore (1533, B ii recto), page 305 of the present edition. Translation by the authors, cf. Moody and Clagett (1960, 156–157).

[3]de Nemore (1533, B ii verso), page 306 of the present edition.

Guidobaldo has underlined the word "ascensus" in the sentence quoted above and noted in the margin that it has to be replaced by "descensus" to make the argument work following the logic of Jordanus who compared descents on both sides of the balance:

> descensus
>
> descent

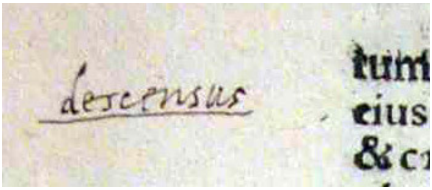

Figure 5.2: Second marginal note to the second proposition.

In a third marginal note on the same page, still related to the second theorem, Guidobaldo remarks:

> [ha]ec demonstratio inutilis est [pr]orsus ut in nostro Mechanicorum libro patet. [s]equens vero recte concludit
>
> this demonstration is completely useless as is clear from our book on mechanics, but in the following he reasons correctly

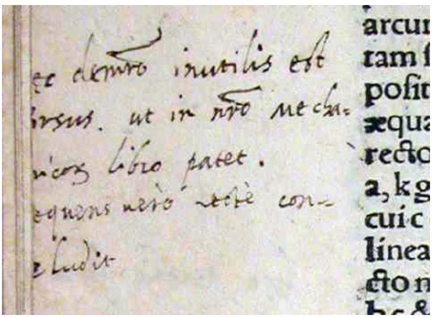

Figure 5.3: Third marginal note to the second proposition.

Guidobaldo claimed here, not that Jordanus' proof is wrong, because geometrically it is correct, but that it is useless. In fact the geometrical reasoning of the commentator about arcs and what they capture from the vertical is mathematically correct.

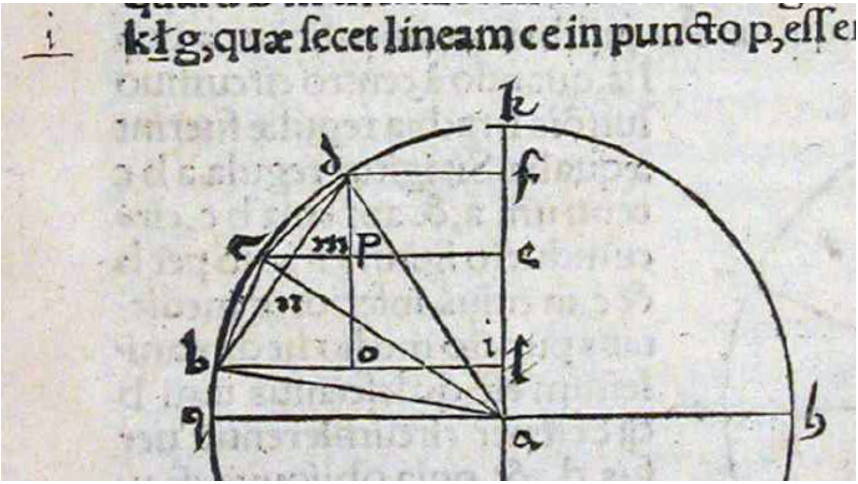

Figure 5.4: Correction of a typographical error.

On the same page there is a very brief fourth marginal note and a correction in the lettering of a figure. The letter *i* replaces the letter *l* in the text which is crossed out. The same is done in the drawing. This just corrects a sloppy designation of a point in the diagram.

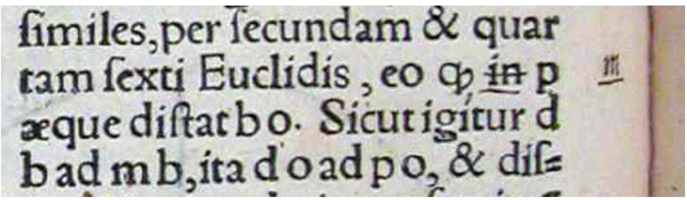
ſimiles, per ſecundam & quar
tam ſexti Euclidis, eo ꝙ in p m
æque diſtat b o. Sicut igitur d
b ad m b, ita d o ad p o, & diſ=

Figure 5.5: Correction of another typographical error.

A similar error in the same paragraph is also corrected by Guidobaldo. The word *in* in the text is crossed out and replaced by the letter *m*.

This shows that Guidobaldo worked carefully through Apianus' edition, even correcting typographical errors, and that he followed, equally carefully, the intrinsic logic of Jordanus' arguments.

5.2 Fourth theorem: the neglect of the cosmological context

The next marginal note refers to the fourth theorem which claims that a weight becomes positionally lighter when it is removed from the horizontal position:

> Quodlibet pondus in quamcumque partem discedat secundum situm fit levius.
>
> In whichever direction a weights descends, it becomes positionally lighter.[4]

Guidobaldo's comment may address either this claim or the first statement of the commentary to this theorem which refers to the fourth postulate, the definition of positional heaviness by the obliqueness of descent:

> Manifestum est hoc per suppositionem quartam.
>
> This is evident by the fourth postulate.[5]

Guidobaldo noted in the margin, close to both of these statements, in large script:

> Falsa
>
> False

Most probably, this comment refers to Guidobaldo's claim, discussed extensively in his book (see section 3.8.4), that, when one takes into account the cosmological context, it is not the horizontal position but a different one, in dependence on the distance of the balance from the center of the world, in which a weight reaches its maximal positional heaviness.

[4] de Nemore (1533, B iii verso), page 308 in the present edition. Translation in Moody and Clagett (1960, 156–157).

[5] de Nemore (1533, B iii verso), page 308 in the present edition. Translation in Moody and Clagett (1960, 156–157).

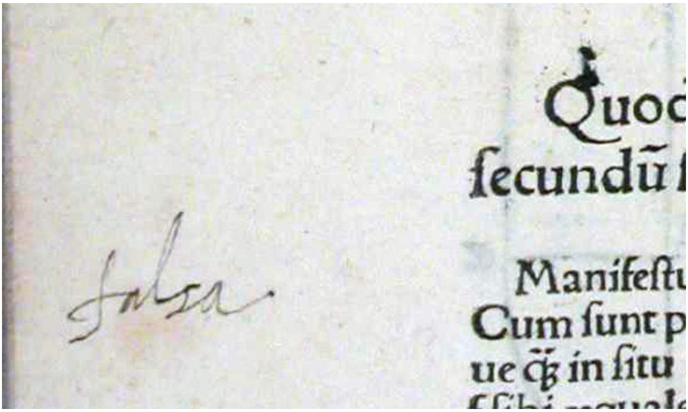

Figure 5.6: Marginal note to the fourth proposition.

5.3 Fifth theorem: failure to recognize the authority of Archimedes in mechanics

The fifth theorem claims, as we have discussed, that a balance with equal weights but unequal arms will descend on the side of the longer arm. In the middle of the extensive second commentary justifying this proposition, which, as we have also discussed, contains many technical references, Guidobaldo just underlined the word *Archimedes*:

> [...] sed sicut circumferentia ad circumferentiam, ita semidiameter ad semidiametrum per quintam Archimedis de curvis superficiebus.
>
> [...] but as circumference to circumference such is diameter to diameter by the fifth proposition of Archimedes' *De curvis superficiebus*.[6]

Possibly this underlining simply meant that Guidobaldo found it remarkable that Archimedes is just quoted as an authority in mathematics but not in mechanics by the author.

[6] de Nemore (1533, C i recto), page 311 in the present edition. Translation by the authors, cf. Moody and Clagett (1960, 158).

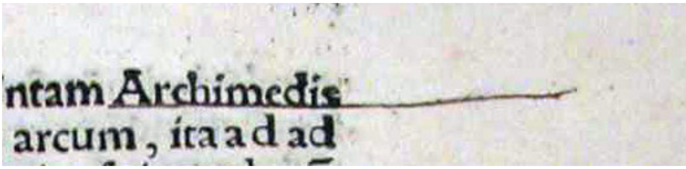

Figure 5.7: Underlining a word in the proof of the fifth proposition.

5.4 Seventh theorem: on the erroneous treatment of an equilibrium problem

The seventh theorem is one of the problematic theorems concerning the bent lever, the other being theorem 6 on which Guidobaldo left no annotation. Theorem 7 concerns the balance with one fixed and one freely moving suspension, claiming that the freely swinging weight is positionally heavier.

Guidobaldo's first note is written in the margin of the beginning of the commentary to this theorem, but it may also refer to the theorem as a whole. Indeed, as we have discussed above, the first part of the commentary is hardly illuminating, while the enunciation of the theorem offers itself to criticism from the viewpoint of Guidobaldo's mechanics. Guidobaldo's approach crucially involves the concept of center of gravity, as we have discussed above. Now the question raised by the present theorem, namely what happens to such a balance when it is displaced from the horizontal position, can be addressed with the help of this concept, leading to the conclusion that the balance is in stable equilibrium. But Guidobaldo may have also focused on a statement in the commentary which is obviously problematic:

> […] tunc illud quod est circumvolubile, maiorem circulum constituit in causa [should be casu] quia plus declinat propter circumvolutionem, et sic pondus ibi gravius est secundum situm cum eius descensus sit rectior.
>
> Then the arm which can swing freely will describe a greater circle in its descent, because it has a greater declination on account of the rotation; and thus the weight there is positionally heavier, since its path of descent is straighter.[7]

[7] de Nemore (1533, C iii verso), page 316 in the present edition. Translation in Moody and Clagett (1960, 158–159).

In any case, Guidobaldo wrote:

> falsa [i]mmo sequitur oppositum [s]i per principia vera fiat demonstratio
>
> false; in fact the opposite follows if the demonstration is performed according to true principles

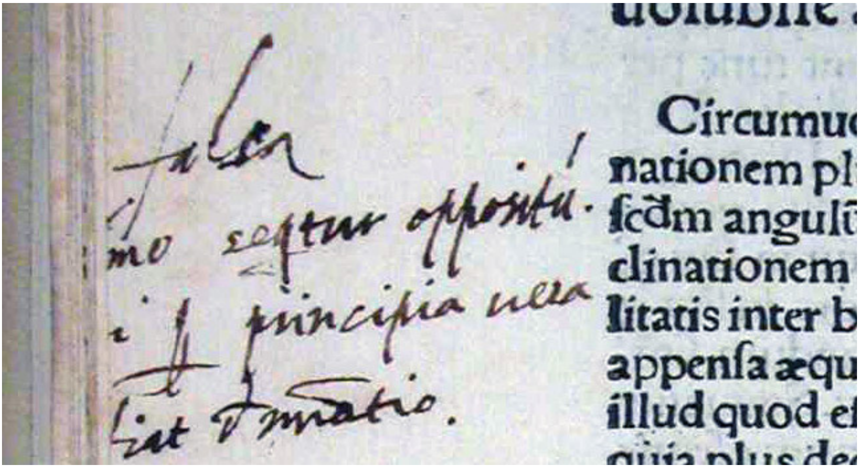

Figure 5.8: Marginal note to the seventh theorem.

The next marginal note simply corrects a typographical error at the end of the commentary to theorem 7, replacing *ho* in the text which is underlined by the correct word *hoc*.

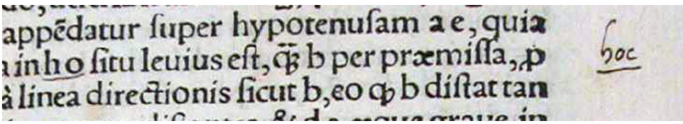

Figure 5.9: Correction of a typographical error near the end of the commentary to Jordanus’ seventh theorem.

5.5 Eighth theorem: rejecting Jordanus' proof of the law of the lever

Theorem 8 constitutes, as we have discussed, a proof of the law of the lever. Guidobaldo left two marginal notes to this theorem and its discussion in the commentaries. The first note refers to the statement of the theorem, expressed in terms of an equality of positional heaviness:

> Si fuerint brachia librae proportionalia ponderibus appensorum, ita, ut in breviori gravius appendatur, aeque gravia erunt secundum situm.
>
> If the arms of the balance are proportional to the weights suspended, in such manner that the heavier weight is suspended on the shorter arm, then the suspended weights will be of equal positional heaviness.[8]

Guidobaldo wrote:

> propositio quidem vera. Demonstrationum vero sequentium nulla ex necessitate concludit
>
> the proposition is true indeed, but he derives nothing of the following demonstrations by necessity

While the claim – the law of the lever – is evidently correct, Guidobaldo questioned the method of proof based on Aristotelian principles rather than on the concept of center of gravity.

Guidobaldo's second marginal note to this theorem on the same page refers to the proof given in the second commentary which he evidently worked through. It refers, in particular, to the following statement (see figure 5.11):

> Dico, quod non faciet motum in aliquam partem regula recta, ascendat primo *B* et descendat *C*, ita ut *DAE* sit quasi regula, et *D* quasi pondus *C*, sint *DM* et *EF* perpendiculares super *BC* palam est igitur […], quod triangulia *ADM* et *AEF* sunt similes.
>
> I say that, without the balance moving in any direction, let first *B* rise and *C* descend so that *DAE* becomes like the balance

[8] de Nemore (1533, C iv recto), page 317 in the present edition. Translation adapted from Moody and Clagett (1960, 160–161).

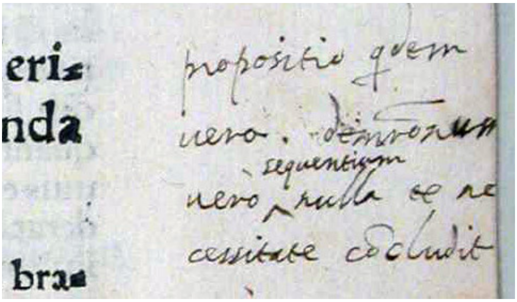

Figure 5.10: Guidobaldo criticizes Jordanus' proof.

> and *D* like the weight *C*, and let *DM* and *EF* be perpendicular lines on *BC* then it is obvious [...] that the triangles *ADM* and *AEF* are similar.[9]

Here the words "D quasi pondus C" ("D like the weight C") are underlined.

In the margin next to it Guidobaldo wrote:

> *DE* quasi pondera [*BC*]
>
> *DE* like the weights [*BC*]

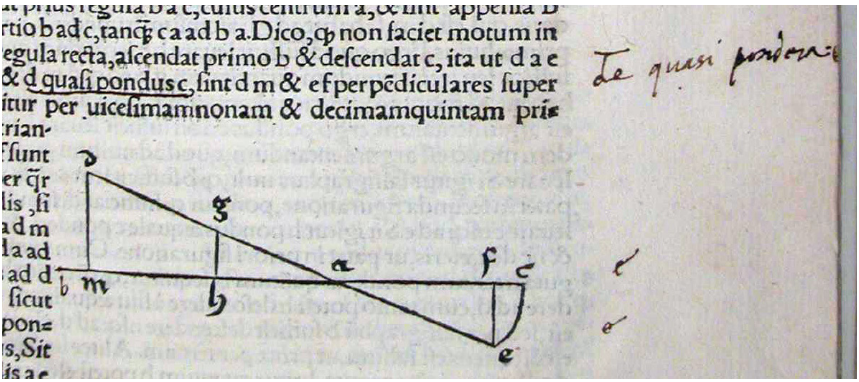

Figure 5.11: Guidobaldo clarifies a sentence of Jordanus' proof.

[9] de Nemore (1533).

Furthermore, he added labels to the diagram. Evidently, Guidobaldo attempted to follow the proof concerning imaginary displacements of weights that allow the application of Aristotelian dynamical principles. In particular, the weight *D* corresponds to the weight originally labelled *B* but then designated as *C* in the text quoted above. The lettering of the text and of the diagram was apparently so confusing to Guidobaldo that he strove for greater clarity by correcting labels both in the diagram and in the text.

5.6 Ninth theorem: the problematic attribution to Euclid

The last sentence of the commentary to the ninth theorem ascribes the first nine theorems not to Jordanus but to Euclid. Guidobaldo evidently found this remarkable and underlined the statement that here the book of Euclids ends:

> Hic explicit secundum aliquos liber Euclidis de ponderibus.
>
> Here ends, according to some, Euclid's book on weights.[10]

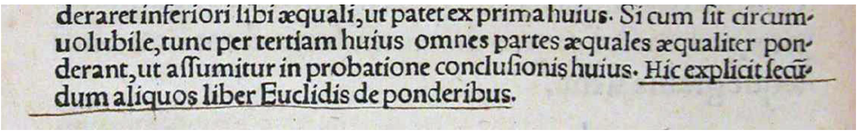
deraret inferiori ſibi æquali, ut patet ex prima huius. Si cum ſit circum-
uolubile, tunc per tertiam huius omnes partes æquales æqualiter pon-
derant, ut aſſumitur in probatione concluſionis huius. Hic explicit ſecū-
dum aliquos liber Euclidis de ponderibus.

Figure 5.12: The ascription of the book to Euclid.

[10] de Nemore (1533, D i verso), page 320 in the present edition.

Chapter 6
The Treatise *De Mechanicis* in Benedetti's Book

The *Diversarum speculationum mathematicarum et physicarum liber* by Benedetti is a collection of six quite different treatises, as has been mentioned in the beginning (see section 1.2). Only one of these treatises entitled *De mechanicis* was in the focus of Guidobaldo del Monte's marginalia and will be presented here in some detail, summarizing and complementing what has been said in section 3.9.

This treatise on mechanics is divided into twenty-five chapters. There are some more references to mechanics in the letters that are also part of Benedetti's book. His discussion of the motion of fall through media and of hydraulic problems are not part of this treatise. The treatise starts with a brief preamble in which Benedetti claimed that he treats topics that have never been dealt with before or have not been sufficiently well explained.

6.1 The oblique position of the beam of a balance

Chapters 1 to 6 contain a systematic account of the foundation on which Benedetti built his mechanics. He presupposed the theory of Archimedes but also incorporated the concept of *positional heaviness.*

Chapter 1 clarifies qualitatively how the variable weight changes depending on the obliqueness of the beam of a balance. While a body attached to the end of the beam has a maximum weight if the beam is in a horizontal position, it vanishes when the beam is in a vertical position. Benedetti explained this behavior as a consequence of the different extent to which the attached weight rests on the center of the balance. If the position of the beam is close to the vertical, the weight of a body attached to the end of the beam is close to zero since it rests nearly completely on the center of the balance.

Chapter 2 clarifies the positional changing of the weight quantitatively. Benedetti related the balance with an oblique position of the beam to a bent lever with one horizontal and one oblique arm, thus providing the precondition for a generalization of his result. A generalization of this kind is indeed required if the lines of inclination of the bodies at the end of a balance are conceived as being directed to the center of the earth and hence no longer as being parallel to each other. Benedetti mentioned this possibility at the end of his chapter, but considered the angle between the two directions as being too small to be measured and thus need not be taken into account.

In chapter 3 Benedetti generalized from the downward inclination of a body attached to the beam of a balance to forces acting upon the body not vertically but making an acute or obtuse angle with the horizontal beam. Accordingly, he replaced the bodies at the end of the beam of a balance with two weights or two moving forces (*duo pondera, aut duae virtutes moventes*), as he formulated somewhat ambiguously. His derivation of their quantities was based on a reinterpretation of the horizontal distances between the center of the balance and the vertical projections of the bodies at the end of a beam in an oblique position. He interpreted these distances as perpendicular distances from the center of the balance to the lines of inclination, and was thus able to apply the result he achieved for vertically descending weights also to lines of inclination caused by forces that are not vertical.

In the following, Benedetti maintained that his arguments in chapters 1 to 3 clarify all the causes operating on balances and levers. To demonstrate this, he discussed in chapters 4 and 5 the validity of his results if applied to material balances and levers, taking into account that they have a beam with finite extension. This, however, does not imply that he calculated the influence of the weight of the beam itself. His discussion was rather restricted to a justification of his claim that the geometry of a rectangular beam does not require a modification of his propositions. In chapter 5 he treated the case of a lever whose fulcrum is at one of its ends.

Finally in chapter 6 Benedetti added the description of an instrument used in bakeries for treating the dough. He explained the function of the instrument by applying his proposition of chapter 3.

The systematic approach of Benedetti in this first part of his treatise is complemented by chapter 9 in which he justified the division of the scale of a steelyard into equal intervals.

6.2 Benedetti's criticism of Tartaglia and Jordanus

In chapters 7 and 8 Benedetti criticized theorems of his former teacher Tartaglia, in particular those Tartaglia adapted from Jordanus de Nemore. Both chapters deal exclusively with some propositions of Book VIII of

Tartaglia's *Quesiti, et inventioni diverse*,[1] which is concerned with the science of weights and is entitled, accordingly, *Sopra la scientia di pesi.* In those cases in which Tartaglia's propositions are adapted from Jordanus, Benedetti mentioned explicitly the corresponding proposition in the edition of Jordanus' *De ratione ponderis*, corrected and illustrated by Tartaglia, and published under the title *Iordani opusculum de ponderositate Nicolai Tartaleae studio correctum novisque figuris auctum.*[2]

Chapter 7 starts with some brief critical remarks on Tartaglia's propositions 2 to 5. Tartaglia's proposition 2 essentially paraphrases and modifies the Aristotelian claim that the speed of moving bodies is proportional to the driving force. Following Jordanus, Tartaglia maintained that the velocities of descending heavy bodies of the same kind are proportional to their power (Italian: *potentia*) while in the case of ascending bodies their velocities are inversely proportional to their power. For bodies of the same kind their power is conceived here as proportional to their sizes, that is, to their weights. Descending bodies are thus simply falling bodies with velocities proportional to their weights, while in the case of ascending bodies their weight acts as a resistance. Tartaglia's proposition 3 generalizes proposition 2 for bodies with equal weights but unequal positional heaviness. His proposition 4 maintains that in the latter case the power of bodies attached to a balance is proportional to the distances from the center.

Benedetti's critical remarks are somewhat eclectic. He argued that Tartaglia, in his proposition 2, did not take into account the quantity of external resistance (*quanti momenti sint extrinsecae resistentiae*). With regard to Tartaglia's proposition 3 Benedetti pointed to its assumptions, namely that the bodies have to be homogenous and must have the same shape. He criticized that Tartaglia's proof does not actually require these assumptions but would be true also for heterogeneous bodies or for bodies with differing shapes. Concerning proposition 4 he criticized that Tartaglia did not prove what he claimed to prove. He should have rather followed Archimedes' proof of the law of the lever.

Benedetti's chapter 7 continues with a detailed discussion of the second part of Tartaglia's proposition 5 and the following two corollaries and is thus directly concerned with the equilibrium controversy. Tartaglia maintained in this proposition that a balance that is in equilibrium in a horizontal position will necessarily return to this horizontal position when moved into an oblique position. In a first corollary he claimed that the

[1] Tartaglia (1546).
[2] de Nemore (1565).

more the beam of a balance is brought into an oblique position, the more the bodies attached to it become positionally lighter. In a second corollary he claimed that while both bodies in this case become positionally lighter, the lifted body loses less of its positional heaviness than the body moving down. He concluded that the beam will return to a horizontal position. Benedetti questioned Tartaglia's approach by referring to the first three chapters of his own treatise, arguing in particular that Tartaglia's second corollary must be wrong. He discussed once more the beam of a balance in an oblique position, but now without the assumption that the lines of inclination of bodies attached to the beam of a balance are parallel. He rather considered the case that these lines are directed to the center of the world, showing, as we have discussed above in section 3.9, that not the lifted body but rather the body that is moved down loses less of its positional heaviness.

Benedetti continued in chapter 8 with critical comments on Tartaglia's propositions 6, 7, 8, and 14. Tartaglia's proposition 6 contains the proof of his fallacious claim that the lifted body of an oblique beam of a balance loses less of its positional heaviness than the body moving down, now modified by the further claim that the difference is smaller than any finite quantity. Tartaglia claimed:

> […] che la differenzia ch'è fra le gravità de questi dui corpi egli è impossibile a poterla dar, over trovar' fra due quantità inequali.
>
> […] that the differences between the heaviness of these two bodies is impossible to give or find between two unequal quantities.[3]

Like Guidobaldo had done before him, but with different results, Benedetti criticized Tartaglia for not taking into account that the lines of inclination are not parallel.

Tartaglia's proposition 7 contains the simple statement that if the arms of a balance are unequal and bodies with equal weights are attached to the ends of the beam the balance will tilt on the side with the longer arm. Benedetti criticized that Tartaglia again did not take into account that the lines of inclination are not parallel and that in any case Tartaglia did not give the correct cause of the effect.

Tartaglia's proposition 8 formulates, following Jordanus, the law of the lever in terms of positional heaviness, stating that if the lengths of the parts

[3] Tartaglia (1546, 91r). Translation in Drake and Drabkin (1969, 130).

of the beam of a balance with unequal arms are inversely proportional to the weights of the bodies attached to them, their positional heaviness will be equal. Benedetti criticized, just as Guidobaldo had done in his marginal notes to Jordanus, that this proposition is much better demonstrated by Archimedes. He added that therefore all the proofs of the propositions 9 to 13 are invalid.

Finally, Tartaglia's propositions 14 and 15 concern Jordanus' proof of the law of the inclined plane which, from a modern perspective, is essentially correct. Benedetti criticized, as we have also discussed in section 3.9, Tartaglia's argument by imputing to it an interpretation of the inclined plane as a balance, with the top of the plane being its center. His criticism based on the propositions of his chapters 1 to 3 thus completely missed the point of Tartaglia's argument.

6.3 Benedetti's criticism of Aristotle

Benedetti's treatise on mechanics continues mainly with critical notes on the Aristotelian *Mechanical Problems*[4] which constituted a key reference for mechanical arguments at the time.[5] His notes are as diverse as the Aristotelian *Mechanical Problems* themselves.

Before he embarked on this criticism, Benedetti dealt, in chapter 9, with the problem of why a steelyard carries a linear gradation.[6] He took into account the weight of the beam and that of the scale by postulating the equilibrium of the balance when no extra weight is added. Then he added weights of one pound on both sides, arguing that, by common science (*scientia communis*),[7] the balance stays in equilibrium if they are placed at equal distances from the fulcrum. He had thus found the mark on the beam that indicates a magnitude of one pound. He then successively placed further weights onto the scale, now arguing from the law of the lever that they must be compensated by distances proportional to their number. He thus avoided the problem of applying the law of the lever directly to a material steelyard, just as one does in practice when gauging such a balance.[8]

[4] Aristotle (1980). See subsection 3.4.1.
[5] See Rose and Drake (1971) and also the introduction to Nenci (2011a).
[6] Benedetti (1585, 152), page 339 in the present edition, see also Drake and Drabkin (1969, 178).
[7] In the sixteenth century the term *scientia communis* was used to designate knowledge common to all mathematical sciences, its core being the Euclidean theory of proportions. See Sepper (1996, 153–154).
[8] See the discussion in Damerow et al. (2002).

In chapters 10 and 11, Benedetti started with critical remarks on Aristotle's first problem. Aristotle asked why larger balances are more accurate than smaller ones.[9] Actually, this concrete physical question is not the focus of the extensive answer the author gave to this problem. He rather provided a long proof of the basic explanatory principle which plays a major role in the whole treatise (see section 3.4.1). At the end of the proof Aristotle argued that the same load will move faster on a larger balance thus making such balances more accurate.[10]

The criticism Benedetti passed on Aristotle's argument has two parts. In chapter 10 Benedetti began by rejecting Aristotle's claim that the circumference of a circle combines concavity with convexity. He then argued against a specific part of Aristotle's proof of his principle which involves the superposition of motions. In this part Aristotle showed that:

> Quandoquidem igitur in proportione fertur aliqua id, quod fertur, super rectam ferri necesse.
>
> […] whenever a body is moved in two directions in a fixed ratio it necessarily travels in a straight line.[11]

He concluded:

> Si autem in nulla fertur proportione secundum duas lationes nullo in tempore, rectam esse lationem est impossibile.
>
> […] if a body travels with two movements with no fixed ratio and in no fixed time, it would be impossible for it to travel in a straight line.[12]

For the Aristotelian author this proposition served as a means to describe circular motion as a result of two movements with no fixed ratio. Benedetti, however, did not relate his criticism to this context. He argued only that Aristotle's inference concerning movements in two directions is not sufficient since a straight movement can result from two quite different motions, a criticism which does not really relate to the Aristotelian argument, other than showing that his entire attempt to derive the behavior of a balance from a principle of circular motion is misguided.

In the same vein Benedetti's criticism in chapter 11 then deals directly with Aristotle's answer to the question of why larger balances are more

[9] Aristotle (1980, 337–347).
[10] Aristotle (1980, 347).
[11] Aristoteles (1585, 507). Translation in Aristotle (1980, 337).
[12] Aristoteles (1585, 508). Translation in Aristotle (1980, 339).

accurate than smaller ones. He argued that Aristotle's argument is not well founded since the greater accuracy has nothing to do with the motion of the beam of the balance but only with the geometrical constellation.[13] To conclude he added a consideration of material balances, arguing according to his own principles that a weight on the larger balance will be positionally more effective.

Benedetti's chapter 12 concerns problems 2 and 3 of the Aristotelian *Mechanical Problems*.[14] Problem 2 raises the question that forms the starting point of the equilibrium controversy:

> Cur siquidem sursum fuerit spartum, quando deorsum lato pondere, quispiam id admouet, rursum ascendit libra: si autem deorsum constitutum fuerit, non ascendit, sed manet?
>
> If the cord supporting a balance is fixed from above, when after the beam has inclined the weight is removed, the balance returns to its original position. If, however, it is supported from below, then it does not return to its original position. Why is this?[15]

Aristotle implicitly assumed that the beam of the balance has a certain thickness and weight. It follows as a result of the geometry of the balance in an oblique position that if the beam is fixed from above, a greater part of the beam is on the lifted side of the perpendicular line across the suspension point (see figure 3.37). Consequently the beam will move back by itself into the horizontal position. The opposite is true for a beam fixed from below. In this case the greater part of the beam is on the lower side so that it cannot move back into a horizontal position by itself.

Benedetti criticized that in the first case it is not only the weight of the beam that causes it to return to the horizontal position but also the different distances of the weights in an oblique position from the vertical through the point where the beam is fixed. According to his theory of the dependency of the weight on the obliqueness of the beam, the weights must be different on both sides. Benedetti thus generalized Aristotle's argument to the case of a balance without a material beam carrying weight itself.

In the second case of a beam supported from below, he argued that Aristotle is completely mistaken. He rightly maintained that the beam

[13] Benedetti (1585, 153), page 341 in the present edition; Drake and Drabkin (1969, 180–182).
[14] Aristotle (1980, 347–355); Drake and Drabkin (1969, 182–183).
[15] Aristoteles (1585, 511). Translation in Aristotle (1980, 347–349).

will not remain in its oblique position, but that the lower part will further move down until the beam is below the point where it is fixed.

Problem 3 of the Aristotelian *Mechanical Problems*[16] concerning an explanation of the effect of a lever is, to Benedetti, not worth the effort of a detailed criticism. He only briefly notes that Aristotle did not give the true cause which one will find in his own theory presented in chapters 4 and 5.[17]

In the very short chapter 13, Benedetti criticized problem 6 of the Aristotelian *Mechanical Problems*:

> Cur quando antenna sublimior fuerit, iisdem velis, et vento eodem celerius feruntur navigia?
>
> Why is it that the higher the yard-arm, the faster the ship travels with the same sail and the same wind?[18]

The Aristotelian answer provided in the *Mechanical Problems* is based on the interpretation of the yard-arm as a lever having its base where the yard-arm is fixed as the fulcrum. Benedetti maintained that this interpretation of the yard-arm as a lever:

> [...] verum non est. Huiusmodi enim ratione navis tardius potius, quam velocius ferri deberet, quia quanto altius est velum, vi venti impulsum, tanto magis proram ipsius navis in aquam demergit.
>
> [...] does not give the true explanation. For on this kind of explanation the ship would have to move more slowly rather than more swiftly. For the higher is the sail that is struck by the force of the wind, the more is the ship's prow submerged in the water.[19]

Benedetti added one sentence with his own explanation according to which the ship with a higher sail moves more swiftly because the wind blows more strongly in the higher region.

Chapter 14 provides a long discussion of problem 8 of the Aristotelian *Mechanical Problems*. The question posed in this problem is why round

[16] Aristotle (1980, 353–355).

[17] Benedetti (1585, 154), page 342 in the present edition; Drake and Drabkin (1969, 183).

[18] Aristoteles (1585, 515). Translation in Aristotle (1980, 361).

[19] Benedetti (1585, 155), page 343 in the present edition. Translation in Drake and Drabkin (1969, 183).

and circular bodies are easiest to move. Three examples are mentioned and later discussed: the wheels of a carriage, the wheels of a pulley, and the potter's wheel. Benedetti claimed that Aristotle's answer to the question he posed is not sufficient. Nevertheless Benedetti himself argued essentially in a similar manner, only somewhat more extensively. Both of them argued that the circle, contrary to differently shaped bodies, touches a plane only at one point which can be considered as the fulcrum of a lever. But Benedetti added a further argument which is not given by Aristotle. He argued that a circle can be pulled along a plane without difficulty and resistance:

> […] quia huiusmodi centrum ab inferiori parte ad superiorem, nunquam mutabit situm respectu distantiae seu interualli, quae inter ipsum lineamque *AD* intercedit.
>
> […] because in such a case the center will never change its position by moving upward from below, i.e., will never change its position with respect to the distance or interval which lies beweeen it and line *AD*.[20]

At the end of the chapter Benedetti discussed the question of why a potter's wheel set into motion by an external force will continue to rotate for a time, but not forever. In his response he took into account the friction with the support of the wheel and with the surrounding air. But he also discussed reasons that are more deeply concerned with the nature of such motion. He claimed, in particular, that the rotational motion is not a *natural motion* of the wheel, evidently making reference to the Aristotelian distinction between natural and violent motions. He also claimed that a body moving by itself because an *impetus* has been impressed upon it by an external force has a natural tendency to move along a rectilinear path. This statement comes close to the principle of inertia of classical physics, although it actually deals with rectilinear motion as a forced motion and does not involve any assertion about its uniformity. Benedetti seems to suggest, in any case, that this natural tendency is in conflict with the forced rotational motion of the wheel, thus slowing it down, and the more so, the smaller the wheel and the more its parts are constrained to deviate from the rectilinear path.[21]

In chapters 15 and 16 Benedetti dealt with issues of scale as they are brought up by the Aristotelian *Mechanical Problems*. In chapter 15,

[20] Benedetti (1585, 155), page 343 in the present edition. Translation in Drake and Drabkin (1969, 184).

[21] For the historical context, see Büttner (2008).

consisting merely of one short sentence, Benedetti referred to his own earlier treatment of Aristotle's question of why larger balances are more exact (erroneously citing chapter 10 instead of chapter 11 of his treatise) in order to deal with the ninth problem of the Aristotelian *Mechanical Problems* which reads:

> Cur ea, quae per maiores circulos tolluntur et trahuntur, facilius et citius moveri contingit [...]?
>
> Why is it that we can move more easily and more quickly things raised and drawn by means of greater circles?[22]

In chapter 16 he discussed the tenth problem of the Aristotelian *Mechanical Problems* which reads:

> Cur facilius quando sine pondere est, movetur libra, quam cum pondus habet?
>
> Why is a balance moved more easily when it is without a weight than when it has one?[23]

In his detailed response to this problem – indeed much more detailed than the one found in the Aristotelian text – Benedetti compared two like balances with different sets of weights on their scales, one with two weights of one ounce, the other with two weights of one pound. He then added a half-ounce weight on one side of each balance and observed that the balance with the smaller weights moves more rapidly. He explained this effect by referring to the dynamical assumption that one always has to consider *the ratio of the moving force to the body moved.*

In chapter 17 Benedetti addressed the twelfth problem of the Aristotelian *Mechanical Problems* which reads:

> Cur longius feruntur missilia funda, quam manu missa [...]?
>
> Why does a missile travel further from the sling than from the hand?[24]

Benedetti's response is based on the concept of *impetus*, conceived as an intrinsic cause of motion originally acquired by the action of an external force that then gradually decreases after separation from the original

[22] Aristoteles (1585, 517). Translation in Aristotle (1980, 365).
[23] Aristoteles (1585, 517). Translation in Aristotle (1980, 365).
[24] Aristoteles (1585, 518). Translation in Aristotle (1980, 367).

mover. He argued that a greater impetus can be impressed by the sling due to the repeated revolutions which evidently lead to an accumulation of this intrinsic force. He observed that the impetus would lead, if not impeded by the sling or the hand, to a straight motion of the projectile along the tangent to the circle of its forced motion. He also noted – distancing himself from a claim made by Tartaglia – that the motion due to the impressed force can mingle with the projectile's natural motion downward, thus leading to a curved trajectory. It may well be the case that it was this claim that later induced Galileo and Guidobaldo to perform their experiment on projectile motion from which they drew the conclusion that such a mixture of motions indeed takes place.[25]

In chapter 18 Benedetti considered problem 13 of the Aristotelian *Mechanical Problems* dealing with the question of why larger handles can be moved more easily around a spindle than smaller ones.[26] In his short response Benedetti simply referred to the fourth and fifth chapters of his own treatise, stressing that everything depends on the lever and was evidently convinced that the Aristotelian reduction of such problems to properties of the circle is superfluous if not misguided.

In chapter 19 he handled in the same way problem 14 of the Aristotelian *Mechanical Problems* which reads:

> Cur eiusdem magnitudinis lignum facilius genus frangitur, si quispiam aequi diductis [deductis] manibus extrema comprehendens fregerit, quam si iuxta genu?
>
> Why is a piece of wood of equal size more easily broken over the knee, if one holds it at equal distance far away from the knee to break it, than if one holds it by the knee and quite close to it?[27]

Again Benedetti just referred to the earlier chapters of his treatise.

In chapter 20 Benedetti reconsidered problem 17 of the Aristotelian *Mechanical Problems* which reads:

> Cur a parvo existente cuneo magna scinduntur pondera, et corporum moles, validaque sit impressio?
>
> Why are great weights and bodies of considerable size split by a small wedge, and why does it exert great pressure?[28]

[25] See the discussion in Renn et al. (2001).
[26] Aristotle (1980, 367).
[27] Aristoteles (1585, 518). Translation in Aristotle (1980, 369).
[28] Aristoteles (1585, 520). Translation in Aristotle (1980, 371).

The answer is based on interpreting the wedge as two levers opposite to each other, their fulcra being placed at the entry points of the wedge into the wood. Benedetti disagreed with the identification of the two levers allowing the action of the wedge to be interpreted in terms of force, fulcrum, and resistance. He claimed that the fulcrum is actually placed just underneath the deepest point of the opening produced by the wedge entering a block of wood.

In chapter 21 Benedetti claimed to provide the true explanation of compound pulleys. He reduced a compound pulley to a chain of balances by appropriately identifying forces and fulcra, each wheel of the pulley corresponding to one balance.

In chapter 22 Benedetti discussed Aristotle's wheel, i.e. problem 24 of the Aristotelian *Mechanical Problems* which reads:

> Dubitatur quam ob causam maior circulus aequalem minori circulo convolvitur lineam, quando circa idem centrum fuerint positi.
>
> A difficulty arises as to how it is that a greater circle when it revolves traces out a path of the same length as a smaller circle, if the two are concentric.[29]

While the author of the *Mechanical Problems* referred to dynamical reasons in explaining this apparent paradox, Benedetti resorted to a kinematic argument, a pointwise reconstruction of the trajectory of the motion of a point on the circumference, arguing that it results from a superposition of two motions. In the case in which the motion is controlled by the larger circle, a point on the circumference of the smaller circle traverses a path resulting from an *addition* of two motions. In the case in which the motion is controlled by the smaller circle, a point on the circumference of the larger circle traverses a path resulting from a *subtraction* of two motions.

Chapter 23 of Benedetti's treatise does not exist.[30] In chapter 24 Benedetti discussed problem 30 of the Aristotelian *Mechanical Problems* which reads:

> Cur surgentes omnes, femori crus ad acutum constituentes angulum, et thoraci similiter femur, surgunt?

[29] Aristoteles (1585, 525). Translation in Aristotle (1980, 387).

[30] In Drake and Drabkin (1969, 193) chapter 22 is erroneously numbered as chapter 23.

> Why is it that, when men stand up, they rise by making an acute angle between the lower leg and the thigh, and between the trunk and the thigh?[31]

In his response Benedetti suggested that the reason for this behavior is to create an equilibrium of the body with regard to the line that serves as support underfoot.

In chapter 25 Benedetti addressed the last problem, problem 35 of the Aristotelian *Mechanical Problems* which reads:

> Cur ea quae in vorticosis feruntur aquis, ad medium tandem aguntur omnia?
>
> Why do objects which are travelling in eddying water all finish their movement in the middle?[32]

Benedetti's answer simply referred to the fact that whirlpools are depressed in their middle without giving an explanation of this phenomenon. He could thus restrict himself to arguing that the motion of an object to the center of such a whirlpool is simply its natural downward motion. Remarkable is the final comment by Benedetti, concluding his criticism of Aristotle as well as his treatise on mechanics:

> [...] a quo aliarum omnium quaestionum, quas ego omisi rationes sunt bene propositae.
>
> But in the case of all those other problems that I have omitted, Aristotle's explanations are correct.[33]

[31] Aristoteles (1585, 532). Translation in Aristotle (1980, 403–405).
[32] Aristoteles (1585, 533). Translation in Aristotle (1980, 409).
[33] Benedetti (1585, 167), page 355 in the present edition. Translation in Drake and Drabkin (1969, 196).

Chapter 7
Guidobaldo's Marginal Notes in Benedetti's Book

Guidobaldo's marginal notes in Benedetti's book illustrate a case where different conceptual frameworks are applied to similar mechanical problems and thus are testimony to a competition taking place within the same territory. More generally, these marginal notes reveal the potential tensions and conflicts inherent in the bodies of knowledge carried over from antiquity when these are elaborated, integrated with each other, and applied to the challenging objects of preclassical mechanics (see sections 3.3 and 3.4.3). From an analysis of the marginal notes it becomes clear that Guidobaldo considered Benedetti's entire approach as being misguided. On the one hand, he repeatedly claimed that Benedetti had taken over propositions from his own book on mechanics, such as the assertion that a balance with equal arms would, when deflected from the horizontal position, not return to its original position. On the other hand, he found that the conceptual foundation of Benedetti's approach, based on determining positional heaviness by a perpendicular to the line of inclination, is untenable, leading him to false or problematic claims. In summary, according to Guidobaldo, everything that Benedetti had accomplished was either plagiarized or simply wrong.

The different conceptual foundations of Guidobaldo's and Benedetti's mechanics also account for the different status of certain problems within the theoretical frameworks of their treatises. Thus, while the case of a bent lever and the case of a balance with a weight on one side and a force acting in an arbitrary direction on the other side are rather central to Benedetti, such situations receive at best a marginal treatment in the deductive part of Guidobaldo's treatise. Benedetti's framework was evidently more apt than Guidobaldo's to deal with cases in which force and lever arm are not perpendicular to each other, or more generally, with cases in which the acting force and the path of the motion constrained by a mechanical device are not parallel as is the case, for instance, for the inclined plane. While this potential was not fully exploited in Benedetti's treatise, not least because of the sketchiness of some of his proofs, it became essential to Galileo's more rigorous and systematic treatment of mechanics modeled after that of Guidobaldo, as well as to the theory of motion built upon it (see section 3.10). Guidobaldo's marginal notes on Benedetti's treatise thus not only illustrate the clash of their different perspectives, but also the developmental potential inherent in this clash. They allow us to witness the beginnings of a process in which the heterogeneous conceptual traditions of early modern mechanics were eventually fused to constitute the framework of classical mechanics in the context of a scientific controversy (see section 1.3).

Guidobaldo's notes are presented here in the context of those passages of Benedetti's text to which they refer. Several parts of the marginalia have been deleted by Guidobaldo himself. Other parts have later been cut off by a book binder. Also, we have been unable to read all of his handwriting.[1]

7.1 First chapter: the general charge of plagiarism

Benedetti started the part of *Diversarum speculationum [...] liber* that deals with mechanics after a short introduction with a first chapter entitled:

> De differentia situs brachiorum librae.
>
> On the difference in the position of the arms of a balance.[2]

It has been mentioned above that Benedetti sets out in this first chapter to explain the idea of positional heaviness by stating:

> Omne pondus positum in extremitate alicuius brachii librae maiorem, aut minorem gravitatem habet, pro diversa ratione situs ipsius brachii.
>
> Every weight placed at the end of an arm of a balance has a greater or a lesser heaviness depending on differences in the position of the arm itself.[3]

The rest of the chapter elaborates on this idea.

In the introduction which precedes this chapter Benedetti claimed that he presents material that has either never been dealt with previously or has

[1] The transcriptions take into account the samples given by Anthony Grafton in the prospectus of the auction house (Catalogue 38 of Martayan Lan). For an analysis of the deletions, see the appendix.

[2] Benedetti (1585, 141–142), pages 329–330 in the present edition. Translation in Drake and Drabkin (1969, 166–167).

[3] Benedetti (1585, 141), page 329 in the present edition. Translation in Drake and Drabkin (1969, 166).

not been sufficiently explained. It is to this bold claim that Guidobaldo reacts in his first marginal note, written after the beginning of the first chapter. The note is placed in the right margin of the page at the height where the text of the first chapter begins. It refers to the chapter as a whole, claiming that it had been taken entirely from his own book:

> Hoc primum caput to[tum] desumptum est a n[ostro] mechanicorum libr[o] tractatu de lib[ra].
>
> This entire first chapter is taken from our book on mechanics, from the treatise on the balance.

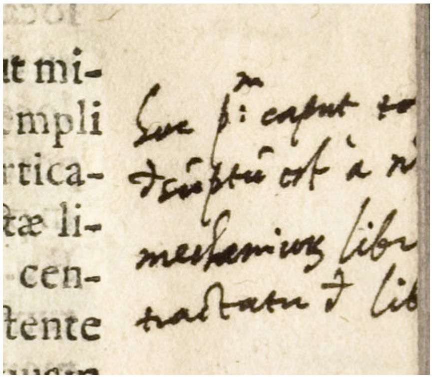

Figure 7.1: The first marginal note.

As this marginal comment suggests, Guidobaldo was evidently convinced that not only had he derived all the relevant theorems in his own book but also addressed what he saw as the problematic character of the concept of *positional heaviness* in Jordanus, Tartaglia, and Cardano. As Benedetti's approach corresponds to one of the options of Cardano (see section 3.7), he was apparently under the erroneous impression that he had thus dealt with Benedetti's approach as well.

7.2 Second chapter: the neglect of the cosmological context

Benedetti's second chapter is entitled:

> De proportione ponderis extremitatis brachii librae in diverso situ ab horizontali.

> On the ratio of the weight at the extremity of the arm of a balance in various positions with respect to the horizontal.[4]

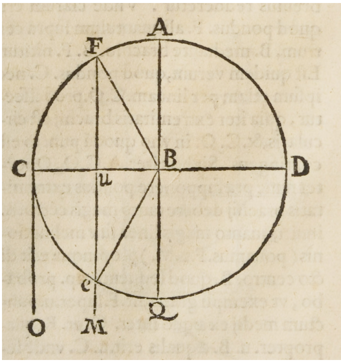

Figure 7.2: Figure at the beginning of the second chapter.

The chapter considers, as we have discussed, the changing effect of a weight in different positions of the arm of a balance. Guidobaldo began his marginal note on the lower left margin of the page opening the chapter and continued it at the bottom. The note refers to the concluding sentence on the page:

> Unde fit ut hoc modo pondus magis aut minus a centro pendet, aut eidem nititur: atque haec est causa proxima, et per se, qua fit ut unum idemque pondus in uno eodemque medio magis aut minus grave existat.
>
> Hence it results that in this way a weight hangs more or less from the center or is sustained by it. And this is the proximate and essential cause why it happens that one and the same weight in one and the same medium is more or less heavy.[5]

In his comment, Guidobaldo questioned the basic geometrical set-up of Benedetti's argument because it does not take into account that the perpendicular lines are not parallel but have to meet at the center of the

[4]Benedetti (1585, 142–143), pages 330–331 in the present edition. Translation in Drake and Drabkin (1969, 168–169).

[5]Benedetti (1585, 142), page 330 in the present edition. Translation modified from Drake and Drabkin (1969, 169).

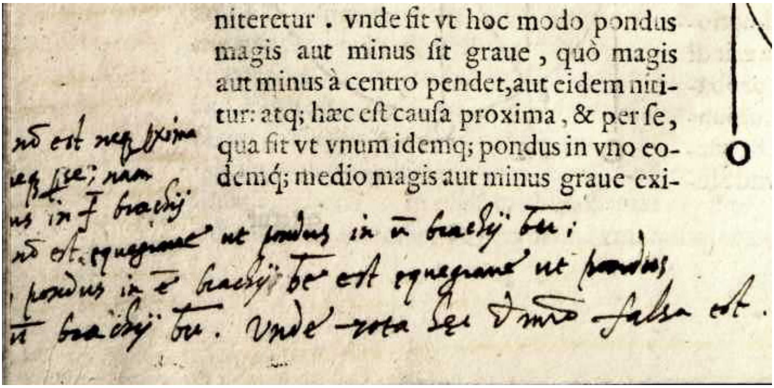
niteretur . vnde ſit vt hoc modo pondus
magis aut minus ſit graue , quò magis
aut minus à centro pendet,aut eidem niti-
tur: atq; hæc eſt cauſa proxima , & per ſe,
qua ſit vt vnum idemq; pondus in vno eo-
demq; medio magis aut minus graue exi-

Figure 7.3: Marginal note to the second chapter.

world. As we have discussed, in his own book, he had extensively criticized Tartaglia's approach in a similar way – not because of an overemphasis on precision, but for reasons of logical consistency enforced by Tartaglia's conceptualization of oblique descents.

> non est neque proxima neque per se; nam [pond]us in *F* brachii [*BF*] non est equegrave ut pondus in *U* brachii *BU* ; [nec] pondus in *E* brachii *BE* est equegrave ut pondus [in] *U* brachii *BU*. Unde tota haec demonstratio falsa est.
>
> because that [i.e. the greater or smaller extent to which a weight rests on the center] is neither the next [cause] nor the [cause] by itself. For the weight at *F* of the arm *BF* is not equally heavy as the weight *U* of the arm *BU*; nor is the weight at *E* of the arm *BE* equally heavy as the weight at *U* of the arm *BU*. Whence this entire demonstration is false.

Guidobaldo thoroughly examined what he considered to be the problematic foundation of Benedetti's mechanics also in his research notebook, the *Meditatiunculae*, under the heading:[6]

> Contra Cap. 2 Jo. de Benedicti de Mechanicis
>
> Against chapter 2 of Giovanni Benedetti's [treatise] on Mechanics

[6] See DelMonte (1587, 145).

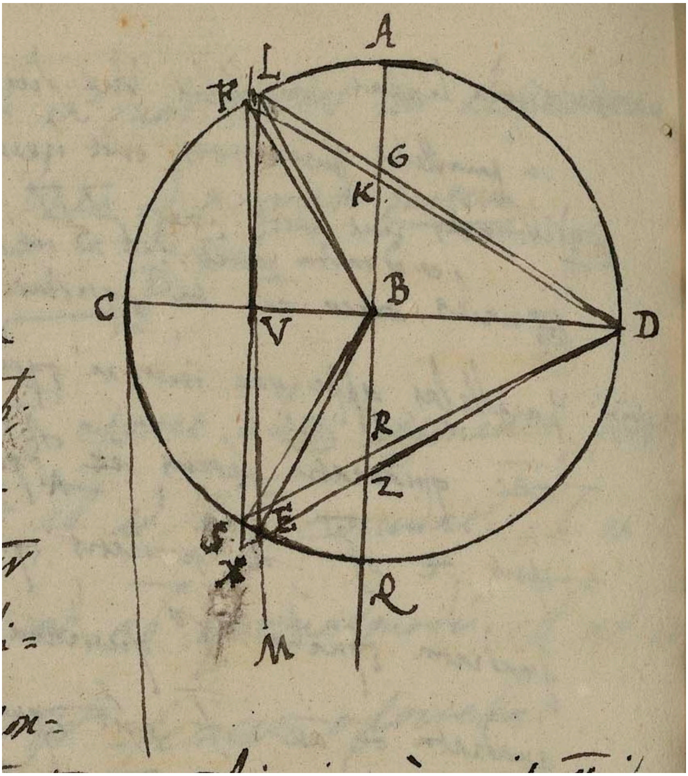

Figure 7.4: In his notebook Guidobaldo reconsidered Benedetti's analysis of the bent lever, conceived as a balance with one horizontal arm *BD* and an oblique arm in the positions *BF* or *BE*. He stressed the difference between Benedetti's treatment and a treatment that takes into account the finite distance of the bent lever from the center of the world. For this purpose, Guidobaldo compared the line *LUS* parallel to the line *AQ*, connecting the fulcrum *B* of the balance with the center of the world, with the line *FM* connecting the weights on the beam with the center of the world. He concluded that it is the weight at *S*, at the point where the line *LUS* meets the circle the beam describes around the fulcrum, and not, as claimed by Benedetti, the lower weight at *E*, that will be equally heavy as the weight at *U*.

He addressed Benedetti's claims by reconstructing them from the perspective of his own conceptual framework based on the concept of center of gravity. As indicated in his marginal notes, Guidobaldo rejected Benedetti's approach because it supposedly did not take into account the finite distance of the weights from the center of the world and hence the fact that the plumb lines are not parallel to each other.

In his diagram, Guidobaldo compared the line LUS parallel to the line AQ through the fulcrum with the line FM connecting the upper weight F with the center of the world (see figure 7.4). S is the point where the line LUS meets the circle the beam describes around the fulcrum, which is above the position of the lower weight E. He next considered a bent lever made of the oblique arm BS, rigidly connected to the straight arm BD, assuming that BU is half BD.

If now a weight is placed at S which is double the weight at D, the bent lever will be in equilibrium, as Guidobaldo showed with reference to his book, because the center of gravity of the weights at S and at D will be at the point R, which will be in its lowest place on the vertical line BQ.

He then concluded that it is the weight at S, but not the lower weight E, that will be equally heavy as the weight at U. He proceeded to demonstrate this in greater detail by considering the proportions in which the line connecting the two weights of the bent lever is cut by the perpendicular BQ for the two cases, i.e. the weight being placed at S and weight being placed at E.

Guidobaldo concluded that the same weight is heavier at S than at E. He then turned to a closer consideration of the upper weight F. Again he constructed a bent lever LBD in equilibrium in order to compare it with the bent lever formed with the upper weight F. And again he showed that the weight is heavier at L than at F, concluding:

> Et quibus etiam constat idem pondus in F, et in U, et in E, diversi modo gravitare. Gravius est enim in situ E quam in U et in F. In U vero gravius, quam in F.
>
> From this it is also clear that the weight at F, at U, and at E gravitates in a different way. It is namely heavier in the position E than it is at U and at F. But at U it is heavier than at F.[7]

Finally, he summarized in almost the same words as in his marginal comment quoted above:

[7] DelMonte (1587, 145).

> Veluti quoque falsum est propter filum pondus in *E* est aequegrave, ut pondus in *U* brachii *BU*. Non est igitur haec vera et proxima causa, et per se harum gravitatum. Ut ipse profitetur.
>
> In the same way it is also false that because of the thread the weight at *E* is equally heavy as the weight at *U* on the arm *BU*. This is therefore not the true and next cause, nor the essential [cause] of these gravities. As he himself admits.[8]

7.3 Third chapter: the pitfalls of determining positional heaviness

The third chapter is entitled:

> Quod quantitas cuiuslibet ponderis, aut virtus movens respectu alterius quantitatis cognoscatur beneficio perpendicularium ductarum a centro librae ad lineam inclinationis.
>
> That the magnitude of one given weight or the magnitude of one motive force in comparison with another can be found by means of perpendiculars drawn from the center of the balance to the line of inclination.[9]

The title summarizes the gist of Benedetti's procedure for determining positional heaviness. Guidobaldo left two comments in the right margin of the page opening the chapter – the first short, the second long and with deletions.

Guidobaldo's first comment refers to the figure on the preceding page of Benedetti's treatise to which in turn the first sentence of chapter 3 refers (see figure 7.2):

> Ex iis, quae a nobis hucusque sunt dicta, facile intellegi potest, quod quantitas *BU* quae fere perpendicularis est a centro *B* ad lineam *FU* inclinationis, ea est, quae nos ducit in cognitionem quantitatis virtutis ipsius *F* in huiusmodi situ, constituens videlicet linea *FU* cum brachio *FB* angulum acutum *BFU*.
>
> From what we have already shown it may easily be understood that the length of *BU*, which is virtually perpendicular from

[8]DelMonte (1587, 145).

[9]Benedetti (1585, 143), page 331 in the present edition. Translation in Drake and Drabkin (1969, 169–170).

the center B to the line of inclination FU, is the quantity that enables us to measure the force of F itself in a position of this kind, i.e., a position in which line FU constitutes with arm FB the acute angle BFU.[10]

The point of Guidobaldo's first short comment is probably the same as that of the preceding comment: to stress that Benedetti's diagram fails to take into account that the vertical lines have to actually converge at the center of the world.

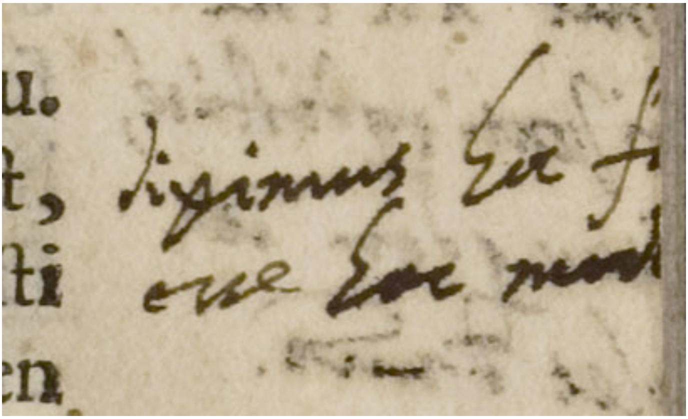

Figure 7.5: Note at the beginning of the third chapter.

Guidobaldo's first comment reads:

diximus hoc f[iguram] esse hoc mod[o]

We said that this figure is in this way

Guidobaldo's second comment, beginning in the lower right margin and continuing at the bottom of this page, refers to the interpretation of the figure at the bottom of the page and to the argument beginning with the second sentence of the chapter which reads (see figure 7.6):

Ut hoc tamen melius intelligamus, imaginemur libram BOA fixam in centro O ad cuius extrema sint appensa duo pondera,

[10] Benedetti (1585, 143), page 331 in the present edition. Translation in Drake and Drabkin (1969, 169).

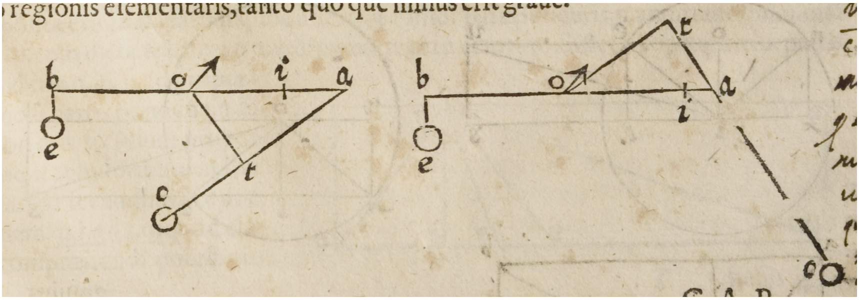

Figure 7.6: Benedetti's bent lever with forces acting along the oblique lines *AC*.

> aut duo virtutes moventes *E* et *C* ita tamen quod linea inclinationis *E* idest *BE* faciat angulum rectum cum *OB* in puncto *B* linea vero inclinationis *C* idest *AC* faciat angulum acutum, aut obtusum cum *OA* in puncto *A*. Imaginemur ergo lineam *OT* perpendicularem lineae *CA* inclinationis [...]
>
> To understand this better, let us imagine a balance *BOA* fixed at its center *O*, and suppose that at its extremities two weights are attached, or two moving forces, *E* and *C*, in such a way that the line of inclination of *E*, that is, *BE*, makes a right angle with *OB* at point *B*, but the line of inclination of *C*, that is, *AC*, makes an acute angle or an obtuse angle with *OA* at point *A*. Let us imagine, then, a line *OT* perpendicular to the line of inclination *CA* [...][11]

Guidobaldo's second comment thus refers to Benedetti's analysis of the case of a balance with a weight on one side and a force acting in an oblique direction on the other side:

> si intelligamus p[ondus] in *C*, ut supponi p[otest] ex verbis ipsius, intelligendum est *C*[*T*] quoque consolidatam consolidatis *TO* [...]. Unde si intelligamus *C* pondus et non movens, falsa est i[ta]que si intelligatur *C* movens ut homi[...] vera esse pote[st] quod [deleted: non] moveat non esse pondus s[i...] ipse

[11] Benedetti (1585, 143), page 331 in the present edition. Translation in Drake and Drabkin (1969, 169–170).

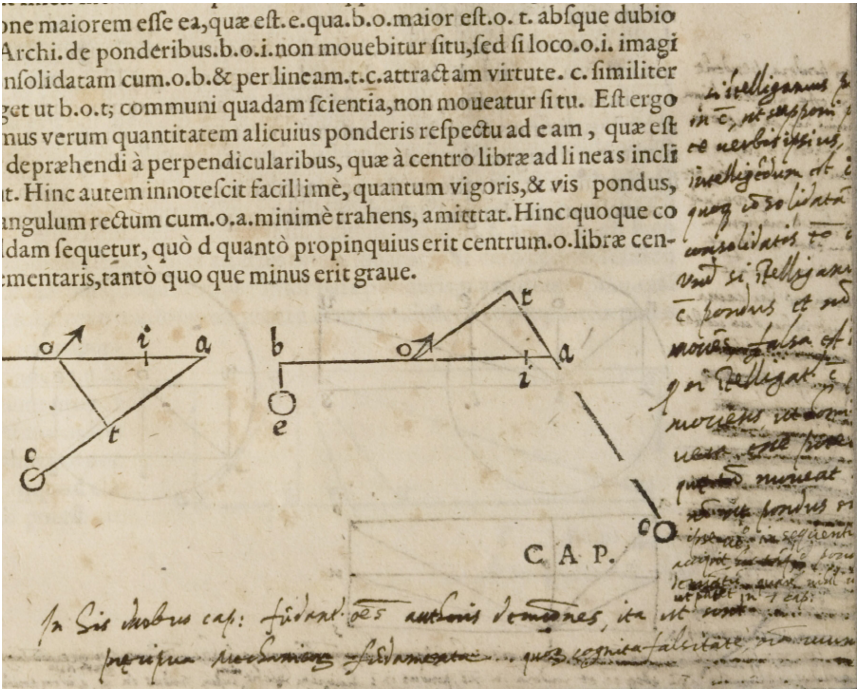
ne maiorem eſſe ea,quæ eſt.e.qua.b.o.maior eſt.o. t. abſque dubio
Archi.de ponderibus.b.o.i.non mouebitur ſitu,ſed ſi loco.o.i. imagi
nſolidatam cum.o.b.& per lineam.t.c.attractam virtute. c. ſimiliter
get ut b.o.t; communi quadam ſcientia,non moueatur ſitu. Eſt ergo
nus verum quantitatem alicuius ponderis reſpectu ad eam , quæ eſt
depræhendi à perpendicularibus, quæ à centro libræ ad li neas incli
t. Hinc autem innoteſcit facillimè, quantum vigoris,& vis pondus,
ngulum rectum cum.o.a.minimè trahens, amitttat.Hinc quoque co
dam ſequetur, quò d quantò propinquius erit centrum.o.libræ cen-
mentaris,tantò quo que minus erit graue.

Figure 7.7: Note beside drawings of bent levers in the third chapter.

[vero] in sequenti accipiat [hoc atque ponderi?] posse demonstratum quare nihil [...] ut patet in 7 cap.

In his duobus cap. fundantur omnes authoris demonstrationes ita ut sunt praecipua mechanicorum fundamenta quorum cognita falsitate omnia rem[oventur.]

If we understand that a weight is at C, as we can assume from his own words, then CT must also be understood as being solid [and connected with] the solid lines TO [...] If we hence understand that C is a weight and not moving, [the proposition] is false. If it is understood that C moves as [...] of a man, it can be true, since what moves is not a weight. [But] if he himself assumes in the following that [this] can be demonstrated [also for a weight], nothing [...] therefore as is evident in chapter 7.

> On these two chapters all demonstrations of the author are founded inasmuch as they are the first fundaments of mechanics; once their falsity is recognized, everything is rejected.

This comment illustrates Guidobaldo's difficulties in coping with a subject that was apparently unfamiliar to him. Generalizing from the case of the bent lever treated in chapter 2, Benedetti argued, as we have discussed in section 3.9, that the magnitude of a weight or a force can be found by means of perpendiculars drawn from the center of the balance to the line in which the weight or the force acts, that is, the line of inclination which does not, however, have to be a perpendicular. Now this generalization raised problems for Guidobaldo: must this line of inclination be understood as the solid arm of a bent lever with a weight attached to it at the lower end, thus generating a pull downward to the center of the world? Then Benedetti's conclusion would be wrong. Or can the line of inclination also represent a moving force, for instance, the pull of a man acting on the handles of a wheel? Then Benedetti's conclusion may actually be correct.

In his second marginal comment on this page, as well as on the related page in the *Meditatiunculae*, which we shall discuss immediately below, we see Guidobaldo struggling with these two possibilities. Apparently Guidobaldo believed that while Benedetti's procedure may be applicable to the case of moving forces, it was certainly false for weights tending to the center of the world. In his marginal comment Guidobaldo referred to chapter 7 of Benedetti's treatise, probably because it served as evidence that Benedetti applied this procedure not only to forces but also to weights. As we have discussed above, Benedetti had criticized Tartaglia and Jordanus in this chapter and offered a new analysis of the behavior of a balance removed from its horizontal position. When taking into account that the lines of inclination of the two weights on the balance have to converge at the center of the earth, Benedetti had come to the conclusion – by applying the incriminated procedure – that their positional heaviness must be different, a conclusion with which Guidobaldo could obviously not agree.

Then, in his final comment concluding the second marginal note, Guidobaldo summarized his conviction that the entire foundation of Benedetti's approach, as outlined in the first two chapters of his book, is untenable.

In his research notebook Guidobaldo dealt at even greater length with the same problem. He began his notes with the following comment:

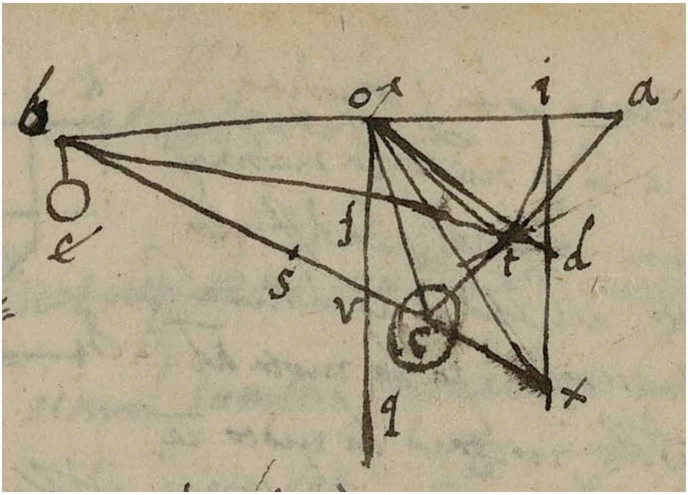

Figure 7.8: In his notebook, Guidobaldo attempted to refute Benedetti's determination of the positional effect of forces acting in an arbitrary direction under the erroneous assumption that such forces can be replaced by weights. Following Benedetti, he considered a broken bent lever *BOAC* with fulcrum *O*. For the case of an acute angle *BAC*, he showed that this broken bent lever cannot be in equilibrium because its center of gravity *S* can never fall on the perpendicular line *OU* through the fulcrum.

> Falsum est igitur ex dictis, quod in principio tertii capitoli inquit. Praeterea demonstratio falsa quoque videtur.
>
> From what has been said, what he claims in the beginning of the third chapter is therefore false. Moreover also the demonstration appears to be wrong.[12]

Guidobaldo extensively refuted Benedetti's procedure under the erroneous assumption that the latter had claimed that forces can indiscriminately be replaced by weights. In particular, Guidobaldo considered a broken bent lever *BOAC* with fulcrum *O*, weights *E* and *C*, straight arm

[12] DelMonte (1587, 146).

BO, and broken arm *OAC*, just as Benedetti had discussed it for the two cases of an acute and an obtuse angle *BAC* (see figures 7.8 and 7.9).

He first recapitulated Benedetti's procedure, assuming that a vertical line *OT* be drawn from the fulcrum to the line *AC* representing the oblique end of the bent lever. He stated that when the weight *C* is instead placed at the end of the horizontal line *OI*, whose length is the same as that of the perpendicular *OT*, it will, according to Benedetti, be in equilibrium with the weight *E*, if the weight *C* is to the weight *E* as is *BO* to *OT* or *OI*. Guidobaldo then summarized that Benedetti claimed that also the bent lever formed by the straight arm *BO* and the oblique arm *OTC*, where a force represented by the weight *C* acts along the line *TC*, will be in equilibrium, which he doubted. Ironically referring to Benedetti's use of the term *common science*, he wrote:

> Fateor me hanc quamdam communem scientiam non intelligere.
>
> I admit that I do not understand this certain common science.[13]

Guidobaldo reformulated Benedetti's claim by stating that the same weight *C* will be in equilibrium with the weight *E*, whether it is placed on the straight balance *BOI* or on the broken bent lever *BOTC*. He thus replaced Benedetti's conception of a force acting along an oblique line with that of a weight always tending downward and necessarily arrived at absurd conclusions.

Guidobaldo showed in particular that the same weight will be heavier on the horizontal at the point *I* than along the bent lever at *T*, demonstrating that the bent lever *TOB* will not be in equilibrium if the straight lever *BOI* is in equilibrium. In order to demonstrate this, he again proceeded by finding the center of gravity of the weights *E* and *C* placed at *T*. More precisely, Guidobaldo determined a position for the weight *C* in which the bent lever is in equilibrium, a position, however, that is distinct from *T*, so that it follows that *T* cannot be the equilibrium position for this weight. For this purpose, he prolongued the line *BT* to *D*, just underneath *I*, so that it is immediately evident that, if the weight *C* is placed at *D*, the center of gravity of the two weights will be just underneath the fulcrum.

He then continued to show by the same pattern that also the bent lever *BOC* cannot be in equilibrium because its center of gravity *S* can never fall on the perpendicular line *OU* through the fulcrum. And finally he extended this argument to the broken bent lever *BOTC*.

[13]DelMonte (1587, 146).

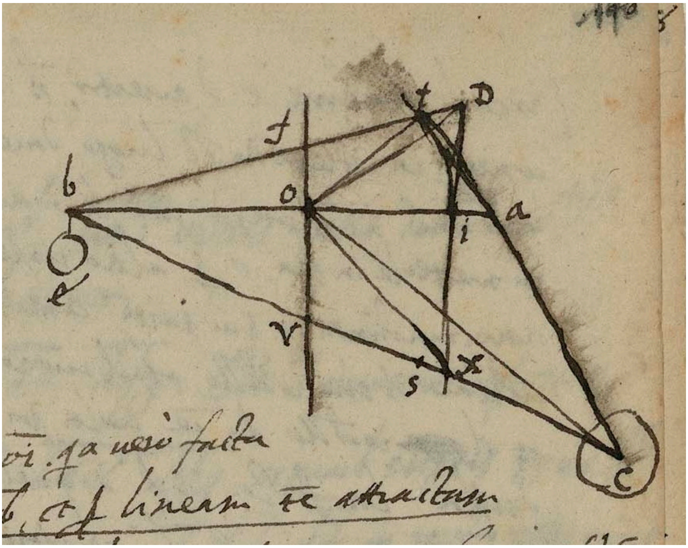

Figure 7.9: Following Benedetti, Guidobaldo considered in his notebook the broken bent lever $BOAC$ also for the case of an obtuse angle BAC, under the assumption that E and C are weights. He again came to the conclusion that their center of gravity S can never fall on the perpendicular line OU through the fulcrum and that hence the lever cannot be in equilibrium.

Guidobaldo next addressed the case in which the bent lever is characterized by an obtuse angle BAC, showing that the weight at T has a smaller heaviness than the weight at I (see figure 7.9).

In his concluding remarks, however, he began to waver. He once again stated that Benedetti is completely in error when applying his procedure to weights. But he admitted that it may be when one is dealing with a force:

> Falsa igitur est demonstratio. Fallacia vero est, cum inquit, continget, ut BOT communi quadam scientia, non moveatur situ.
>
> Et est omnino falsum si intelligatur C esse pondus, quod in centrum mundi sempre tendit. Ut ipse supponere videtur. Et

> ut ipse in seguentibus capitolis accipit hoc tamquam de ponderibus demonstratum.
>
> At vero si intelligatur C potentia movens, ut hominis, qui potest trahere T per rectam lineam TC, tunc vera esse potest demonstratio. Ut patet ex tractatum de axe in peritrochio nostrorum Mechanicorum.
>
> The demonstration is therefore false. But the fallacy is, as he says, that it is the case that BOT by some common science does not change its place.
>
> And it is totally false if C is understood to be a weight which always tends to the center of the world, as he seems to assume, and as he in the subsequent chapters assumes it to be demonstrated as if it holds for weights.
>
> But if C is understood to be a moving power, like that of a man who can draw T along the straight line TC, then the demonstration can be true. As is clear from the treatise on the wheel and axle of our [book] on mechanics.[14]

Remarkably, while the Copernican Benedetti speaks of the center of the region of the elements (*centrum regionis elementaris*), Guidobaldo insists on the center of the world (*centrum mundi*). By way of an afterthought, Guidobaldo once again criticized Benedetti's appeal to common science, remarking that this is not worthy of an expert mathematician:

> Notandum tamen, quod conclusiones per communem quandam scientiam deductae, non sunt periti mathematici cum propriis uti oporteat.
>
> It nevertheless has to be noted that the conclusions which are inferred by a certain common science are not worthy of an experienced mathematician because he should use his own [demonstrations].[15]

And by way of a second after-thought, he constructed an extreme case in which it is immeditaly clear that the broken bent lever cannot be in equilibrium if weights are attached to it, rather than forces (see figure 7.10):

[14] DelMonte (1587, 146).
[15] DelMonte (1587, 146).

> Ex hac etiam figura magis patet absurdum, hoc est pondera *E C* aequeponderare non posse.
>
> From this figure it appears even more absurd, that is, that the weights *E C* cannot be in equilibrium.[16]

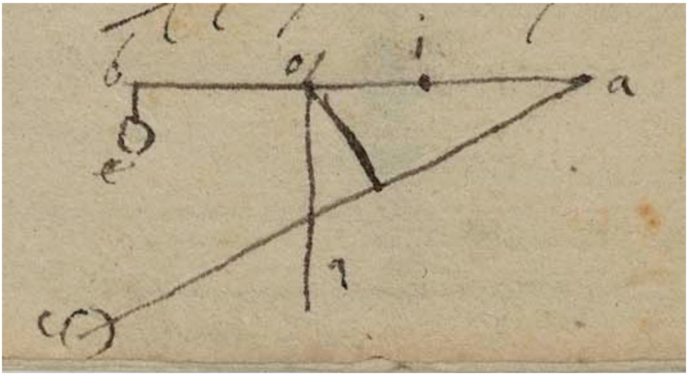

Figure 7.10: In his notebook, Guidobaldo concluded his alleged refutation of Benedetti's treatment of the broken bent lever with the construction of an extreme situation in which the two weights *E* and *C* are found on the same side of the fulcrum *O* so that it is obvious that the lever cannot be in equilibrium.

7.4 Fourth chapter: on the problem of the material beam

The fourth chapter is entitled:

> Quemadmodum ex supra dictis causis omnes staterarum et vectium causae dependeant.
>
> How all causes operating on steelyards and levers depend on the aforesaid causes.[17]

The chapter deals with the fact that the beam of a balance is not a mathematical line but a material body. Benedetti made use of his earlier

[16] DelMonte (1587, 146).
[17] Benedetti (1585, 144–145), pages 332–333 in the present edition. Translation modified from Drake and Drabkin (1969, 171–172).

treatment of the bent lever to take into account the fact that the weights attached to such a material beam do not act along a beam that can be idealized as a horizontal line, but along oblique lines from the fulcrum to the points of suspension of the weights, which are assumed to be placed at the upper part of the material beam. More specifically, Benedetti stated that if two equal weights are attached to the longer and the shorter arm of the balance, the weight attached to the longer arm will overpower the one attached to the shorter arm. He claimed that such an analysis of the material beam has never been dealt with before, a point that Guidobaldo rejected in his notes.

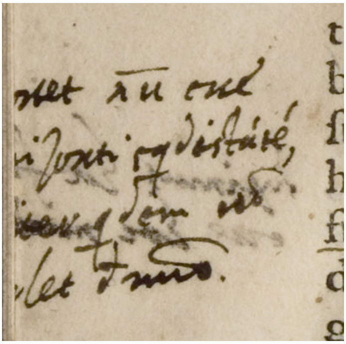

Figure 7.11: Left note to the first paragraph of the fourth chapter.

Guidobaldo left three marginal comments on the page opening the chapter; in addition he referred in a short note in the right margin to a sentence he underlined. The short comment in the left margin includes a drawing by Guidobaldo. The longest comment begins in the lower half of the left margin and is continued at the bottom of the page; it also comprises a drawing. About half of this comment was deleted by Guidobaldo himself; at least one line has later been cut off.

Guidobaldo's first comment reads (see figure 7.14):

> [opo]rtet *NU* esse [hor]izonti equidistantem [ali]ter quidem unde [vo]let demonstrandum
>
> It is necessary that *NU* is equidistant from the horizon, differently from [the way] in which he wanted [it] to be demonstrated.

The hand-drawn diagram in the left margin was evidently added in the same context.

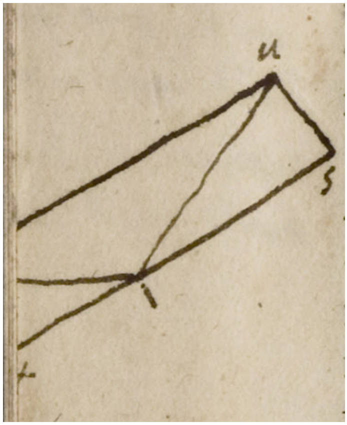

Figure 7.12: Drawing of a material beam at the left side of the first paragraph of the fourth chapter.

The meaning of Guidobaldo's first comment is not entirely clear. It seems to pinpoint the fact that Benedetti designated the upper part of the beam as being horizontal, while, according to Guidobaldo, this is in contrast to what has to be demonstrated. Possibly he referred to Benedetti's own later generalization of his argument from balances to levers in the penultimate sentence of the chapter:

> In stateris, recte et proprie appelari potest *XIS* aut *NOU* orizontalis, sed in omnium vectium specie, hoc tantum per quandam similtudinem dicatur.
>
> In balances with unequal arms, *XIS* or *NOU* can be rightly and properly called horizontal, but in the case of all levers this can be said only with a certain approximation.[18]

Or Guidobaldo wanted to express that this premise implicitly assumes the weights are connected by a horizontal line, in contrast to Benedetti's own detailed analysis which makes reference to oblique lines according to which the weights supposedly act. To stress this point he may have added the diagram in the margin showing a balance with equal arms.

[18]Benedetti (1585, 145), page 333 in the present edition. Translation adapted from Drake and Drabkin (1969, 172).

In any case, Guidobaldo's first comment, his drawing, and also his second comment all refer to the set-up of Benedetti's demonstration in the introduction of the chapter:

> Positis igitur duobus ponderibus aequalibus in extremitatibus brachiorum, experientia innotescit, quod pondus ad *US* appensum, violentiam faciet ponderi appenso ad *NX* sed nos volumus investigare causam huius effectus, quae a nemine unquam literarum monumentis, quod sciam, consignata fuit.
>
> Now if two equal weights are placed at the ends of the arms, it is clear from experience that the weights appended at *US* will overpower the weight appended at *NX*. But we wish to investigate the cause of this effect, which cause has never, so far as I know, been assigned by anyone in the annals of literature.[19]

The last claim is underlined by Guidobaldo as the point of reference of his second note, in the right margin of this page.

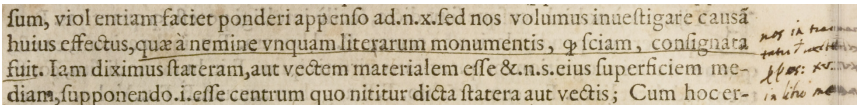
ſum, violentiam faciet ponderi appenſo ad.n.x.ſed nos volumus inueſtigare cauſā
huius effectus,quæ à nemine vnquam literarum monumentis, ꝙ ſciam, conſignata
fuit. Iam diximus ſtateram,aut vectem materialem eſſe &.n.s.eius ſuperficiem me-
diam,ſupponendo.i.eſſe centrum quo nititur dicta ſtatera aut vectis; Cum hoc er-

Figure 7.13: Underlined text with marginal note in the fourth chapter.

> Underlined text:
>
> quae a nemine unquam literarum monumentis, quod sciam, consignata fuit.
>
> which was never, so far as I know, documented by anybody in the annals of literature.[20]
>
> Marginal note:
>
> nos in tractatu de vecte propos. XV in libro me[chanicorum]
>
> We [did] in the treatise on the lever, prop. 15, in the book on mechanics.

[19]Benedetti (1585, 144), page 332 in the present edition. Translation in Drake and Drabkin (1969, 171).

[20]Benedetti (1585, 144), page 332 in the present edition. Translation in Drake and Drabkin (1969, 171).

Guidobaldo thus rejected Benedetti's claim to originality and referred to his own work on mechanics, and in particular to proposition 15 of the part on the lever:

> Problema.
> Quia vero dum pondera vecte mouentur, vectis quoque grauitatem habet, cuius nulla hactenus mentio facta est: idcirco primum quomodo inueniatur potentia, quae in dato puncto datum vectem, cuius fulcimentum sit quoque datum, sustineat, ostendamus.
>
> Problem.
> But since in moving weights with a lever, the lever also has weight, which has not been mentioned up to this point, we shall demonstrate how to find the power which will sustain the lever in a given point, the fulcrum being likewise given.[21]

As becomes clear from the proof of this proposition, Guidobaldo considered the centers of gravity of the two parts of the material beam, as they are divided by the fulcrum of the lever, and treats the entire beam of the lever as being represented by two weights suspended at the distances of these centers of gravity from the fulcrum. Not only is his procedure entirely different from that of Benedetti. Guidobaldo's and Benedetti's conceptual frameworks actually capture different aspects of the material beam. While Guidobaldo managed to take into account the weight of the beam, Benedetti only dealt with its geometrical extension and focused on the direction of the pull of the attached weight, corresponding in modern terms to the torque of the applied force.

The third comment, also in the left margin of this page, refers to Benedetti's construction of the lines according to which weights act within a material balance. It is accompanied by another diagram of Guidobaldo's to which he referred in the last part of this commentary.

> [cu]m pondera pendeant [*S*]*X* estque *XIS* recta linea huius [...] quaerenda a lineis *IN IU*, ut in [...]?, quae quidem sint pro[...] immaginariae. Deinde [poste]a facit mentionem [de gra]vitate vectis, et considerat matem[atice] [...] ex hac causa ut infra duo vectes
>
> [large passage deleted]

[21] DelMonte (1577, 60v), Renn and Damerow (2010, 178). Translation in Drake and Drabkin (1969, 303).

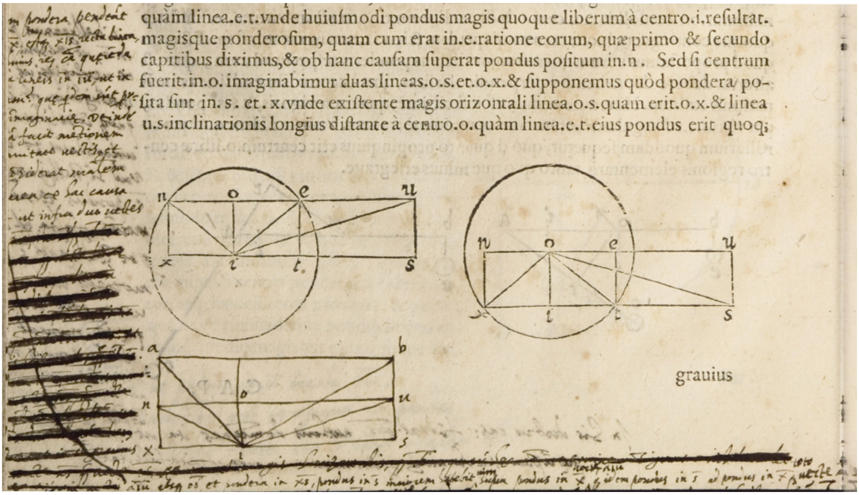
quàm linea.e.t.vnde huiuſmodi pondus magis quoque liberum à centro.i.reſultat.
magisque ponderoſum, quam cum erat in.e.ratione eorum, quæ primo & ſecundo
capitibus diximus,& ob hanc cauſam ſuperat pondus poſitum in.n. Sed ſi centrum
fuerit.in.o. imaginabimur duas lineas.o.s.et.o.x.& ſupponemus quòd pondera po-
ſita ſint in. s. et. x.vnde exiſtente magis orizontali linea.o.s.quam erit.o.x.& linea
u.s.inclinationis longius diſtante à centro.o.quàm linea.e.t.eius pondus erit quoq;
grauius

Figure 7.14: Long note to Benedetti's fourth chapter.

> *AIU* absque *OB* et pondera at *XS*, pondus in *S* maiorem habebit vim, supra pondus in *X*, vecte *AIU*, quod idem pondus in *S* ad pondus in *X* toto vecte [...] [line below cut]
>
> Since the weights hang from *SX* and *XIS* is a straight line whose [...] is to be found from the lines *IN IU*, as in [...], which are though only [...] imaginary. Finally he later mentions the heaviness of the lever and considers in a mathematical way [...] from which cause as the two levers below
>
> [large passage deleted]
>
> AIU if without *OB* and the weights in *XS*, the weight at *S* will have more power over the weight at *X*, by the lever *AIU*, than the same weight at *S* to the weight at *X* with the entire lever [...]

Although much of this commentary remains illegible, two salient points of Guidobaldo do emerge: He considered the oblique lines along which, according to Benedetti, the weights attached to a material beam act as being purely imaginary. And he apparently attempted to construct a contradiction within Benedetti's framework by considering the weights being attached to different heights of the material beam. Guidobaldo's drawing shows indeed a beam of approximately twice the height of the original one, with the original one inscribed. In the last legible line of his note he considered the weight on the longer right-hand side of the balance being attached from the original height of the beam, while the weight on the shorter left-hand side is suspended from the beam with double height. Guidobaldo concluded with an argument that he evidently later rejected that, in this constellation, the weight on the longer right-hand side has more power than if it were suspended from the same height as the weight on the left-hand side.

7.5 Fifth chapter: on the problem of the material lever

The fifth chapter is entitled:

> De quibusdam rebus animadversione dignis.
>
> On certain facts worthy of notice.[22]

The chapter deals with levers whose fulcrum is at one end of the lever, while the weight to be lifted by a force acting on the other end is positioned between these ends and somewhere near to the fulcrum. As we have discussed before, Benedetti treated the material lever not with regard to the weight of the beam but only with regard to its geometrical configuration. He hence imagined a rectangular cross-section of such a lever with a weight being placed on top of the beam. One lower corner serves as the fulcrum, the other corner is lifted by the hand. The question then is how the weight exerts a pressure on the corner where the hand is acting. Benedetti claimed that the ratio between that part of the weight that rests on the fulcrum and that part of the weight that rests on the corner where the force is acting is given by the inverse ratio of the horizontal distances of the weight from these two points (see figure 7.15):

> Si vero eadem resistentia posita erit in *U* clarum quoque erit, quod minor pars ponderis *N* annitetur ipsi *U* quam ipsi *O* cum dicta *NI* a centro *U* longius quam a centro *O* distet, et proportio partis ponderis *N* in *O* ad proportionem partis ponderis *N* in *U* non erit secundum proportionem angulorum *UNI* et *ONI* sed secundum proportionem *UI* ad *IO* [this proportion is underlined] quod clare comprehendi potest ab huius effectus converso […]

[22] Benedetti (1585, 145–146), pages 333–334 in the present edition. Translation in Drake and Drabkin (1969, 172–174).

> And if the same resistance is placed at *U*, it will also be clear that a smaller part of weight *N* will press on *U* than on *O*, since *NI* is farther distant from fulcrum *U* than from fulcrum *O*. And the ratio of the part of weight *N* that rests on *O* to the part of weight *N* that rests on *U* will be equal, not to the ratio of angle *UNI* to angle *ONI*, but to the ratio of *UI* to *IO*. This may be clearly understood from the converse of this effect […][23]

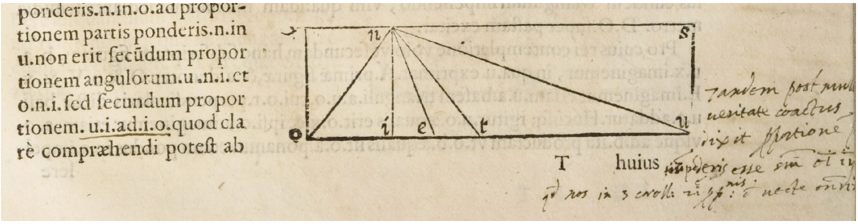

Figure 7.15: Figure and marginal note in the fifth chapter.

In the sequel Benedetti justified his claim by interpreting the situation of the lever according to the model of a balance suspended from the point where the weight is positioned, with the two lower corners now representing weights. From his procedure of determining effective lever arms by horizontal projection his proposition then followed.

Guidobaldo left a marginal note at the bottom right of page 145 and underlined the letters in Benedetti's text referring to the proportion of lengths in the diagram to which his comment refers:

> tandem post mu[lta] veritate coactus dixit proportionem p[artium] ponderis esse secundum *OI*, *IU* quod nos in 3 coroll. secundae propositionis de vecte omnia di[ximus].
>
> Finally he said after many [other things], forced by the truth, that the proportion of the parts of the weight is as *OI*, *IU* which we have said all in the third corollary of the second proposition about the lever.

Guidobaldo's marginal comment refers to the text at the bottom of the page quoted above which in turn refers to the diagram on the same

[23] Benedetti (1585, 145–146), pages 333–334 in the present edition. Translation in Drake and Drabkin (1969, 173).

page. Guidobaldo referred to his own treatment of levers in his book on mechanics, where he also dealt with a lever sustained at its two ends carrying a weight in the middle. He explicitly referred to the third corollary of the second proposition about the lever which reads:

> Ex hoc quoque elici potest, si duae fuerint potentiae, una in *A*, altera in *B*, et utraque sustentet pondus *E*; potentiam in *A* ad potentiam in *B* esse, ut *BC* ad *CA*.
>
> From this likewise it may be deduced that, if there are two powers, one at *A* and the other at *B*, and both sustain the weight *E* [suspended from point *C*], the power at *A* will be to the power at *B* as *BC* is to *CA*.[24]

He also justified his claim by exchanging the roles of fulcrum and force, but he did not take into account any directional effects of these forces, considering the lever without extension.

7.6 Seventh chapter: on the core question of the equilibrium controversy

The seventh chapter is entitled:

> De quibus erroribus Nicolai Tartaleae circa pondera corporum et eorum motus, quorum aliqui desumpti fuerunt a Jordano scriptore quodam antico.
>
> On certain errors of Niccolò Tartaglia on the weights of bodies and their motions, some taken from a certain ancient writer Jordanus.[25]

In this chapter Benedetti criticized Tartaglia's account of the variation of the positional heaviness of a body on a balance changing its position. As we have discussed extensively above (see section 6.2), he rejected, in particular, Tartaglia's claim that a balance would return to its original horizontal position because the weight that has moved upward becomes positionally heavier, while the weight that has moved downward becomes positionally lighter. Benedetti first pointed out that Tartaglia should not have compared the descents of the two weights but the descent of one weight with

[24] DelMonte (1577, 40v), Renn and Damerow (2010, 138). Translation in Drake and Drabkin (1969, 299).
[25] Benedetti (1585, 148–149), pages 336–337 in the present edition. Translation in Drake and Drabkin (1969, 174–176).

the ascent of another. In his annotations Guidobaldo did not fail to notice that, in his own book, he had already drawn attention to this circumstance. Then Benedetti reconsidered, as we have also discussed, the entire situation from a cosmological perspective, concluding that the weight that has moved upward actually becomes positionally lighter, while the weight that has moved downward becomes positionally heavier. Benedetti's argument is based on his procedure of determining effective lever arms by drawing perpendiculars to the lines of inclination of the two weights (see section 3.9). It is remarkable that his first criticism seems to suggest, in agreement with Guidobaldo's opinion, an indifferent equilibrium of the balance (indeed, under terrestrial circumstances it necessarily leads to that conclusion). In contrast, his second criticism (taking into account the cosmological perspective) implies that the balance would actually proceed to the vertical. Benedetti did not, however, actually make this explicit.

Page 148 has two comments by Guidobaldo, one in the middle of the page in the left margin, the other further below, also beginning in the left margin and continuing at the bottom of the page; the latter comment refers to a line in Benedetti's text underlined by Guidobaldo. His two marginal notes address the ambiguity of Benedetti's text. They end in the definitive rejection of Benedetti's method of determining positional heaviness, which – in the eyes of Guidobaldo – is in conflict with his fundamental insight into the indifferent nature of the equilibrium of a balance.

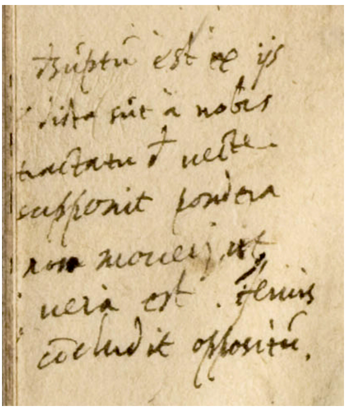

Figure 7.16: Marginal note to Benedetti's seventh chapter.

Guidobaldo's first comment reads:

> desumptum est ex iis [quae] dicta sunt a nobis [in] tractatu de vecte [...] supponit pondera non moveri ut [re] vera est. Serius concludit oppositum.
>
> This is taken from what has been said by us in the treatise on the lever. He assumes that the weights do not move which is true. Later he concludes the opposite.

The comment refers to the passage in which Benedetti pointed out that the descent of one weight should be compared to the ascent of the other (see figure 3.11):

> Sed in secunda parte quintae propositionis non videt quod vigore situs eo modo, quo ipse disputat, nulla elicitur ponderis differentia. Quia si corpus *B* descendere debet per arcum *IL* corpus *A* ascendere debet per arcum *US* aequalem, et similem eadem quoque rationem situatum, ut est arcus *IL* unde ut est facile corpori *B* descendere per arcum *IL* difficile ita erit corpori *A* ascendere per arcum *US*. Haec autem quinta propositio Tartalea est secunda quaestio a Iordano proposita.
>
> And in the second part of the fifth proposition, he fails to see that no difference in weight is produced by virtue of position in the way in which he argues. For if body *B* must descend on arc *IL*, body *A* must ascend on arc *US*, equal and similar to arc *IL* and placed in the same way. Therefore, just as it is easy for body *B* to descend on arc *IL*, it will be difficult for body *A* to ascend on arc *US*. And this fifth proposition is the second question proposed by Jordanus.[26]

Guidobaldo's second note at the bottom of the page refers to the diagram on the following page 149 (see figure 7.17).

It also refers to a passage in the text on page 148 that has been underlined by Guidobaldo:

> Pondus igitur ipsius *A* in huismodi situ, pondere ipsius *B* gravius erit.

[26] Benedetti (1585, 148), page 336 in the present edition. Translation in Drake and Drabkin (1969, 174–175).

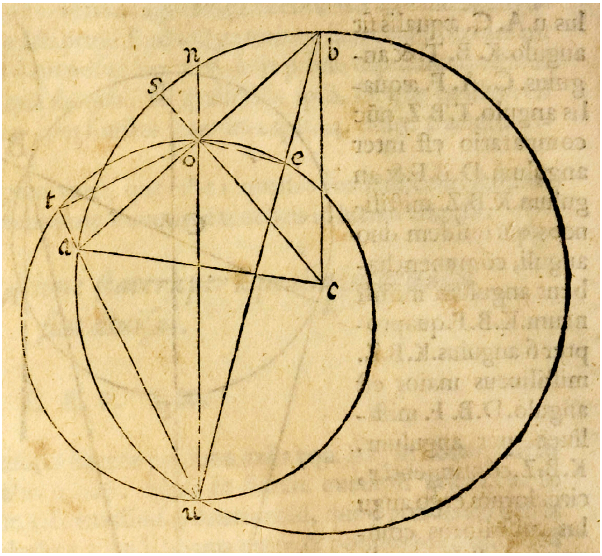

Figure 7.17: Drawing of a balance in a cosmological context in the seventh chapter.

> Therefore the weight of *A* in this position will be heavier than the weight of *B*.[27]

This is the conclusion of Benedetti's consideration of two weights on a balance in an oblique position from a cosmological perspective, amounting to the statement that the weight *A* that has been lowered has become positionally heavier than the weight *B* that has been lifted. Guidobaldo's second comment reads:

> [suppo]nit pondera *AB* non moveri. Hac demonstratione pondus *A* gra[vius] [e]st pondere *B* quia haec gravi[tates] metiuntur ex lineis perpendicularibus *OT*, *OE* quarum *OT* maior est, sequitur pondus *A* in hoc situ gravius esse pondere *B* in hoc situ. Dico igitur quod subterfugiet [p]ondus *A* deorsum non moveatur et *B* sursum? Libra ergo *AB* non manebit ut supposuit, et ut re vera manet. [Qua]re si volens errores Iordani

[27] Benedetti (1585, 148), page 336 in the present edition. Translation in Drake and Drabkin (1969, 176).

et Tartaleae (quorum errorum nec solvit contradictiones) incidit, et si non in peiora, tamen in aequalia [abs]urda. Unde perspicuum est, quod sit inanis, et falsa haec consideratio suis perpendicularibus facta, quam [...]

He assumes that the weights AB are not moved. By this demonstration A is heavier than weight B, because these gravities are measured by the perpendicular lines OT, OE, from which OT is the greater, it thus follows that weight A is in this position heavier than weight B in this position. I therefore say what escapes him: does not weight A move downward and weight B upward? The balance AB will therefore not remain as he assumes and as it truly remains. If he therefore willingly cuts into the errors of Jordanus and Tartaglia (whose contradictions he does not resolve), and if he does not make them worse then nevertheless to an equal extent absurd. From which it is evident that this consideration of his which he makes about the perpendiculars is empty and false as [line cut off]

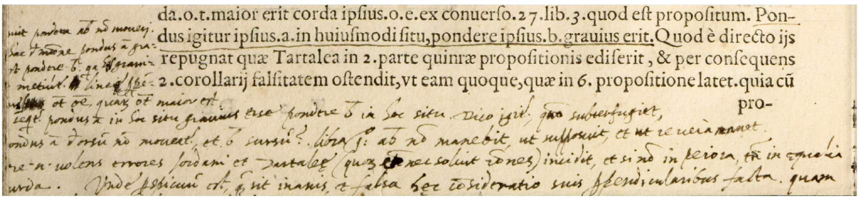
da.o.t.maior erit corda ipsius.o.e.ex conuerso.27.lib.3.quod est propositum. Pondus igitur ipsius.a.in huiusmodi situ,pondere ipsius.b.grauius erit.Quod è directo ijs repugnat quæ Tartalea in 2.parte quintæ propositionis ediserit, & per consequens 2.corollarij falsitatem ostendit,vt eam quoque,quæ in 6. propositione latet.quia cũ pro-

Figure 7.18: Bottom note of the seventh chapter.

As we have discussed above, Guidobaldo had, in his own book, similarly considered the case of a balance in an oblique position from a cosmological perspective, also using the concept of *positional heaviness*.[28] He had arrived at the conclusion, in agreement with his general conviction, that the two weights on such a balance are equally heavy positionally. Against this background, Benedetti's method of determining positional heaviness, necessarily in contradiction with this conclusion, must have appeared entirely unacceptable to Guidobaldo as he indeed clearly stated in this marginal note.

[28]Drake and Drabkin (1969, 282). See section 3.8.8.

7.7 Eighth chapter: on plagiarizing the criticism of Jordanus and Tartaglia

The eighth chapter is entitled:

> Quod autem idem Tartalea in 6. propositione, et Iordanus in secunda parte secundae propositionis scribunt, maximum quoque errorem in se continet.
>
> What Tartaglia writes in proposition 6 and Jordanus in the second part of proposition 2 also contains a most serious error.[29]

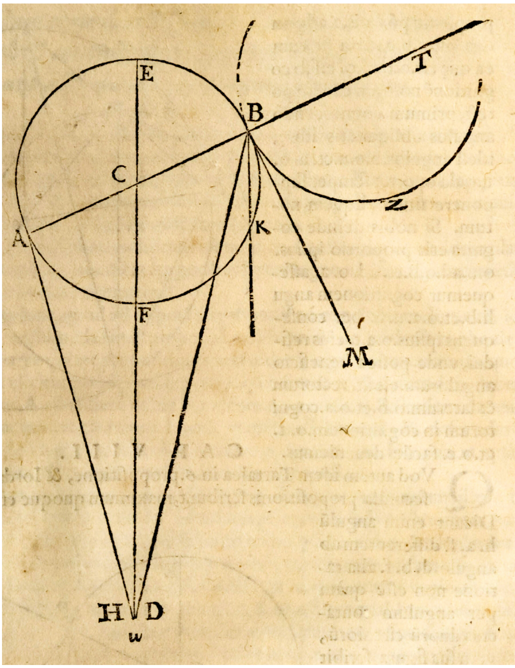

Figure 7.19: Drawing of the convergence of perpendiculars in the eighth chapter.

[29]Benedetti (1585, 149–151), pages 337–339 in the present edition. Translation modified from Drake and Drabkin (1969, 176–178).

In the beginning of this chapter Benedetti criticized, as we have discussed, Tartaglia's way of determining positional heaviness by means of angles of contact between the curved path of a weight and a perpendicular. He showed that this procedure leads to a contradiction when the convergence of these perpendiculars at the center of the world is taken into account, as he indicates in his drawing. Benedetti concluded his analysis with a general rejection of the method of Tartaglia and Jordanus.

> Omnis autem error in quem Tartalea, Iordanusque lapsi fuerunt ab eo, quod lineas inclinationum pro parallelis vicissim sumpserunt, emanuit.
>
> Now the whole error into which Tartaglia and Jordanus fell arose from the fact that they took the lines of inclination as parallel to each other.[30]

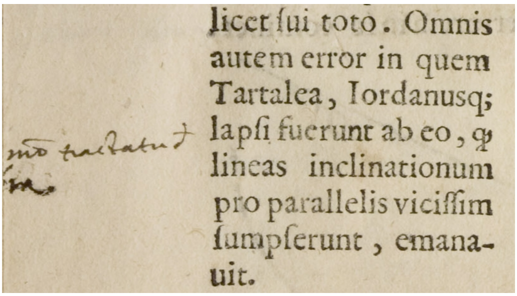
licet sui toto. Omnis
autem error in quem
Tartalea, Iordanusq;
lapsi fuerunt ab eo, quod
lineas inclinationum
pro parallelis vicissim
sumpserunt, emana-
uit.

Figure 7.20: Marginal note in the eighth chapter.

Benedetti's argument is strikingly similar to those of Guidobaldo against this method, as we have discussed above. Page 150 has a single short comment by Guidobaldo in the middle of the left margin, next to a text passage in which Benedetti finally rejected the method of Tartaglia and Jordanus. In his marginal comment Guidobaldo pointed to the fact that Benedetti's argument, in his view, has been taken from his own book:

> [ex] meo tractatu [de lib]ra
>
> from my treatise on the balance

[30]Benedetti (1585, 150), page 338 in the present edition. Translation in Drake and Drabkin (1969, 177).

7.8 Tenth chapter: on Aristotle and the composition of motions

The tenth chapter is entitled:

> Quod linea circularis non habeat concavum cum convexo coniunctum, et quod Aristoteles circa proportiones motuum aberraverit.
>
> That the circumference of a circle does not have a concavity joined with a convexity, and that Aristotle was mistaken in the ratios of motions.[31]

The chapter deals with the introductory part of the Aristotelian *Mechanical Problems* in which the curious properties of the circle and the composition of motions are treated. Benedetti disputed the claim of the Aristotelian author that the circle seems to unite the convex with the concave, essentially arguing that one should distinguish between the circular surface included by the circumference and the plane with a circular hole that is also delimited by that circumference. Guidobaldo left two rather long comments in the left margin of this page. In his first marginal note Guidobaldo rejected Benedetti's distinction because its application would require, according to him, an intervening space which, however, is not given. In his discussion of the Aristotelian analysis of the composition of motions Benedetti questioned the alledged claim of the Aristotelian author that if a body moves along a given line, it moves according to one definite proportion rather than according to another one. In particular, Aristotle maintained that when an object moves along the diagonal it will always move in the ratio of the sides of the parallelogram.[32] Benedetti showed instead that the same trajectory can be generated by motions following different proportions. In his second marginal comment Guidobaldo pointed to the fact that Benedetti failed to understand Aristotle's argument and that his objection is therefore irrelevant. Guidobaldo stressed in this comment that what matters to Aristotle is only the fact that, when a body is moved in a fixed ratio it necessarily travels in a straight line, independently from the fact that the same straight line may be traversed also by a motion that is given by a different ratio.

More specifically, Guidobaldo's first comment refers to the beginning of the chapter:

---

[31] Benedetti (1585, 152), page 340 in the present edition. Translation in Drake and Drabkin (1969, 179–180).

[32] Aristotle (1980, 339).

Aristoteles in principio quaestionum Mechanicorum ait lineam, quae terminat circulum videtur convexum habere coniunctum cum concavo, quod falsum est: quia huismodi linea partes nullas secundum latitudinem habet, (ut ipse etiam confirmat) sed est idem convexum circuli: linea vero quae terminus est superficiae ambientis, et amplectentis circulum est eadem concavitas dictae superficiae eundem circulum ambientis, quae nullam convexitatem habet et haec duae sunt lineae, quarum una diversa est ab alia, neque altera alterius, quod ad convexum, et ad concavum attinet.

Aristotle at the beginning of *Questions of Mechanics* says that the line which bounds the circle seems to unite the convex with the concave. But this is false. For a line of this kind has no thickness (as Aristotle himself also asserts), but is identical with the convex boundary of the circle. On the other hand, the line that bounds the surrounding surface and encloses the circle is identical with the concavity of the surface that surrounds the circle, a surface which has no convexity. And these are two lines of which one is different from the other, and not part of the other, so far as pertains to convexity and concavity.[33]

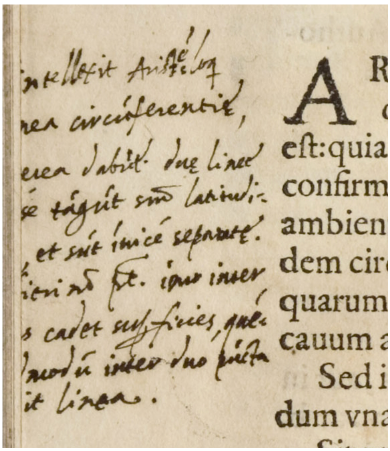

Figure 7.21: First marginal note in the tenth chapter.

[33]Benedetti (1585, 152), page 340 in the present edition. Translation in Drake and Drabkin (1969, 179).

Guidobaldo commented:

> intellexit Aristotelem loqui [quod li]nea circumferentiae area dabitur; duae linee se tangunt secundum latitudi[nem], et sunt invicem separatae. [Hoc] fieri non potest. Imo inter [ea]s cadet superficies quaem[ad]modum inter duo puncta [s]it linea
>
> He understood that Aristotle claimed that the line of the circumference will be given by the area. Two lines touch each other along their latitude and are separate from each other; this cannot be. Therefore a surface falls in between just as there is a line between two points

Guidobaldo's second comment refers to Benedetti's criticism of Aristotle's proof of the composition of motions and to the figure on page 152 (see figure 7.22).

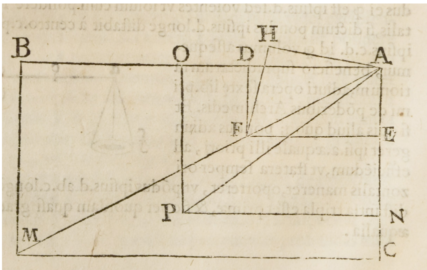

Figure 7.22: The composition of motions discussed in the tenth chapter.

Benedetti wrote:

> Cui respondeo, punctum A quod movetur in linea AM ab A versus M usque ad F non moveri ab aliqua proportione determinata magis quam ab alia: unde non solum possumus imaginari dictum punctum A moveri ab A usque ad F eiusdem velocitatis sub alia quadam proportione, sed etiam sub alia, quae iam datae contraria sit, ut est proportio ipsius AC ad AB imaginantes moveri A versus C et AC versus BM delatam. [...] Huiusmodi igitur consideratio ab Aristotele facta, nullius est momenti.
>
> To Aristotle I reply that the fact that point A moves on line AM from A towards M as far as F does not mean that it moves

according to one definite proportion rather than some other. Thus we can suppose that point *A* moves from *A* to *F* not only according to one ratio of the same velocity, but also some other which is the very opposite of the first ratio – e.g. the ratio of *AC* to *AB*, it being imagined that *A* moves toward *C* and *AC* toward *BM*. [...] Hence the discussion on this point by Aristotle is of no value.[34]

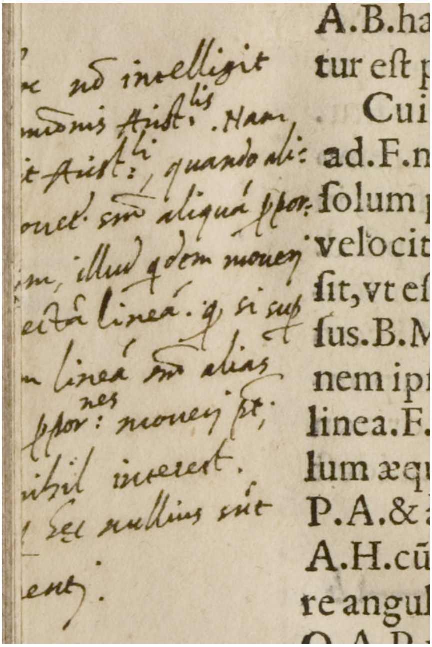

Figure 7.23: Second marginal note in the tenth chapter.

Guidobaldo commented:

[ho]c non intelligit [demonstr]ationis Aristotelis. Nam [paru]it Aristoteli quando ali[quid] [m]ovetur secundum aliquam propor[tione]m, illud quidem moveri [secundum r]ectam lineam, quod si sup[ponere ea]m lineam secundum alias proportiones moveri potest, [n]ihil interest [se]d haec nullius sunt [mom]enti

[34]Benedetti (1585, 152), page 340 in the present edition. Translation in Drake and Drabkin (1969, 180).

> He does not understand that part of Aristotle's proof. In fact, Aristotle held that, if something is moved according to some proportion, it will surely move according to a straight line; but, if it is assumed that the line can move according to other proportions, that makes no difference, but these things are of no value.

7.9 Twelfth chapter: saving Aristotle in the equilibrium controversy

The twelfth chapter is entitled:

> De vera causa secundae, et tertiae quaestionis mechanicae ab Aristotele non perspecta.
>
> On the true cause not perceived by Aristotle of the *Mechanical Questions* 2 and 3.[35]

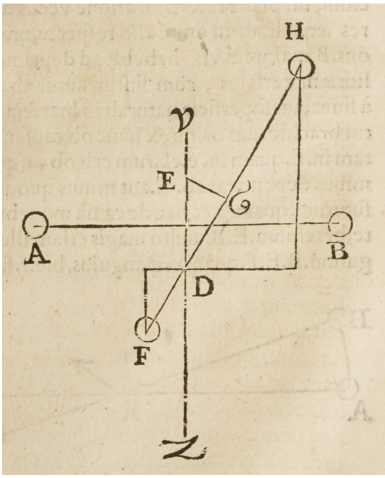

Figure 7.24: Drawing of a balance in an oblique position in the twelfth chapter.

[35]Benedetti (1585, 154), page 342 in the present edition. Translation modified from Drake and Drabkin (1969, 182–183).

The chapter deals with the question of how a balance either supported from above or from below behaves when it is removed from the horizontal position (see figure 7.24). For the case in which the balance is supported from above Benedetti agreed with the Aristotelian conclusion that it will return to the horizontal position, but justified this behavior with his technique for determining positional heaviness. For the case in which it is supported from below he disagreed with Aristotle who seemed to suggest that the balance will stay in its position. On this point Guidobaldo was of the same opinion, as the following quotation from his book on mechanics shows:

> Nam cum in secunda parte secundae quaestionis proponit, cur libra, trutina deorsum constituta, quando deorsum lato pondere quispiam id amouet, non ascendit, sed manet? non asserit adhuc libram deorsum moueri; sed manere. Quod in vltima quoque conclusione colligisse videtur.
>
> For in the second part of the second question he asks, 'Why, when the support is below, the balance being carried downward and released, it does not rise again, but remains?' Here he affirms not that the balance moves downward, but that it remains, which he seems to have deduced in the last conclusion.[36]

In contrast to Benedetti, Guidobaldo was convinced, however, that Aristotle's position can be defended, which is also the point of his marginal comment and in line with his appreciation of the ancient heritage, including the Aristotelian work on mechanics. In his own book he argued in fact that the balance does not move further downward because it is prevented from doing so by the support on which it rests.

Guidobaldo's comment refers to the following text at the lower part of the page:

> In secunda deinde huius quaestionis parte, in qui scribit libram in situ, in quo posita est, firmam manere, toto coelo aberrat, quia necessarium est, ut omnino cadat, eousque quo spartum sursum remaneat: ablato tamen omni impedimento, quod nulla eget probatione, cum natura sua clarissime pateat.
>
> Then in the second part of this problem, in which Aristotle writes that a balance remains fixed in the position in which

[36]DelMonte (1577, 26v), Renn and Damerow (2010, 110). Translation in Drake and Drabkin (1969, 290).

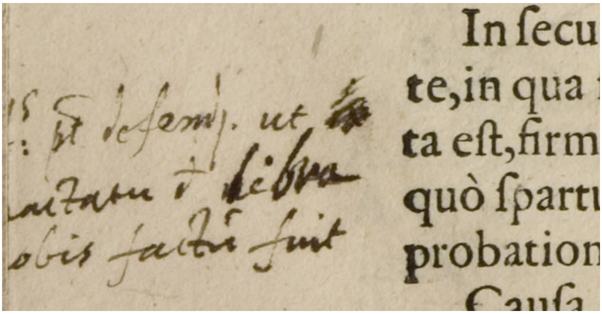

Figure 7.25: Marginal note to Benedetti's twelfth chapter.

it has been placed, he is completely mistaken. For it must continue to fall until the support remains above it, with the assumption, however, that all impediment to this is removed. This proposition requires no proof, since by its own nature it is perfectly clear.[37]

Guidobaldo's comment reads:

[Aristotel]es potest defendi ut [in] tractatu de libra [a n]obis factum fuit

Aristotle can be defended as it was done by us in the treatise on the balance

7.10 Fourteenth chapter: Aristotle's wheel and the problem of infinite limits

The fourteenth chapter is entitled:

Quod rationes ab Aristotele de octava quaestione conficta sufficientes.

That the reasons by Aristotle in *Questions of Mechanics* 8 are not adequate.[38]

[37]Benedetti (1585, 154), page 342 in the present edition. Translation in Drake and Drabkin (1969, 183).
[38]Benedetti (1585, 155–159), pages 343–347 in the present edition. Translation in Drake and Drabkin (1969, 184–187).

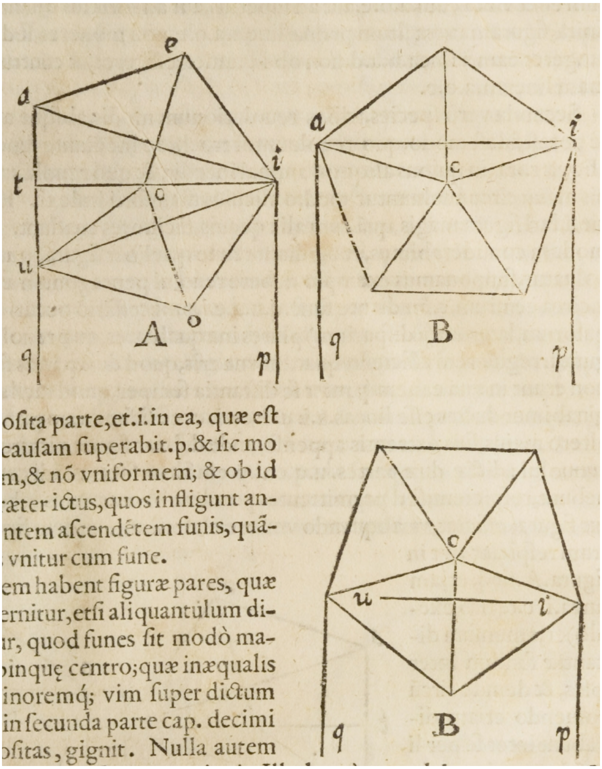

Figure 7.26: The motion of a polygonal shape discussed in the fourteenth chapter of Benedetti's book.

This chapter also deals with the Aristotelian *Mechanical Problems*, here with the question of why bodies of circular shape are easier to roll than others.[39] Benedetti considered various rotational motions, the rotation of carriage wheels, of pulley wheels, and of potter's wheels. He compared the motion of a wheel with that of a polygon and gave reasons why the motion of the former is easier than that of the latter (see figure 7.26). He argued, for instance, that, when a polygon is *rolled* along a plane, its center will go up and down, its upward motion will require an effort, while the center of a wheel will always maintain the same distance from the center, that is, as he formulated, from the goal of heavy bodies. He considered the circular

[39]See the discussion in Büttner (2008).

shape as the limiting case of polygonal shapes with ever more angles. One short comment by Guidobaldo is found in the left margin of this page. In his comment, Guidobaldo expressed his skepticism about this limiting process.

His note refers to the following passage of Benedetti's argument:

> Si ergo quanto plures angolos habebit dicta figura, tanto ad circunvolvendum hoc modo agilior erit. Circularis igitur figura, quae ex infinitis angulis efficitur, omnium agillima erit.
>
> The more angles the said figure will have, therefore the more suitable it will be to rotate in this way. Hence the circular shape, which is constituted from infinite angles, is the most suitable of all.[40]

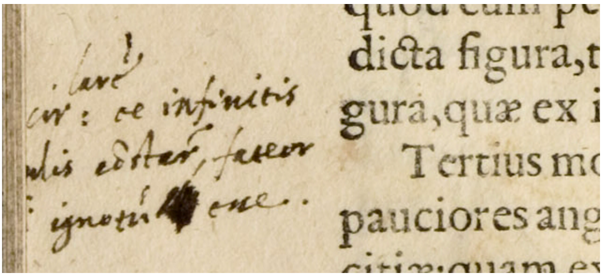

Figure 7.27: Marginal note to the fourteenth chapter of Benedetti's book.

Guidobaldo noted:

> circularem ex infinitis [ang]ulis constare fateor ignotum esse
>
> I confess that it is unknown [to me] that the circular is composed from infinite angles

7.11 Sixteenth chapter: on Aristotle's empty balance

The sixteenth chapter is entitled:

> Quod Aristotelis rationes de decima quaestione sint reiiciendae.

[40]Benedetti (1585, 158), page 346 in the present edition.

> That Aristotle's explanation of *Questions of Mechanics* 10 must be rejected.[41]

The chapter deals with another topic of the Aristotelian *Mechanical Problems*, the greater readiness of an empty balance to move. Benedetti approached the subject by comparing two balances, one carrying two small weights, the other two large weights (see figure 7.28). Now according to him Aristotle wonders about the fact that the balance with the smaller weights moves more rapidly when on one of its arms another small weight is placed than when the same small weight is placed on one of the arms of the balance with the large weights. Benedetti essentially argued that Aristotle would have no reason to wonder had he appropriately taken into account his own dynamical principles (see section 3.4.1).

Benedetti explained:

> quia semper ineunda est ratio proportionis virtutis mouentis super mobile; quod ipse non fecit.
>
> For the ratio of the moving force to the body moved must always be considered; and Aristotle did not do this.[42]

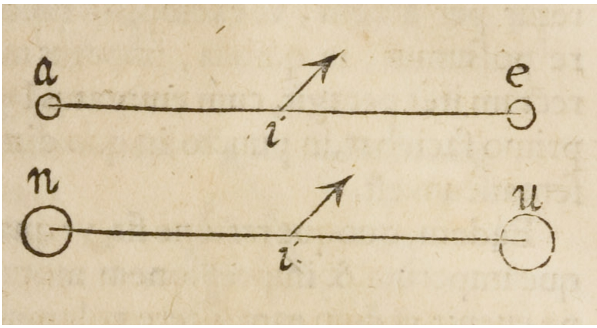

Figure 7.28: Drawing of two balances, one carrying small weights, the other large weights, as discussed in the sixteenth chapter of Benedetti's book.

[41] Benedetti (1585, 159–160), pages 347–348 in the present edition. Translation in Drake and Drabkin (1969, 187–188).
[42] Benedetti (1585, 159), page 347 in the present edition. Translation in Drake and Drabkin (1969, 187).

In his short marginal note Guidobaldo seems to express his surprise at Benedetti's claim that Aristotle was *wondering*, apparently incapable of giving an adequate solution to a problem he had posed himself. The comment refers to the following passage of Benedetti's text (see figure 7.28):

> Sit exempli gratia libra *AIE* quae in utraque extremitate unciam unam solam ponderis obtineat, et sit libra *NIU* aequalis priori, quae pro singula extremitate unam ponderis libram habeat. Aristoteles admiratur, quod addendo ipsi *E* mediam ponderis unciam, brachium *IE* velocius cadat, quam adiiciendo ipsam mediam unciam ipsi *U* brachii *IU*.
>
> Let there be, for example, a scale *AIE* which has at the extremity of each arm merely one ounce of weight; and let there also be a scale *NIU*, exactly like the former one, which has one pound of weight on each end. Aristotle wonders about the fact that, when he adds a half-ounce weight at *E*, arm *IE* falls more rapidly than when he adds that same half-ounce at *U*, the extremity of arm *IU*.[43]

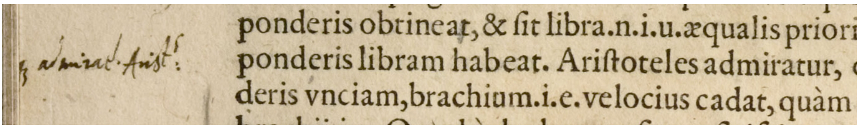
ponderis obtineat, & ſit libra.n.i.u.æqualis priori
ponderis libram habeat. Ariſtoteles admiratur,
deris vnciam, brachium.i.e. velocius cadat, quàm

Figure 7.29: Marginal note to the sixteenth chapter of Benedetti's book.

Guidobaldo noted:

> quod admiratur Aristoteles
>
> what was Aristotle wondering about

[43] Benedetti (1585, 160), page 348 in the present edition. Translation in Drake and Drabkin (1969, 188).

7.12 Twentieth chapter: on reducing the wedge to the lever

The twentieth chapter is entitled:

> De vera ratione 17 quaestionis.
>
> On the true explanation of question 17.[44]

This chapter also deals with the Aristotelian *Mechanical Problems*, here with the question of how the wedge is to be treated according to the model of the lever. Benedetti argued that Aristotle failed to properly reduce the wedge to the lever:

> Decimaseptima quaestio ab Aristotele haud benè percepta fuit, quia is non accommodat partes vectis suis locis.
>
> Question 17 was not correctly understood by Aristotle, for he did not assign the parts of the lever to their correct places.[45]

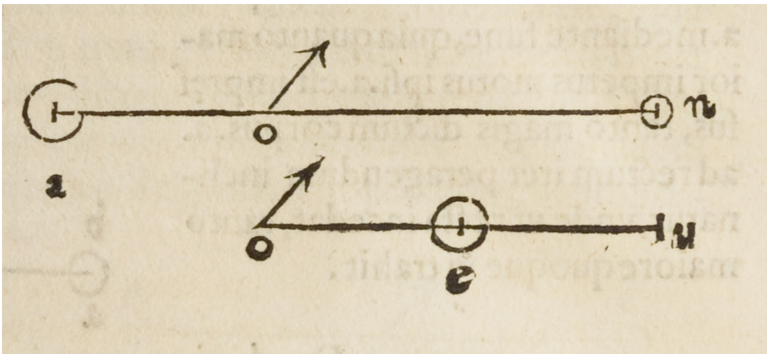

Figure 7.30: A comparison of two levers, one with the fulcrum in the middle, the other with the fulcrum at one end, as discussed in the twentieth chapter of Benedetti's book.

[44] Benedetti (1585, 162), page 350 in the present edition. Translation in Drake and Drabkin (1969, 190–191).

[45] Benedetti (1585, 162), page 350 in the present edition. Translation in Drake and Drabkin (1969, 190).

In order to improve on Aristotle Benedetti began his discussion by comparing two levers, one with the fulcrum in the middle, the weight at one and the force at the other end, the other lever having the fulcrum at one end, the weight in the middle and the force at the other end (see figure 7.30). It is the latter kind of lever that he applied to analyze the wedge, but it first had be reduced to the ordinary lever with the fulcrum in the middle. He argued in fact that when the weights, the distances between weight and fulcrum, and the distances between force and fulcrum are equal in the two levers, a force sufficient to raise the weight with one lever will also be sufficient to raise the weight with the other lever. By way of justification he referred, first of all, to common science (*scientia communis*), and then to his principles treated in chapters 4 and 5. In his marginal note Guidobaldo criticized Benedetti for his all too generous use of the reference to common science (*scientia communis*).[46]

More specifically, his comment refers to the passage:

> Et quia omnia supponuntur aequalia, clarum quoque erit, communi scientia, tantam virtutem in *N* quanta sufficiet ad attollendum *A* in *U* quoque suffecturam ad elevandum *E* oportebit attollere *U*.
>
> And because all are assumed equal, it will also be clear, by common science, that the force at *N* required to raise *A* will also be sufficient at *U* to raise *E*.[47]

Guidobaldo wrote in the upper left margin of this page:

> [...] sua communis [scient]ia multa probat [si]ne demonstratione
>
> His common science demonstrates much without proof

7.13 Twenty-first chapter: the plagiarized pulley

The twenty-first chapter is entitled:

> De vera et intrinseca causa trochlearum.
>
> On the true and intrinsic explanation of compound pulleys.[48]

[46]See also the discussion in section 6.3.

[47]Benedetti (1585, 162), page 350 in the present edition. Translation modified from Drake and Drabkin (1969, 191).

[48]Benedetti (1585, 163–165), pages 351–353 in the present edition. Translation in Drake and Drabkin (1969, 191–193).

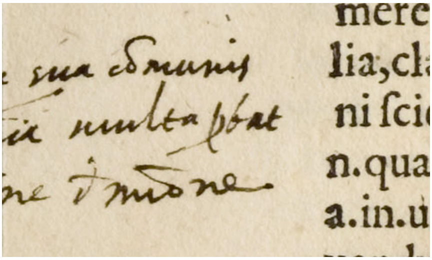

Figure 7.31: Marginal note to the twentieth chapter of Benedetti's book.

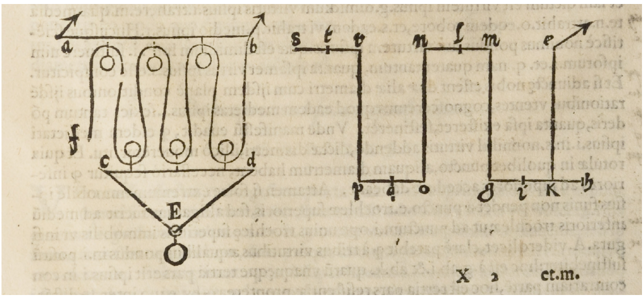

Figure 7.32: Reducing the pulley to the lever, as discussed in the twenty-first chapter of Benedetti's book.

The chapter deals with the explanation of the pulley, and in particular with the way it can be reduced to the lever or the balance (see figure 7.32). In the middle of the right margin of page 163, Guidobaldo left a short comment. Another related comment is found on the subsequent page. Guidobaldo's first comment refers to the passage in the middle of page 163.

> Imaginemur separatim stateram *GH* cuius centrum sit *K* ita situm, ut brachium *GK* sit duplum ad brachium *KH* supponendo igitur in puncto *G* pondus aut virtutem moventem unius librae, et in *H* duarum librarum, absque dubio haec duae virtutes in huismodi distantiis a centro aequales invicem erunt, ob rationes prioribus capitibus iam allatas, et statera orizontalis manebit.

> Let us consider, separately from the preceding figure, a balance *GH* with fulcrum *K* so situated that arm *GK* is double the arm *KH*. Now if we assume a weight or moving force of one pound at point *G*, and of two pounds at *H*, clearly these two forces at these distances from the center will be equal to each other for the reasons already set forth in previous chapters, and the scale will remain horizontal.[49]

In his two comments on this chapter Guidobaldo is, in a sense, less critical of Benedetti than in his other notes. He remarked, however, that Benedetti should have referred to Aristotle when mentioning the law of the lever in his first comment instead of referring to his own work.

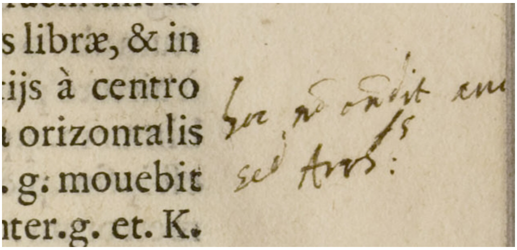

Figure 7.33: First marginal note to the twenty-first chapter of Benedetti's book.

His first comment reads:

> hoc non [con]dit auc[tor] sed Aristoteles
>
> This foundation is not laid by the author but by Aristotle

In the second comment to this chapter – in the lower left margin of the next page – Guidobaldo criticized Benedetti for the lack of acknowledgement that his own treatment of the pulley receives. He accused him of wrongly pretending to add something new when dealing with the compound pulley and its reduction to the balance. Benedetti mentally replaced the compound pulley with a sequence of connected balances, arriving at the conclusion that for a pulley with four wheels, a force that amounts to one fourth suffices to lift a given weight. Up to this point Guidobaldo

[49]Benedetti (1585, 163), page 351 in the present edition. Translation in Drake and Drabkin (1969, 192).

agreed with him and admitted that this chapter is rather clear, even if Benedetti added unnecessary complications. In Guidobaldo's eyes what is true comes anyway from his own work on the pulley which, however, is not mentioned by Benedetti.

More specifically, the comment on the lower part of page 164 refers to the following passage of Benedetti's text and to the figure on page 163 (see figure 7.32):

> Hucusque scientifice novimus pondus, aut virtutem ipsius *S* quae est dimidium ipsi *I* sustinere vim ipsorum *I* et *Q* nam quater tantum, quanta ipsamet virtus ipsius *S* esse conspicitur.
>
> Up to here we have come to know that the weight or the force of *S* which is half of that of *I* sustains the force of *I* and *Q* namely four times as much as the force of *S* is considered to be.[50]

Guidobaldo commented:

> […]dum hucusque verum est usque […]nem confuse immo sine [confusi]one totum hoc caput[…] est. Noluit [vero?] earum quae distincte in tractatu trochlea condimus repe[rire] sed ut aliquid novi af[figer]e videatur per ambages [ill]as communes species, et per communes conceptus ali[quid] [a]ttingit; tandem vero [aliquan]do aliqua vera profert [e]x nostro tractatu de [troc]hlea rescripsit. Quaequae hoc tractatum cap., neminem credo […] demonstrationemque trochlearum […]ere posse
>
> […] up to here everything is true and [not] confused. Even this entire chapter is without confusion. But he does not want to recover [anything] from that for which we have concisely provided the foundation in the treatise on the pulley but rather, in order to appear to add something new, he attacks something with the help of ambiguous ideas and common notions; nevertheless occasionally he advances something true [that] he has rewritten from our treatise on the compound pulley. Whatever […] is treated in this chapter, I believe that nobody can […] the proof of the compound pulleys

[50] Benedetti (1585, 164), page 352 in the present edition.

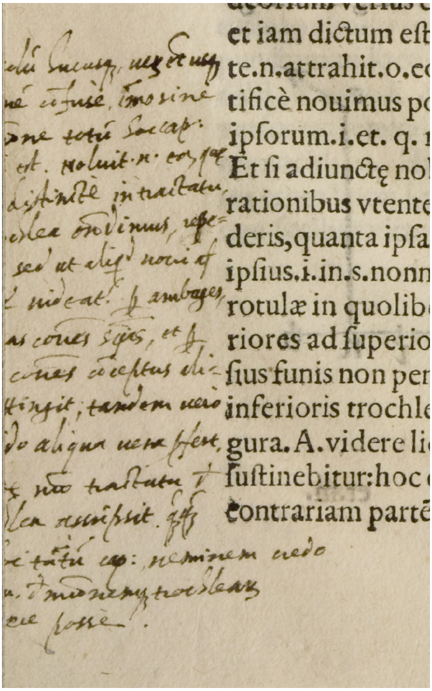

Figure 7.34: Second marginal note to the twenty-first chapter of Benedetti's book.

7.14 Letter to Pizzamano: simplifying the solution of a geometrical problem

The chapter is a letter entitled:

> Qualiter circulus designari possit alios duos circulos propositos includens. Clariss. Petro Pizzamano.
>
> How a circle can be designed so that it includes two other given circles. To the Most Brilliant Petrus Pizzamanus.[51]

This chapter belongs to the part of Benedetti's treatise in which he collected letters to illustrious personalities. The present letter is directed

[51] Benedetti (1585, 262–264), pages 356–358 in the present edition.

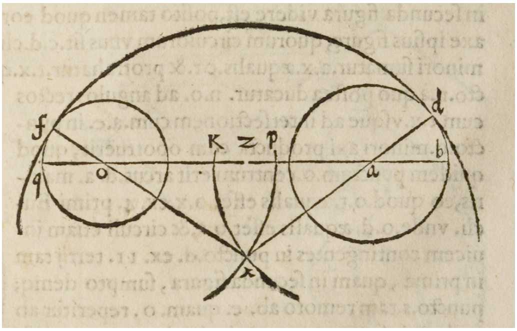

Figure 7.35: Drawing of a geometrical problem in Benedetti's book on which Guidobaldo commented.

to Pietro Pizzamano who was, between 1559 and 1580, an official in Bergamo, Trevigi, and Mercanzia.[52] It deals with a rather trivial geometrical problem to which Benedetti offered various solutions depending on the position of the two circles for which an encompassing circle is being searched (see figure 7.35). In his note at the bottom of the second page dealing with this problem Guidobaldo suggested one brief solution to the problem raised by Benedetti.[53] While Benedetti's solution allows for a variable diameter of the surrounding circle, Guidobaldo's construction does not. Guidobaldo's solution works by simply dividing in half the line connecting the centers of the given circles and extending from one diameter to the other. He then concluded that a circle around this midpoint with this extension will touch the two given circles.

Guidobaldo's comment refers to the diagram at the bottom of page 263 and to Benedetti's text starting in the middle of the page:

> Si vero distantia duorum propositorum circulorum tanta fuerit, quod secundi circuli nequeant se invicem tangere, vel secare, tunc alia via incedendum erit, quae talis est et generalis. Dividatur tota QB per aequalia in puncto Z circa quod signentur duo puncta ab ipso aequidistantia K et P. Distantia vero AK facta sit semidiameter esse unius circuli KX circa centrum A. Distantia autem OP semidiameter alterius circuli PX circa centrum O qui quidem circuli se invicem secent in puncto X a quo

[52]See the discussion in Bordiga (1985, 634).

[53]Compare also Guidobaldo's discussion in his notebook DelMonte (1587, 148).

cum ductes fuerint *XAD* et *XOF* per centra dictorum circulorum, ipse erunt invicem aequales, eo quod cum *BK* aequalis sit *QP* igitur *XD* et *QP* erunt invicem aequales, sed *FX* aequalis est *QP* quare *XF* aequalis erit *XD* tunc si *X* centrum fuerit unius circuli, cuius semidiameter sit una dictarum, problema solutum erit.

Talis etiam solutio commoda erit ad inveniendum dictum circulum cuiusvis magnitudinis, dato tamen quod eius diameter, maior sit *BZ* cum in nostra potestate sit accipere puncta *K* et *P* proxima vel remota ab ipso *Z* ad libitum. Unde absque ulla divisione ipsius *QB* per medium, satis erit signare puncta *K* et *P* duabus distantiis mediantibus *BK* et *QP* invicem aequalibus, et etiam propositis.

But if the distance of the two given circles were such that the second circles cannot mutually touch or cut each other, then one has to proceed by another way which is as follows and which is general. Let the entire line *QB* be equally divided at the point *Z* around which two points *K* and *P* are marked which are equally distant from it. The distance *AK* shall be made the radius of one circle *KX* around the center *A*. But the distance *OP* shall be the radius of another circle *PX* around the center *O* which circles cut each other in the point *X* from which two lines *XAD* and *XOF* shall be drawn through the centers of the said circles, then these will be equal to each other, so that since *BK* is equal to *QP* therefore *XD* and *QP* are equal to each other, but *FX* is equal to *QP* so that *XF* is equal to *XD*, whence if *X* were the center of one circle whose diameter shall be that of one of those mentioned, the problem will be solved.

This solution will also be convenient to find the said circle of arbitrary magnitude, provided that its diameter is larger than *BZ* because it is in our power to assume the points *K* and *P* to be as close or distant from *Z* as we wish. Whence without any division of *QB* in half it will be sufficient to assign points *K* and *P* by two distances *BK* and *QP* equal among each other and also to the given ones.[54]

[54] Benedetti (1585, 263), page 357 in the present edition.

Guidobaldo commented:

> In omnibus casibus, divisa BQ bifariam, quod quidem punctum fiat centrum, circulus descriptus per BQ transiens semper datos circulos in punctis BQ continget
>
> In all cases, if BQ is divided in half, what makes, of course, that point to be the center, the circle described going through BQ will always touch the given circles in the points BQ

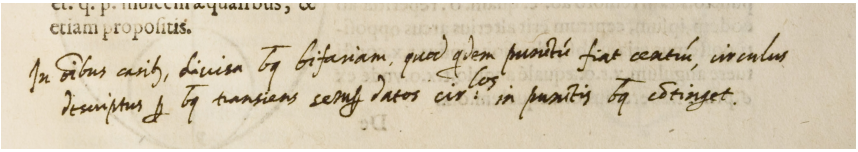

Figure 7.36: Marginal note at the bottom of the first letter.

7.15 Letter to Mercato: rejecting an attempt to improve on Archimedes

The chapter is a letter entitled:

> Considerationes nonnullae in Archimedem. Doctissimo atque Reverendo Domino Vincentio Mercato.
>
> Some notes on Archimedes. To the most Learned and Reverend Lord Vincenzio Mercato.[55]

The chapter also belongs to that part of Benedetti's treatise in which he collected letters to illustrious personalities. In the present letter Benedetti dealt with

> [...] duas Archimedis propositiones, quae in translatione Tartaleae sunt sub numeris .4. et .5. & in impressione Basileae sub numeris .6. et .7. [...]
>
> [...] two propositions of Archimedes that appear under the numbers 4 and 5 in Tartaglia's translation and under the numbers 6 and 7 in the Basel edition [...].[56]

[55] Benedetti (1585, 380–396), pages 359–375 in the present edition. Translation in Drake and Drabkin (1969, 235–237).

[56] Benedetti (1585, 380), page 359 in the present edition. Translation in Drake and Drabkin (1969, 235–236).

He aimed at improving Archimedes' arguments with which, as he also wrote,

> […] the mind cannot rest altogether satisfied.[57]

In his first argument he found a way of suggesting a derivation of the law of the lever by a procedure well known from Galileo's later treatise on mechanics, i.e. by redistributing weights with the help of a suspension mechanism that does not change the center of gravity.[58] Imagine, to begin with, a balance suspended from its midpoint carrying two equal weights on its arms (see figure 7.37). In a first step Benedetti considered all parts of the weights evenly distributed along the entire length of the beam, with the center of gravity remaining in the middle. Then he imagined a construction by which the beam is suspended from a point right above the center of gravity by means of two lines (which Galileo later visualized as strings) which are positioned at unequal distances from the center of gravity. In a second step Benedetti considered the beam to be cut in such a way that each of the strings carries the broken parts of the balance from their centers of gravity. From the geometry of the situation and the initial assumption of the uniform distribution of weight the rest then follows, and is indeed left to the reader.

The second arguement deals with the steelyard and how its equilibrium is disturbed by moving the counterpoise. On this second argument Benedetti wrote:

> Illa vero propositio, quam tibi dixi Archimedem tacuisse in huiusmodi materia est, quod si duo pondera aequilibrant ab extremis alicuius staterae, in certis praefixis distantiis a centro. Tunc dico si eorum uno manente alterum moveatur remotius ab ipso centro quod illud descendet, et si vicinius ipsi centro appensum fuerit ascendet.
>
> The proposition about which I told you that Archimedes was silent deals with the subject of two weights in equilibrium at the ends of a steelyard at certain predetermined distances from the fulcrum. I say that, if one of these weights remains stationary and the other is moved farther from the fulcrum, that second

[57] Drake and Drabkin (1969, 235).

[58] Compare Favaro (1968, vol. 2, 161–163) and Galilei (1960a, 153–154) and the discussion in section 3.10.

> weight will fall; while if that weight is appended nearer the fulcrum, it will rise.[59]

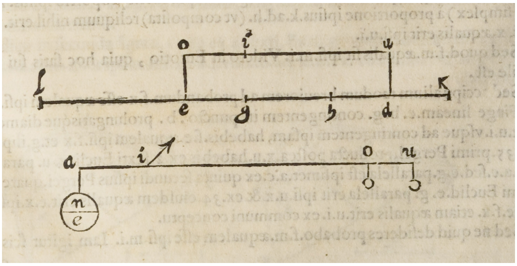

Figure 7.37: Drawing for a proof of the law of the lever in the second letter of Benedetti's book with marginal notes of Guidobaldo.

In his marginal comments Guidobaldo expressed little understanding for Benedetti's approach. Probably he was skeptical about Benedetti's pretension to improve on Archimedes. In his first comment Guidobaldo argued against Benedetti that it is impossible for both the midpoint of the beam of the balance and the point right above it to be centers of gravity of the weights under consideration. This was clearly a misunderstanding triggered by Benedetti's somewhat sloppy use of the word *center* for the point of suspension above the proper center of gravity.

More specifically, Guidobaldo's first comment refers to the passage at the top of page 381:

> imagineris etiam *OU* quae sit parallela ipsi *LK* quae divisa sit in puncto *I* supra *G*. Hinc nulli dubium erit, cum *G* fuerit centrum totius ponderis appensi ipsi *LK* quod *I* similiter erit centrum cum directe locatum sit supra *G* hoc est in eadem directionis linea, quod quidem non indiget aliqua demonstratione, cum per se satis pateat.

> Imagine also *OU*, parallel to *LK* and divided at point *I* above *G*. Thus no one can doubt, since *G* was the center of the whole weight suspended from *LK*, that *I* similarly will be the center, since it is situated directly above *G*, that is in the same line of

[59]Benedetti (1585, 381), page 360 in the present edition. Translation modified from Drake and Drabkin (1969, 236).

direction. And this needs no demonstration, since it is quite clear by itself.[60]

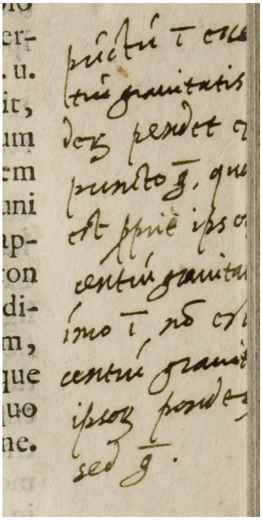

Figure 7.38: First marginal note to the second letter of Benedetti's book.

Guidobaldo commented:

> punctum *I* esse [cen]trum gravitatis. Deorsum pendet e[x] puncto *G*, quod est proprie ipso[rum] centrum gravita[tis] immo *I* non est centrum gravitat[is] ipsorum ponderum sed *G*.
>
> [he claims] that the point *I* is the center of gravity [of the weights on the balance]. It hangs down from the point *G* which properly is their center of gravity therefore *I* is not the center of gravity of these weights but *G*.

In his second comment Guidobaldo caught another oversight by Benedetti. He criticized Benedetti for not correctly expressing the inverse proportion in the law of the lever, which is indeed the case. The second comment refers to the passage in the penultimate paragraph of the page (see figure 7.37):

[60]Benedetti (1585, 381), page 360 in the present edition. Translation in Drake and Drabkin (1969, 169).

Sit exempli gratia statera AU cuius centrum sit I et pondera U A appesa, se invicem habeant ut IU et IA se invicem habent.

Suppose, for example, that there is a steelyard AU, with fulcrum I and weights U and A appended, and suppose that they are to each other as IU to IA.[61]

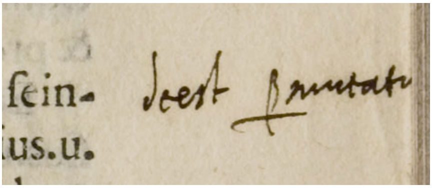

Figure 7.39: Second marginal note to the second letter of Benedetti's book.

Guidobaldo noted:

deest permutatio

the permutation is lacking

As mentioned above, Benedetti's idea of how to improve Archimedes' demonstration of the law of the lever was later taken up and elaborated by Galileo.

[61] Benedetti (1585, 381), page 360 in the present edition. Translation modified from Drake and Drabkin (1969, 236).

Chapter 8
Conclusion

In this book we have presented two hitherto unknown, seemingly "marginal" sources, in the true sense of the word: Guidobaldo del Monte's marginal annotations to two key texts of medieval and Renaissance mechanics by Jordanus de Nemore and Giovanni Battista Benedetti, respectively. These annotations shed new light on a long forgotten and seemingly insignificant controversy about the indifferent equilibrium of a balance. This controversy in fact played a crucial role in the emergence of central concepts of mechanical knowledge. At the same time, it constituted a *transformation of antiquity* in the sense of the construction of an authoritative reference culture – in this case for mechanical arguments – under premises that were themselves created by this culture. These included the transmission of practical and theoretical knowledge about the balance.[1]

In the course of the controversy, the reference to an authoritative and encompassing tradition such as Aristotelian natural philosophy – whether affirmative or negative – provided a model and a reservoir of theoretical knowledge. It sharpened and multiplied conceptual tensions and embedded the specific issue of the equilibrium of a balance in a wider network of knowledge such as the cosmological question of the shape of the earth. Paradoxically, the controversy also resulted in a challenge to the canonical status of this framework by creating a plurality of perspectives on the ancient heritage.

In our historical analysis of this controvery, we have followed an unconventional approach. Rather than concentrating on a "thick description," in the sense of providing a rich context to render a distant historical situation meaningful for the observing historian, we have attempted to develop what one may call a "long description," in the sense of tracing into the deep past the chain of historical dependencies that lie behind a specific event. In this way, we have been able to describe the fate of a

[1] For the concept of *transformation of antiquity* we are indebted to joint work with Hartmut Böhme, Georg Töpfer and other colleagues in the context of the SFB 644 Project, see Böhme et al. (2011). For a specific treatment of the transformation of ancient mechanics from this viewpoint, see Damerow and Renn (2010).

crucial component of mechanical knowledge as the result of a connected – but not continuous – history in which long-term cumulative effects, the intermittent transmission of material culture and written documents, contingent events, varying cultural contexts, and perspectival changes left a lasting imprint on the conceptual organization of this knowledge.

The changing effect of a weight or a force in dependence on the mechanical constellation in which this effect is exerted was at the core of our study. Formulated in this way, it is immediately evident that our theme touches both on fundamental human experiences with mechanical devices and on a variety of modern physical concepts such as the vector component of a force, the torque, or the mechanical work. Not only did we study how our historical actors were able to deal with their mechanical problems without such modern concepts at their disposal, we were also interested in reconstructing the long-term learning and unlearning experiences connected with the historically changing capability to address this challenge and in the conditions on which this capability depended.

In order to describe these learning processes, we have referred to some of the core experiences underlying mechanical knowledge. In particular, we have identified a small set of core experiences that have shaped early mechanical knowledge, such as basic human experiences with force and motion, with the equilibrium of equal-arms balances, and the novel experiences made possible by the invention of the balance with unequal arms in Greek antiquity. Two fundamental concepts of shared theoretical mechanical knowledge played a central role in this study: the concepts of *center of gravity* and of *positional heaviness*. These were shown to result from alternative reflective abstractions based on the same core experiences with the equilibrium of a balance.

The analysis made clear, first of all, that the competence in dealing with the changing effect of a weight was strongly shaped by specific historical contexts, including the existence of overarching conceptual systems such as that of Aristotelian natural philosophy. But it has also become evident that an equally important role was played by the existence and character of the long-term historical transmission processes that bridged the various contexts and historical periods. In addition, the mode of appropriation of the transmitted knowledge has turned out to be of crucial importance for its formation.

Thus, the Aristotelian *Mechanical Problems* gave a written record of insights emerging from the identification of the lever and the unequal-arms balance. They appeared in a specific historical context in which mechanical devices posed a philosophical problem since they seemed to contradict the principle that large effects require large forces. The mechanical writings of Archimedes, which established the concept of *center of gravity*, were part of Greek mathematical literature that emphasized virtuosity in solving abstract problems. Heron's treatise on mechanics, containing a compilation of elements of prior Greek mechanical knowledge, was probably representative of Alexandrian technical literature. The works of Thābit, al-Isfizari, and al-Khāzini were written in an Islamic empire in which far-reaching commercial relations and Greek philosophical and scientific education were key factors. Accordingly, they focused on the balance as an important commercial instrument, conceptualizing it within an Aristotelian context. Jordanus, writing at the beginning of Latin medieval scholasticism, followed in their footsteps and, by adding a sophisticated Aristotelian twist to the understanding of the effect of a weight, formulated the concept of *positional heaviness.* Early modern authors such as Tartaglia and Cardano addressed the challenging objects of their time on the basis of whatever they could assemble in terms of transmitted sources. Later early modern writers such as Guidobaldo and Benedetti attempted to reconcile and systematize this fragmented intellectual heritage, and in particular the concepts of *positional heaviness* and *center of gravity.* Galileo and his followers began to create a new synthesis based on their work.

While this summary may seem to suggest a cumulative, if not progressive historical process, this is not what we wish to claim. On the contrary, with regard to the question of the behavior of an equilibrated balance removed from its horizontal default position, Archimedes himself, in the third century BCE may well have been in a position to argue in favor of its indifferent equilibrium. We cannot know this for certain as too much of his work has been lost. But what is clear is that this insight had been formulated and then forgotten several times in the course of the long history of mechanical knowledge. Al-Khāzini in the twelfth century included it in his work; Leonardo da Vinci probably found it independently more than three hundred years later. And Guidobaldo del Monte, about another century later, claimed it as a key insight of his own research in mechanics. Ironically, this alledged key insight ultimately turned out to be untenable when confronted with the implications of a spherical earth. What had been transmitted across different cultures and long historical distances were evidently not specific insights, but rather some of the means to gain them. Indeed, for all we know, there was a continuous tradition of the practical knowledge needed to produce and use balances with equal and with unequal arms, from Greek antiquity via the Arabic world to the early modern period, probably even reaching China via the Silk Road.

Also some elements of theoretical knowledge, such as the law of the lever, basic features of Aristotelian dynamics, and the Euclidean model of organizing scientific knowledge have been more or less continuously, even if sometimes rather tenuously, transmitted in the larger Mediterranean world. This knowledge substratum was evidently sufficient to provide the stimulus for tackling over and again similar mechanical problems such as that of the equilibrium of a balance and to offer solutions to them. These may have differed considerably and were, in any case, often lost in transmission.

But such losses in transmission constituted a remarkable force of innovation in the development of mechanical knowledge. In the ninth century, Thābit ibn Qurra attempted to clarify an unidentified Greek work on the balance, evidently unaware of the Archimedean proof of the law of the lever. In the course of this attempt, he introduced an approach that justified the law of the lever on the basis of Aristotelian dynamics. This would have far-reaching consequences, in particular, for the later formulation of the concepts of *positional heaviness* and for the *work principle*. In the thirteenth century, Jordanus tried to rebuild a science of weights according to the Euclidean model and Aristotelian standards, although he lacked knowledge of Archimedes and the more sophisticated Arabic treatises on the balance. In consequence, he introduced the concept of *positional heaviness*, which enabled him to address novel problems such as that of the inclined plane. Finally, the fragmentary character of mechanical knowledge transmitted to the early modern period forced its protagonists to concentrate their efforts on probing its consistency and establishing innovative connections among its basic concepts. This gave rise to attempts such as those of Maurolico and later Galileo to build a science of mechanics around the concept of *momentum*.

With regard to our central issue, the equilibrium controversy, the early modern situation differed significantly from that of earlier periods. This was not due to special new insights or approaches concerning, for instance, the experimental method or the increased interest in phenomena of motion. The fact that so many scholars dealt with this controversy in one century, probably more so than in all the preceding historical periods, made a major difference, as did the unprecedented ease of communication facilitated by the availability of paper and print. As we have seen, what had been accumulated over previous centuries was not necessarily the knowledge, which had often been lost, but rather the potential for knowing. This was embodied in such means for representing knowledge, but also in a material culture that offered ever more challenging objects on which to probe and develop whatever fragments of knowledge had become available.

Whatever appears to us in hindsight as major breakthroughs, such as the establishment of the indifferent equilibrium by Guidobaldo, the successful treatment of the bent lever by Benedetti, or the connection that Galileo established between the bent lever and the inclined plane, thus result as much from the long and fragmented history of mechanical knowledge as from the intense exploitation of this history in the era of preclassical mechanics. In rapid succession, Guidobaldo revived the Archimedean treatment of the balance, Benedetti extracted from Jordanus a generally applicable method for treating the bent lever, and Galileo combined such advances in his own pivotal work on mechanics. The fact that Galileo did not properly acknowledge his sources, banning Benedetti's name from his work, is as much a characteristic of this time of intense scholarly competition as the cumulative advance that was nevertheless achieved.

In the early modern period, the network of knowledge spanned by such insights and their connections became ever more dense. This led to an unprecedented stabilization of this network, and also to its transformation into a system of knowledge in which these insights could be derived from the fundamental principles of classical mechanics, which matured in the following centuries. This advancement made it much less likely, albeit not impossible, that insights such as those incorporated in the different contributions to the equilibrium controversy could again be lost. The future long-term stability of this achievement will also depend, however, on whether and to what extent the other equilibria mentioned in the introduction, concerning the relation between humanity and its environment, for instance, or the equilibrium within human society usually designated as "justice" and symbolized by a balance in equilibrium, can be maintained and supported by the development of science.

Berlin, December 2011.


CENTVRIA. I. 38

A·MAS·DISTANCIA·MAS PESO

EMBLEMA 38.

La fê, la lealtad, cordura, y ſeſo,
Se prueua comunmẽte en el auſencia,
Biẽ como en la romana al mucho peſo,
La peſa va huyendo ſu preſencia:
Y en diametro pueſta, tanto exceſſo,
Contraſta, en la diſtante diferencia,
Que eſtãdo en vn fiel, las haze iguales
Aqui vna libra, y alli diez quintales.

F 2 Eſte

Figure 8.1: Emblem showing a steelyard, illustrating that belief, sincerity, intelligence, and understanding are best gauged at a distance. From Covarrubias Horozco (1610).

Chapter 9
Timeline

Early third millenium BCE
The balance with equal arms is introduced in Mesopotamia and Egypt
(Damerow et al., 2002, 93)

Late fifth century BCE
First mention of a balance with unequal arms in Greek literature
(Damerow et al., 2002, 95)

Fourth century BCE
The question of the return of a balance to its original position is raised in the Aristotelian *Mechanical Problems*
(Aristotle, 1980)

Third century BCE
The concept of *center of gravity* is introduced in Archimedes' *Equilibrium of Planes* and employed in a proof of the law of the lever
(Archimedes, 1953)

First century
The problem of the bent lever is treated in Heron's *Mechanics*
(Heron of Alexandria, 1900)

Early fourth century
Part of Heron's work is quoted in Pappus' *Collection*
(Pappus of Alexandria, 1588)

Before 901
The law of the lever is proven from Aristotelian dynamic principles in Thābit ibn Qurra's *Book on the Steelyard*
(Abattouy, 2001)

1048–1116
Thābit's work is elaborated in Al-Muzaffar al Isfizari's *Guiding the Learned Men in the Art of the Steelyard*
(Abattouy, 2001)

1121–1122
Abu al-Fath Khāzini argues for the indifferent equilibrium of a balance in his *Book on the Balance of Wisdom*
(Abattouy, 2001)

After the late eleventh or early twelfth century
Manuscripts of the Aristotelian *Mechanical Problems* begin to spread to the Latin West from Byzantine sources
(see section 3.4.2)

Twelfth century
A version of Thābit's work, suggesting that the balance returns to its original position, is translated into Latin, probably by Gerard of Cremona, under the title *Liber Karastonis*
(Moody and Clagett, 1960)

Thirteenth century
Jordanus de Nemore argues, with the help of the newly introduced concept of *positional heaviness*, that the equilibrated balance returns to its original position in his contributions to the *science of weights*
(Moody and Clagett, 1960)

The work of Archimedes, and in particular the concept of *center of gravity*, becomes known in the Latin Middle Ages through the translations of Willem of Moerbeke
(Clagett, 1984)

1452–1519
In his manuscript notes Leonardo da Vinci argues, using the concept of *center of gravity*, for the indifferent equilibrium of the equilibrated balance
(see section 3.4.2)

1495–1498
The first printed edition of the Aristotelian corpus, including the *Mechanical Problems*, is published by Aldo Manuzio
(Aristoteles, 1498)

1533
Petro Apianus publishes Jordanus' *Liber de Ponderibus* containing the claim that the balance returns to its original position
(de Nemore, 1533)

1546
Niccolò Tartaglia publishes his *Quesiti, et inventioni diverse* exploiting the work of Jordanus and defending the claim that the balance returns to its original position
(Tartaglia, 1546)

1548
Francesco Maurolico defines the concept of *momentum* in his *Archimedis de momentis aequalibus.* His work remains unpublished until 1685
(Maurolico, 1685a)

1550
Girolamo Cardano proposes various measures of *positional heaviness* and defends the claim that the balance returns to its original position in his *Hieronymi Cardani medici mediolanensis de subtilitate libri XXI*
(Cardano, 1550)

1565
Niccolò Tartaglia publishes an edition of Jordanus' *De ratione ponderis* containing an argument pointing toward a new measure of *positional heaviness*
(de Nemore, 1565)

1577
Guidobaldo del Monte argues in his *Mechanicorum Liber*, using the concept of *center of gravity*, that the balance does not return to its original position and claims this insight into its indifferent equilibrium as his own major contribution
(DelMonte, 1577)

1581
In the Italian edition of the *Mechanicorum Liber* Guidobaldo del Monte refers to experimental evidence in favor of his claim
(DelMonte, 1581) (see page 87ff.)

1585
Giovanni Battista Benedetti introduces, in his *Diversarum speculationum mathematicarum et physicarum liber*, a "new measure" for the positional effect of a weight or a force and argues for an indifferent equilibrium of the balance under terrestrial circumstances and for the claim that the balance tilts into the vertical if the spherical shape of the earth is taken into account
(Benedetti, 1585)

1588
Guidobaldo del Monte insists on a strictly Archimedean approach to the treatment of the balance in his *In duos Archimedis aequeponderantium libros paraphrasis*
(DelMonte, 1588)

After ca. 1592
Galileo Galilei takes over Benedetti's measure of positional heaviness and introduces the concept of *momento*. Together with the concept of *center of gravity*, defined in terms of *momento*, this becomes the basis for his treatment of mechanical problems such as the inclined plane
(see his treatise *Le mechaniche* (1909b), written in the 1590s and later published in French as *Les méchaniques* (1634))

Chapter 10
Online sources

The ECHO project (European Cultural Heritage Online) of the Max Planck Institute for the History of Science is continuously extending its collection of sources. In collaboration with other institutions, these are made freely accessible as text files in XML format and/or as high quality images via the website: echo.mpiwg-berlin.mpg.de. The sources mentioned in the present publication and listed below are currently accessible in this way. For their cooperation and support, we are particularly grateful to the Bayerische Staatsbibliothek in Munich, the Biblioteca Nazionale Centrale in Florence, the Bibliothèque Nationale de France, the Biblioteca Oliveriani in Pesaro, the Bibliothek Werner Oechslin in Einsiedeln, the Linda Hall Library in Kansas City, the Museo Galileo in Florence, the Museo Leonardiano and Biblioteca Leonardiana in Vinci, the Niedersächsische Staats- und Landesbibliothek in Göttingen, and the University of Oklahoma Libraries.

10.1 The first editions of Benedetti's *Diversarum speculationum mathematicarum et physicarum liber* and of Guidobaldo del Monte's *Mechanicorum liber*

DelMonte 1577
Benedetti 1585
– supplemented with Guidobaldo's handwritten marginalia

10.2 Early modern printed treatises on mechanics

Tomeo 1525
de Nemore 1533
– supplemented with Guidobaldo's handwritten marginalia
Apianus 1541
Archimedes 1543a
Archimedes 1543b

Tartaglia 1546
Cardano 1550
Benedetti 1553
Commandino 1565
de Nemore 1565
Piccolomini 1565
DelMonte 1581
Aristoteles 1585
Stevin 1586
DelMonte 1588
Stelliola 1597
DelMonte 1615
Baldi 1621
Galilei 1655a
Galilei 1655b
Maurolico 1685a

10.3 Other printed Renaissance sources

Taisnier 1562
Vitruvius 1567
Cardano 1570
Benedetti 1574
Benedetti 1579
DelMonte 1600
Pappus of Alexandria 1660
Maurolico 1685b

10.4 Renaissance manuscript sources

Galileo 1589
Galilei 1602
Galilei 1634
Baldi 1707

Chapter 11
Appendix: Analyses of iron gall inks by means of X-ray fluorescence analysis

Oliver Hahn and *Timo Wolff*

11.1 Introduction

A copy of the first edition of Giovanni Battista Benedetti's *Diversarum speculationum mathematicarum et physicarum liber* exhibits some notes in the margins by Guidobaldo del Monte that were written with iron gall ink. Some of these comments were covered with deletions also carried out with iron gall ink. The study presented here was carried out with the aim to find out if it would be possible to read the original comments underneath the deletions.

The application of band pass filter infrared reflectography technique or fast scan X-ray fluorescence mapping requires a distinct difference between both ink materials. By means of X-ray fluorescence analysis (XRF) we have tried to find out the elemental composition fingerprint to distinguish between both inks.

11.2 Iron gall ink

Iron gall ink is the most used drawing and writing material in Western history.[1] In general, it is produced from four basic ingredients: galls, vitriol, gum Arabic as a binding medium and an aqueous medium such as wine, beer or vinegar. By mixing gallic acid with iron sulphate, a water-soluble ferrous gallate complex is formed. Due to its solubility, the ink penetrates the parchment surface, making it difficult to erase. When exposed to oxygen, a ferric gallate pigment is formed. This complex is not water-soluble, which contributes to its indelibility as writing ink. Due to the variety of different recipes[2] and the natural origin of different materials,

[1] Krekel (1999).
[2] Oltrogge (2005).

there is a wide range of different components and impurities in historical iron gall inks.

Vitriol, the main source of iron in the iron gall inks, was obtained from different mines and by various techniques.[3] Therefore, inks contain many other metals, such as copper, aluminium, zinc and manganese, in addition to the iron sulphate. These metals do not contribute to color formation in the ink solution but possibly change the chemical properties of the inks. The determination of different inorganic components in iron gall inks provides the basis for the differentiation of these writing materials.

11.3 The archaeometric fingerprint method

The chemical composition including trace components is a characteristic property of an art object. As mentioned before the qualitative and quantitative investigation of minor or trace components leads to composition fingerprints which allow to differentiate between varying iron gall inks. This is usually not possible by means of visual examination or with further non-destructive techniques.

For a reliable quantitative analysis of the XRF-data, several technical parameters have to be taken into account. Due to the fact that light elements such as carbon, oxygen and hydrogen, which are not detected with energy dispersive XRF, are the main components of the ink as well as of some inorganic additives to paper, an absolute quantification is not possible. However, the determination of a fingerprint which represents the amount of a detectable trace element in relation to the main compound iron enables the characterisation of different iron gall inks. As mentioned before iron gall inks contain many other metals, such as copper, aluminium, zinc and manganese, in addition to the main inorganic component iron sulphate. The fingerprint method relies on the determination of characteristic elemental compositions in samples.

In the fingerprint model, the ink-paper system is regarded as a three-layer-model-system with a top layer of iron gall ink, a diffusion layer with a linear decreasing amount of ink and a bottom layer consisting of paper. For this system of layers a fundamental parameter approach[4] for the primary fluorescence is made. The primary intensity can be expressed as the sum of the contributions from the ink and from the paper. The successive computations take into account all physical phenomena which contribute

[3] Hickel (1963); Lucarelli and Mandò (1996).
[4] Wolff (2009).

to the formation of X-ray fluorescence. After all, this leads to the fingerprint value, which depends on three parameters: the transmission of the entire system, the penetration depth of the ink into the paper and the ratio of the absorption coefficients. For all minor constituents i such as copper, aluminium, zinc and manganese a fingerprint value W_{i} can be specified. This fingerprint value represents the amount of a minor constituent relative to the main compound iron (e.g., W_{Cu} = concentration (Cu) / concentration (Fe)).[5]

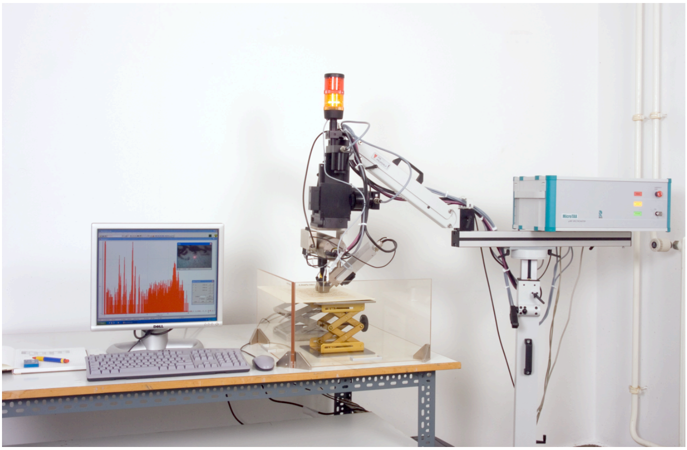

Figure 11.1: Mobile XRF spectrometer.

11.4 Experimental set-up

Analyses were carried out with the mobile energy dispersive micro-X-ray spectrometer ArtTAX® (formerly Röntec-GmbH, Berlin, Germany, see figure 11.1), which consists of an air-cooled low-power molybdenium tube, polycapillary X-ray optics (measuring spot size 70 µm diameter), an electrothermally cooled Xflash detector, and a CCD camera for sample positioning. Furthermore, additional open helium purging in the excitation and detection paths enables the determination of light elements ($11 < Z < 20$) without vacuum. All measurements are made using a 30 W low-power Mo tube, 45 kV, 600 µA, and with an acquisition time of 25 s (live time) to minimize the risk of damage. For better statistics, at least

[5] Malzer et al. (2004).

ten single measurements were averaged for one data point. Further details concerning the method are described elsewhere.[6]

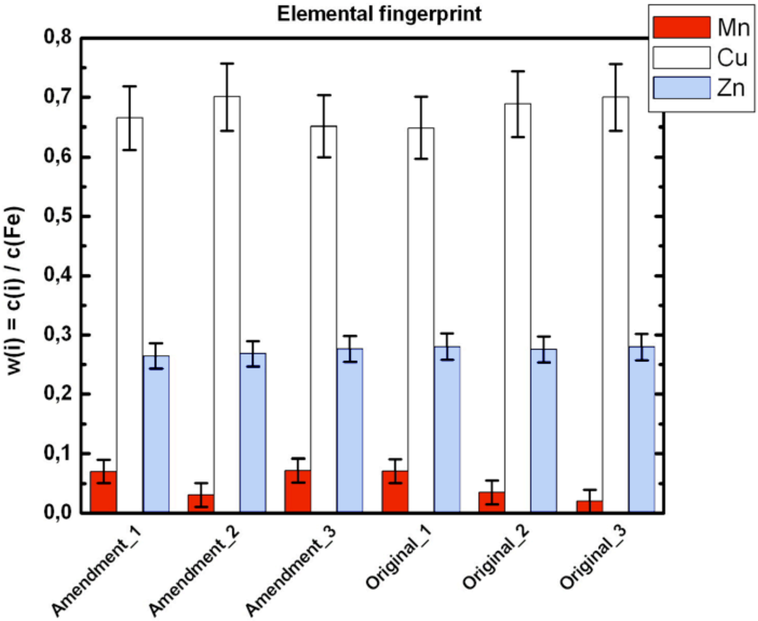

Figure 11.2: Elemental fingerprints (quantification by means of fundamental parameter approach) of "original" iron gall ink and "deletion" iron gall ink.

11.5 Results

The original iron gall inks as well as the deletion inks have the same elemental composition. Taking into account the measuring faults (see error bars in figure 11.2) it is obvious that there is no difference between "original ink" and "deletion ink." Due to this result it will not be possible to distinguish between both ink materials. As a further consequence it will not be possible to read the original comments underneath the deletions by use of non-destructive techniques.

[6]Bronk et al. (2001); Hahn et al. (2004); Wolff (2009).

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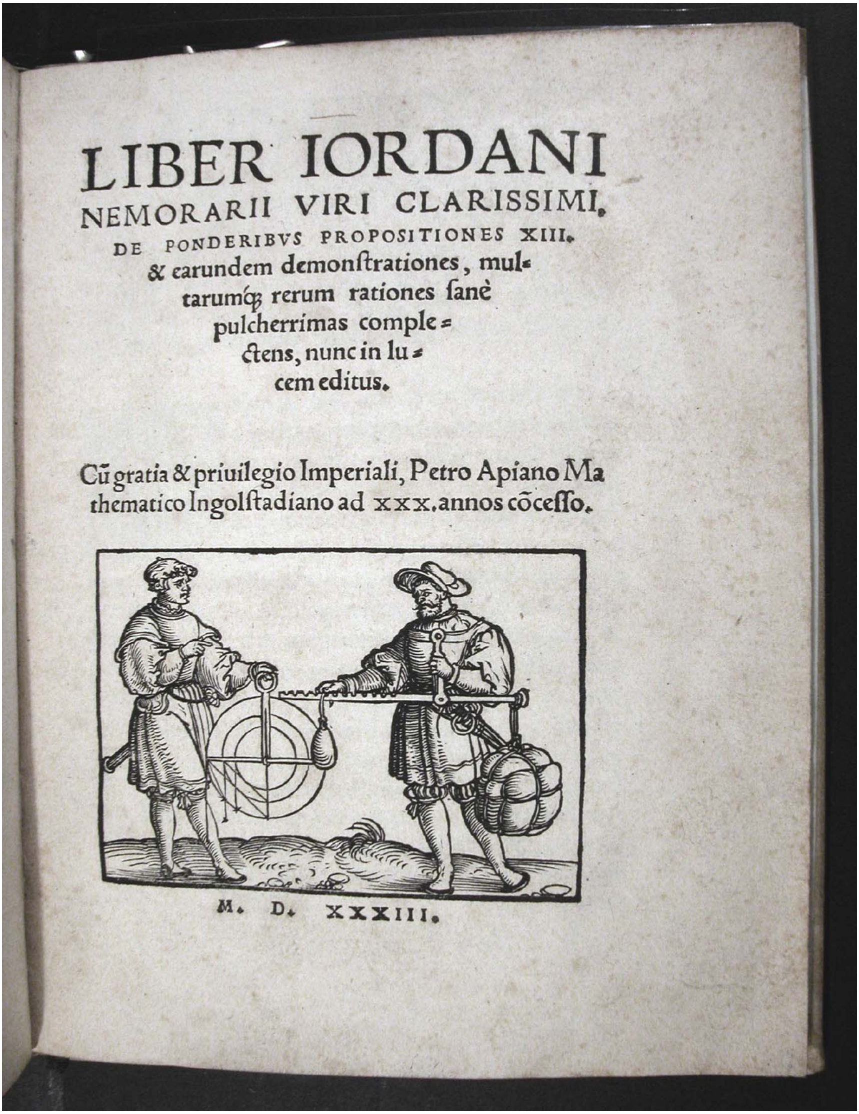

LIBER IORDANI
NEMORARII VIRI CLARISSIMI,
DE PONDERIBVS PROPOSITIONES XIII.
& earundem demonſtrationes, mul-
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cem editus.

Cũ gratia & priuilegio Imperiali, Petro Apiano Ma
thematico Ingolſtadiano ad XXX. annos cõceſſo.

M. D. XXXIII.

MAGNIFICO ET

NOBILISSIMO, ORNATISSIMOQVE VIRO AC D. Leonhardo ab Eck, à Vuolffeck & Randeck, Iuriſcõſulto, Oratori & Philoſopho inſigni, Illuſtriſſimorũ Boiariæ Ducũ ab intimis cõſilijs, uiro undecunq; maximo, humaniſſimoq;, Petrus Apianus ex Leyſnig, ſtudij Ingolſtadieñ. Mathematices profeſſor ordinarius, perpetuam felicitatem precatur & optat.

SEmper fuit ſua rebus literiſq; bonis dignitas, autoritas, et maieſtas, uir modis omnibus ornatiſſime, neq; id hoc noſtro ſeculo tantum, ſed & iam olim in primis bonarum artium fundamẽtis, ita ſolenniter omnibus ſeculis obſeruatum eſt, ut ſi quid eſſet, quo res literaria promoueri queat, ſollicite obſeruaretur, id ne periret facile. Sic factum eſt, ut & optimorum autorum ſcripta, & grauiſſimorum uirorum exempla ad nos uſq; peruenirent incorrupta, nulliſq; temporum iniurijs obnoxia, peritura aliàs, ſi defuiſſent, qui ea ꝓ innato tum ſtudio tum candore, ſtudioſe obſeruarent. Atq; ut olim non magnopere curatum eſt, ſiue quid nouũ ſiue recens eſſet, modo bonum foret, ita nunc uel maxime locum habere debet illa eiuſmodi rerũ æſtimatio, ut neminem bonum ac ſtudioſum à rei bonitate ac commēdatione, uel immodica uetuſtas, uel recens, alijsq́ʒ grata, alijs inuiſa nouitas, deterreat. Et utinã quidē ſic res literæq́ʒ oēs æſtimarentur, ſanè & maneret ſua debita artibus bonis dignitas, & auferret̄ iſta execrabilis hominum mentib. opinio, ſuſpicandi nimium uel antiqua uel noua pro peſſimis. Vſqʒ adeo firmiter inſedit inhæſitq́ʒ quorundam animis perniciosa iſta artib. rebusq́ʒ humanis omnibus ſuſpicio, ut alij uetuſtiora ꝓ peſſimis habeãt, alij uero, ut qcquid recentiſſimum eſt, ita maxime malum iudicent. Et falluntur ſanè atqʒ errãt utriqʒ. Nam nec ita mala fuerũt apud antiquos omnia, ut non quædam apud eos bona, quædam etiã optima inueneris. neqʒ ſic deplorata noſtra noſtrorumq́ʒ ſunt ingenia, ut non nunc quoqʒ reperias multa priſcis æquanda ſeculis, quædam etiã anteferenda. Equidem crediderim, uelimq́ʒ ſanè & optarim omnib. tum bonis tũ doctis perſuaſum eſſe, quatenus ſic conſulant & literariæ & ciuili reipublicæ, ut neqʒ ita nimium ueterum conſilijs inuentis ac monumentis ſint addicti, ut noſtra contemnant ac deſpiciãt oĩa, neqʒ ſic res huius ſeculi oēs obſeruēt, ut ſuſpectũ habeãt, quidquid uetuſtatē ſapit. Ita temperanda ſunt in rebus humanis omnia, omniaq́ʒ iuſto diſcrimine obſeruanda, ut ex ueteribus nouisq́ʒ deſumantur optima quæqʒ, ut nec illis antiquitas ſua, neqʒ ijs quicqʒ ſua incommodet nouitas. Id adeo nunc quoqʒ in

men-

mentem uenit mihi, ut inconsultum uideretur in hac parte quoq; communi deesse utilitati, persuasus itaq; sum facile, ut optimi hominis Iordani meliorē adhuc commentarium de ponderib. in lucem ederem, neq; diutius studiosos utilitate, quam ex hoc libello percipient, defraudarem. Igitur cum de libro in publicum edendo mecū deliberarem, operæpreciū uisum est, ut de patrono aliquo circunspicerem, de quo ubi diu multumq; mecū consultarem, tu præ multis Patrone omniū optime merite, animo meo occurristi unus, tibiq; adeo præcipue uisum est illū dedicare, qp & multa tuæ amplitudinis sunt de me, tale nil merito, merita, & ita tibi sunt studiosi oēs addictissimi, ut uel tuo nomine habituri sint libellum hunc commendatiorē, quem & si nullus tibi dedicassem, poteras tuo tibi iure uelut proprium uendicare. Tantum itaq; tuam oro amplitudinem, & genuinam humanitatem obtestor, ut quicquid hoc est munusculi, hilari lætaq; fronte suscipias, remq; literariam porrô promoueas. Valeat iam tua excellentia diu incolumis & salua, ut habeant studiosi doctiq; semper ad quem confugiant. Iterum uale ex Ingolstadio viij. Kalendas Martias, Anno M. D. XXXIII.

LIBER DE PONDERIBVS IORDANI NEMORARII.

CVm ſcientia de ponderibus ſit ſubalternata tam Ge-
ometriæ q̃ naturali Philoſophiæ, oportet in hac ſci-
entia quædam geometrice, quædam phyſice proba-
re. Primū ergo oportet ſcire, ꝙ brachium deſcenden
do in libra, deſcribit circulū, cuius circuli ſemidia-
meter, eſt ſemper æqualis brachio libræ. Secundo
oportet oſtendere, ꝙ maior arcus eiuſdem circuli,
eſt magis curuus minori, & ꝙ talis minor plus cur-
uatur, q̃ in circulo maiore. Primū probatur, quia minus de corda, quæ
eſt recta linea, correſpondet proportionaliter arcui maiori, q̃ minori,
non em̄ arcui duplo correſpondet corda dupla, ſed minus ea. Secun-
dum patet ſic, quia ſi ſumantur de circulo maiori & minori arcus æqua-
les, corda arcus maioris circuli longior eſt, ꝓpterea poſſet ex hoc oſten-
di, ꝙ pondus in libra tanto ſit leuius, quanto plus deſcendit in ſemicircu
lo. Incipiat igitur mobile deſcendere à ſummo ſemicirculi, & deſcendat
continue, dico tunc ꝙ maior arcus circuli plus contrariatur rectæ lineæ
q̃ minor, & caſus grauis per arcum maiorē, plus contrariatur caſui gra
uis, qui per rectam fieri debet, q̃ caſus per arcū minorē, patet, ergo ma-
ior eſt uiolentia in motu ſecundū arcum maiorē, q̃ ſecundum minorē,
aliter em̄ fieret motus magis grauis. Cum ergo plus in aſcenſu aliqd mo
uetur uiolentie, patet, qm̄ maior eſt grauitas ſecundū hunc ſitum, et quia
ſecundū ſituationē talium ſic fit, dicatur grauitas ſecundū ſitum in futu-
ro proceſſu. Ita em̄ ſyllogiſando de motu, tanq̃ motus ſit cauſa grauita-
tis & leuitatis, potius contrariū concludimus per cauſam huius contrari
etatis, plus contrariam, id eſt plus habere uiolentiæ, ꝙ ſi graue deſcen-
dat, hoc eſt à natura, ſed per lineam curuam, hoc eſt contra naturam, ideo
iſte deſcenſus eſt mixtus ex deſcenſu naturæ & uiolento. In aſcenſu uero
ponderis, cum ibi nihil ſit ſecundum naturam, licet argumentari ſicut
de igne, qui naturaliter aſcendit. De igne enim argumentatur in aſcenſu,
ſicut de graui in deſcenſu, ex quo ſequitur, Quanto graue plus ſic aſcen-
dit, tanto minus habet de leuitate ſecundum ſitum, & ſic plus habet de
grauitate ſecundum ſitum. Diceret forte aliquis, ꝙ nō oportet propter
prædicta, graue in parte circuli inferiori fieri ſecundum ſitum leuius, pa
tet unū non eſſe motum, ſed quietem, tunc nihil contrariū naturæ acqui-
ritur. Sed contra illud obijcitur ſic, poſſibile fuit hanc quietem fuiſſe ter-
minum intrinſecum motus, ſicut albationis albedo, cum igitur motus
non

non contrarientur, nisi quia termini contrariantur eorum. Et est propor
tio quietum inter se, & motuum inter se per locum à proportione, sequi-
tur tantam esse contrarietatem in quiescendo, sicut in mouendo. In termi
no enim cuiuscunq; motus intenditur, intenditur & uiget tota natura
in actu, qui in motu fit quasi in potentia, secundum quem fiebat contra-
rietatis suæ oppositio. Graue igitur in parte inferiori, siue moueatur si-
ue quiescat, leuius est secundum situm. Atq; eodem syllogismo necesse
est pondus grauius esse quodammodo & uelocius descendere, ꝙ moue
tur in circulo maiori, quia ut prius probatur, minus obliquatur, q̃ in
circulo minori, & per consequēs minus habet uiolentiæ, quia igitur mi-
nus distat descensus in circulo maiori à descensu naturali, qui fit per rectā
lineam, q̃ qui est in circulo minori. Dicatur descensus rectior, id est plus
tendens ad rectitudinem, atq; in circulo minori, ob rationem oppositā,
obliquior descensus. Quare uero superius dictum est in quiete esse con-
trarietatem, sicut in motu potest esse dubitatio, quia in eodem situ, ubi
est illa dependentia quietis obliquitatis, potest & rectitudinis, Sicut si la
pis suspendatur in tecto domus ad locum ponderis, & ꝙ pendeat in li-
bra. Sed dicendum ad hoc, ꝙ uarietas uiolentiæ, facit uarietatem quietū
secundum formam, quod manifestum est ex motuum ad quietes uaria-
tione. Ex eadem enim uiolentia fit totus ad quietem motus, & ipsa quies,
sicut patet ex prædictis, unde idem forte fit locus quietum naturaliter di-
uersarum. Istis igitur notis, sequuntur suppositiones libri Ponderum
& dicuntur suppositiones, quia per istam scientiam non debēt probari,
sed supponuntur, probari tamen ex iam dictis quædam indigent proba-
tione, sicut post apparebit. Sunt itaq; suppositiones septem. Prima
est, Omnis ponderosi motum ad medium esse. Secunda, Quāto gra-
uius tanto uelocius descendere. Tertia, Grauius esse in descendendo,
quanto eiusdem motus ad medium est rectior. Quarta, Secundum si-
tum grauius esse, quanto in eodem situ minus obliquus est descensus.
Quinta, Obliquiorem autē descensum minus capere de directo, in eadē
quantitate. Sexta, Minus graue aliud alio esse secundum situm, quan-
to descensus alterius consequitur contrario motu. Septima, Situm
æqualitatis esse æquidistantiam superficiei orizontis. Omnes autem
suppositiones sunt satis manifestæ, sicut patet per prædicta, et ideo pro-
positiones prosequi licet, & dicuntur propositiones, quia, ut probentur,
proponuntur. Sunt itaq; tredecim.

Propo-

PROPOSITIO PRIMA.

Inter quælibet duo grauia est uelocitas descenden do proprie, & ponderum eodem ordine sumpta pro portio, descensus autem, & cõtrarij motus, proportio eadem, sed permutata.

Dicitur proprie, ut excludãtur omnes uelocitates, quoquo modo præter naturam acquisitæ. Prima pars patet, quia cum uelocitatis pro-prie precisa causa sit pondus, patet, quo ad multiplicationem ponderis sequitur uelocitatis multiplicatio. Secunda pars patet, quia eadem est proportio descensus & ascensus, sed contrarie sumitur proportio hic & ibi, propter quod dicitur permutata. Sicut enim se habet in descensu pondus, ita aliud pondus in ascensu, quia eiusdem proportionis est di-stantia grauis in descendendo, in circulo superiori, sicut ascensus ab infe-riori, eadẽ igitur est ꝓportio, sed permutata. Oportet.n. quanto illud exce-dit, tanto id isto excedi. Et per consequens, quanto illud quod est graui-us, uelocius ascendit, tanto leuius mouetur contrarie.

Sequitur aliud commentum. Sint duo pondera, a maius, b minus. Sit etiam descensus a ab e in c, & descensus b. à b. in d. Dico ergo, ꝙ ea-dem est proportio a. ad b. quæ est à c. ad b. d. Sin autem, semper erit mi-nor, uel maior. Sit igitur primo minor, & sit e. excessus a. super b. & f. ex-cessus à c. super b. d. Cum ergo minor sit proportio a. ad b. q̃ a. c. ad b. d. erit a. ad e. maior proportio q̃ a. c. ad f. ut postea probatur. Sed f. est de-scensus e, eo ꝙ propter e. excedit descensus ipsius à descensu b. per f. Est igitur maior proportio huius ponderis scilicet a ad e pondus, q̃ descen-sus ad descensum, cum tamẽ Falsigraphus ponat contrarium, uidelicet minorem esse proportionem ponderis, q̃ descensuum. Vnde licet pro-betur contrarium, non tamen in eisdem ponderibus nihilominus stat probatio, eo ꝙ eadem est ratio in quibusdam ponderibus, & in omni-bus, uidelicet, ꝙ si in uno casu fuerit maior uel minor proportio ponde-rum ad pondus, q̃ descensus ad descensum, semper accidit eodem mo-do. Si maior fuerit proportio ponderis a ad pondus b, q̃ descensus a c ad descensum b d, erit minor proportio a ad e, q̃ a c ad f, ut postea pro-babitur. Sed f est descensus, ut prius probatum est, igitur accidit contra-rium ponenti, eo ꝙ concluditur minorem esse proportionem pon-deris ad pondus, q̃ descensus ad descensum. Sic igitur patet prima pars propositionis, ex qua sequitur secunda pars, cuius sensus est, uidelicet, ꝙ sicut a pondus se habet ad b pondus, sic ascensus b ponderis se habet

econ-

econtrario ad aſcenſum a ponderis, quãto enim a pondus ex grauitate
ſua plus inclinat ad deſcenſum, tanto plus ex eadem grauitate declinat
ad aſcenſum. Et quanto b minus ex ſua grauitate declinat ad deſcenſum,
tanto minus ex eadẽ grauitate declinat ad aſcenſum, id eſt, tanto minus
reſiſtit trahenti ipſum ad ſuperius. Igitur eadem eſt proportio deſcenſus
a ad deſcenſum b, quæ eſt aſcenſus b, ad aſcenſum a. Sed deſcenſus a ad
deſcenſum b eſt, ſicut a pondus ad b pondus. Igitur, ſicut a pondus ad b
pondus, ita aſcenſus b ad aſcenſum a, patet igitur ſecunda pars concluſi-
onis. Ex qua conſtare poteſt, ꝙ nõ intendit maior propriam partem, ꝙ
ſcilicet a pondus relictum ſit propriæ naturæ, maiori uelocitate moue-
retur in altero medio, uel aliud pertranſiret iſtius in eodem tempore ſe-
cundum proportionem quam habet b ad a, Sed maior nõ habet deter-
minare, de motu grauis relicti propriæ naturæ, ſed de motu grauis in
æquilibri cum reſiſtentia grauis poſiti in alio brachio æquilibris, hoc
autẽ patet per ſecundam partem concluſionis, in qua loquitur maior de
aſcenſu ponderis, cum tamen nõ aſcendat pondus naturaliter in medio,
in quo naturaliter deſcenderet, ſi ꝑmitteretur naturæ propriæ. Sed aſcen
dit in brachio æquilibris propter uiolentiam, quã inducit pondus alte-
rius brachii in deſcendendo Cum igitur proportio, quã maior innuit
inducere de aſcenſu huius probaretur per primam concluſionis, Iſta
probatio nõ ualeret, niſi ſumeretur deſcenſus in æquilibri in prima par
te concluſionis Et ſi ſic ſumatur, oportet tunc habere reſpectũ ad æqua-
litatem & inæqualitatem brachiorum. Vnde ideo notandum, ꝙ non
poteſt ſic intelligi concluſio, ꝙ ſicut deſcenſus a ad deſcenſum b, ita tota
grauitas a ſimpliciter, & ſecundũ ſitum ad totam grauitatem b ſimplici
ter & ſecundũ ſitum, & hoc debet ſtrictiſſime intelligi. Nam hoc non eſt
uerum, niſi quando eadem eſt proportio totius grauitatis ad totam gra
uitatem b, quæ eſt totius potentiæ a ſuper ſuam reſiſtentiã, & ad poten-
tiam b ſuper ſuam reſiſtentiam, & ſecundum hoc uariaretur uelocitas &
deſcenſus, aliter nõ ualeret propoſitio autoris. Nam ubi aduerſarius po-
nit, ꝙ maior eſt proportio deſcenſus a ad deſcẽſum b, ꝗ̃ a ad b, & autor
nihil aliud concludit, niſi ꝙ non eſt uniuerſaliter uerũ, ꝙ maior eſt pro-
portio deſcenſuũ, ꝗ̃ ponderum. Et hoc non repugnat dicto ab aduerſa-
rio, imò quandoqꝫ ſic eſt, quandoqꝫ econtrario. Et ideo ad hoc ꝙ conclu-
datur propoſitio uniuerſalis ex particulari data, oportet ſic intelligere
concluſionem Quòd in æquilibra b a c d centrum ſit a g, pondus in ſi
tu c ſe habet ad idem pondus g in ſitu d, ſecundũ proportionem totius
deſcenſus, quẽ poteſt habere in ſitu c ad totũ deſcenſum, quẽ poteſt habe
re in ſitu d. Ex quo ergo nõ poteſt ulterius deſcendere, niſi ſecundum
quantitatem ſemidiametri, cuius circumferentiã deſcribit. Sic ſequitur
ex iſta expoſitione, ꝙ g pondus in c ſitum, ſe habet ad idem g pondus in

B ſitu

situ d secundum proportionem c a ad d a, ita, ꝙ g pondus in c situ suffi-
ceret cum maiori pondere, in alio brachio sufficeret descendere in d situ,
ꝙ se haberet ad primum pondus locatum secundum proportionem c a
ad d a. Et hoc pro sensu primæ partis conclusionis. Item pro sensu se-
cundæ partis conclusionis, dico, ꝙ si b pondus sufficeret pondus leuare
in d c situm ad lineam directiōis, unū aliud pōdus ꝙ æque faciliter leua
ret g in d situ ad lineā directiōis, se haberet ad b secundum proportio-
nem d a ad c a. Vnde si ille sensus sit uerus in uno casu, uidetur ꝙ ita erit
in quolibet casu, ita ꝙ secundū illam expositionem non ualet uariatio
grauitatis, nisi propter uariatiōem situum. Igitur si in uno casu uariatur
grauitas eiusdem ponderis secundū ꝓportionem brachiorum, non est
maior ratio, quando ita erit in quolibet casu. Sic igitur intelligendo cō
clusionē, procedit propositio autoris, aliter non. Et sic intelligendo con
clusionē, est ad propositū octauæ cōclusionis, ad cuius probationē alle-
gatur illa cōclusio. Sed uidetur ꝙ ista expositio nō sufficiat pro sensu cō-
clusionis: Nam cōclusio ponit, ꝙ sicut pondus ad pondus, sic uelocitas
ad uelocitatem, cū tamen in ista expositione nō arguitur de uelocitate,
ergo persuaderi potest isto modo. Sit e pondus in eodem situ cum b ad
quē se habet g in situ d, sicut g in situ c se habet ad b, ergo ut prius ꝑ cōsi-
milem uiolentiam sufficit g in d situ agere in e g situ, sicut idem g in eo-
dem situ sufficit agere in b, ergo econtrario sufficit in d situ eleuare e ad
directionem, sicut a in c situ sufficit leuare b ad directionem, sed quia
æque cito deueniet b uel e, uel f g ad directionē, ergo & uelocitas g in d
situ, se habebit ad uelocitatem eius in c situ, secundū proportionem d a
ad c a per quintam Archimedis de curuis superficiebus, eo ꝙ eadem est
proportio diametrorum, uel semidiametrorū, uel circumferentiarum,
ergo etc. Si aūt istud argumentū non faciat fidē, nō est cura, tantū ꝙ ue-
locitas sit proportionalis uel non, dum tamen sequatur, Si g in d sufficit
leuare e, ꝙ g in c sufficit leuare b. Et iam prima conclusio textus Iordani
habet aliam literā, scilicet, ꝙ inter quælibet grauia sit uelocitatis & pon-
deris eodem ordine sumpta proportio. Et hoc etiam sufficit pro octaua
conclusione probanda, ad cuius probationem ista conclusio allegatur,
hoc igitur sufficit ad explicationem conclusionis. Iam igitur restat pro-
bare, ꝙ prius præmittebatur, uidelicet, Quòd si pondus maius se habet
ad minus in minori proportione, q̄ꝫ descensus maioris, ad descēsum mi
norem, pondus maius se habebit ad excessum suū supra minus, maiori
proportione, q̄ꝫ descensus maioris, ad excessum suum supra descensum
minoris. Sit enim pondus maius a b & minus c, & sit a excessus a b su-
per c. Item sit d e f descensus maioris ponderis, & g descensus minoris,
& sit d e f excessus, d e f ad g secundum ꝙ a b ad c, erit autem h f maior
e f per octauam quinti Euclidis. Tunc arguitur sic, Sicut d f ad h f, ita a b
ad

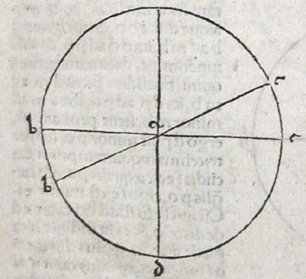
ad b g disiunctim per decimamseptimam quinti Euclidis. Sicut d h ad hf, ita a ad b g, & econtra, sicut b ad a, ita h f ad d h, ergo coniunctim per decimamoctauã quinti Euclidis. Sicut b a ad a, ita d ad h d, sed per octa- uam quinti Euclidis, maior est ipsius f d ad h d, q̃ ad e d. Igitur maior est proportio d b a ad a q̃ f d ad e d, sed a est excessus ponderum, & b æquale c a e d excessus descensuum, & e f æquale g. Sufficienter igitur pa tet intentum, idem igitur probatur ex tricesimaquarta quinti Euclidis, quæ est quinta propositio Archimedis, & hoc sic. Si maior sit proportio d f ad e f, q̃ a b ad b g, euersim per illam conclusionem tricesimam, mi- nor erit proportio d f ad e d, q̃ a b ad a. Eisdem medijs potest probari, Si maior sit proportio ponderis ad pondus, q̃ descensus ad descensum, ꝙ minor erit proportio eiusdem ponderis ad excessum super aliud, q̃ descensus super suũ excessum, q̃ supra alium excessum, & hoc est, quod ab initio promisimus demonstrare.

PROPOSITIO SECVNDA.

hec propositio falsa

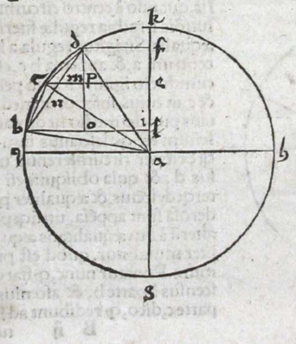
Cum fuerit æquilibris positio æqualis, æquis pon deribus appensis, ab æqualitate non discedet, etsi ab æquidistantia separet̃, ad æqualitatis sitũ reuertetur.

Primum patet, quia sunt æque grauia. Secundum patet per suppositi- onem quartã, uocatur autẽ illud situs, ꝙ circulus dicitur, sicut patet per prædicta. Aliud cõmentũ sequitur. Aequilibris positio dicitur æqua lis, quando à centro circumuo lutiõis brachia regulæ fuerint æqualia. Sit igitur regula a b c centrum a, & appensa b c, cir- cumducto igitur circulo per b & c, in cuius inferioris medie- tatis puncto medio sit d, mani- festum est, ꝙ descensus tam b q̃ c est per circumferentiã uer sus d, & quia obliquus est u- terq; descẽsus, & æqualiter põ derosa sunt appẽsa, utrũq; per alterũ à situ æqualitatis æqua- liter mutabitur, quod est pri- mum. Ponatur nunc, ꝙ fiat de scensus à parte b, & ascensus à parte c, dico, ꝙ redibunt ad si-



descensus

tum æqualitatis. Non enim ulterius descendet b, eo ꝙ descensus eius uersus d magis obliquus est, q̃ ascensus c ad æqualitatem, b enim & c iam æqualiter distant à situ æqualitatis, & c per descensum appropinquabit ad situm æqualitatis continue, & b per suũ descensum continue recedet à situ æqualitatis, igitur quilibet arcus, per quã descendet b, plus distat à situ æqualitatis, q̃ alius arcus per quem descendit c. Sed arcuum æqualiũ inæqualiter distantium à situ æqualitatis, ille minus capit de directo, qui plus distat, ut statim probabitur. Ergo per quãcunq; arcum descẽdit c, eius descensus rectior est, q̃ descensus b, igitur per quartam suppositionem c in illo situ grauius est, q̃ b, igitur per sextam suppositionem c descendit, b ascendit. Iam igitur restat probare, ꝙ arcuũ æqualium, inæqualiter distantiũ à situ æqualitatis, ille minus capit de directo, qui plus distat ab illo situ. Sicut.n. in circulo k q g h, cuius centrũ a, k g linea directionis, & q h linea æqualitatis, & sit arcus b c æqualis arcui c d, si tunc protrahatur d f & c e & b i æquedistanter q a h, dico, ꝙ f e linea, minor est q̃ e i linea, producta .n. linea b d secante liniam c e in puncto m, & a c secante b d in puncto n, erit b n æqualis n d, protractis etiam b c & c d cordis æqualibus, per uicesimamoctauã tertij Euclidis. Et protractis lineis b a & d a, erunt duo trianguli, scilicet, a b c & a c d, quorum angulus a unius, æqualis fit angulo a alterius, p octauã primi Euclidis, sed & duo latera b a & a n, sunt æqualia duobus lateribus n a & a d, trianguli n a d, ergo per quartam primi Euclidis b n & n d sunt æquales, quarũ b m est maior m d. Protracta igitur linea d o æquedistãter lineæ k l g, quæ secet lineam c e in puncto p, essent duo trianguli d m p & d b o similes, per secundam & quartam sexti Euclidis, eo ꝙ in p æque distat b o. Sicut igitur d b ad m b, ita d o ad p o, & disiunctim per decimamseptimã quinti Euclidis. Sicut d m ad m b, ita d p ad p o, sed d m est minor m b, sicut probatũ est, ergo d p est minor p o, sed per tricesimamquartam primi Euclidis f e est æqualis p d, & e i æqualis p o, ergo f e est minor e i. Cũ igitur f e sit illud quod capit c d de directo, & e i sit illud ꝙ c b capit de directo, q plus distat, eodẽ mõ patet, ꝙ minus capit q distat, q̃ sibi æqualis q nõ distat.



Propo-

et dextro inutilis est ... rsus. ut in nostro mecha... nicorum libro patet. ... equens vero recte con... cludit

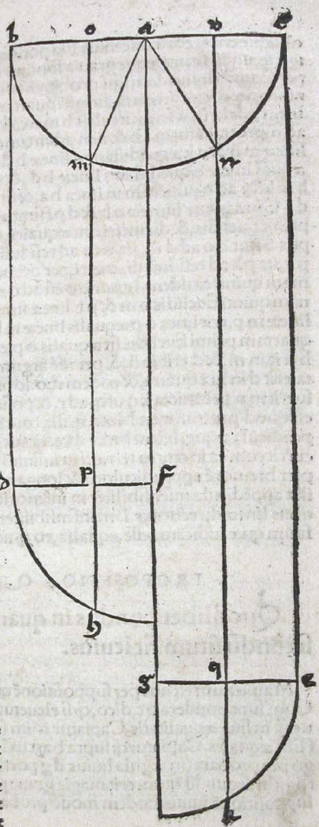
i

m

PROPOSITIO III.

Cũ fuerint appensorum pondera æqualia, non motũ faciet in æquilibri appendiculorum inæqualitas.

Non debet hic sumi inæqualitas appendiculorũ pondere, sed longitudine, probatur sic. Si fiat motus in una parte, ergo pars alia est min9 grauis, per suppositionẽ secundã, sed positum est prius appensorũ pondera esse æqualia, ergo. Sequitur aliud commentum. Sit regula a b c, cuius sit centrum a, & appẽdicula b d c e, longius autẽ c e, & breuius b d, & pondera æqualia appensa d & e. Sitq́ linea directionis a f g, quæ procedat qualibet, ducanturq́ d f & g e lineæ æquedistantes lineæ b a c, positisq́ centris g & f, describantur quartæ circulorũ per d & e, quæ erunt æquales, eo φ d f & g e semidiametri sunt æquales. Propter hoc, φ a b & c a sunt æquales, & d f est æqualis b a, & g e est æqualis a c per tricesimamquartam primi Euclidis, eo φ b d & c e lineæ æquedistant lineæ a f g. Cũ igitur iste quartæ circulorũ sunt æquales, & per istarum circũferentias, erit descensus d & e põderis, ut probabitur, æque



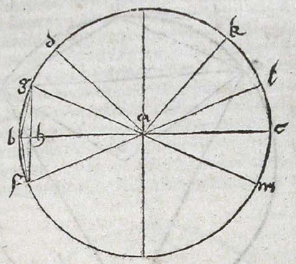
oblique erunt eorū descensus. Ista pondera sunt simpliciter æque grauia
igit̃ secundū situm æque grauia sunt, nõ igitur mutabit̃ regula hinc inde
per secundã huius. Iam igit̃ probandū est, ꝙ descensus d & e ponderis, ue
niunt per circumferentiã dictarū quartarum, ꝙ sic constabit. Circa cen
trum a describat̃ semicirculus b m n c, & descendat b usꝫ ad m, & c usꝫ
ad n, protrahãturꝗ a b & n m ad circumferentias dictarū quartarū, duæ
lineæ m h & n k æquedistantes lineæ b d & a f g & c e. Dico ergo, ꝙ m h
æquat̃ lineæ æquedistanti lineæ b d, & n k æquat̃ lineæ c e. Transeat̃ ẽm
h m usꝫ ad o punctum in linea b a, & sit p punctus in quo secat lineam
d f. Cum igitur lineæ a o b & d p f sunt æquales per tricesimamquartam
primi Euclidis, & diametri sint æquales, et sic residuis diametrorū dem=
ptis. Sicut b o ad d m, ita o m ad residuū diametri, & etiam, sicut d p ad
p h, ita p h ad residuū diametri, per octauã sexti Euclidis, & per tricesi=
mam quinti eiusdem. Igitur b o est ad o m, sicut d p ad p h, quare per no
nam quinti Euclidis o m & p h lineæ sunt æquales, addita igitur utriꝫ
lineæ m p, erit linea o p æqualis lineæ m h. Cum igitur b d per tricesimã
quartam primi Euclidis sit æqualis o p, erit b d æqualis m h. Cum ergo
b erit in m, & d erit in h, & per idẽ argumentū ubicūꝫ erit b in sua quar
ta, erit d in sua quarta, & eodem modo probandū est, ꝙ e sit in k, cum c
fuerit in n, protracta k q usꝫ ad r, & posito ꝙ in q secet lineam g e, & hoc
est quod promisimus. Nota, ꝙ illa conclusio fundatur super hoc, ꝙ ap
pendicula æque distent lineæ directionis, quod tamen est falsum, eo ꝙ cõ
currit cum ea in centro terræ si in infinitū protraherentur, uerū, quia pro
pter breuiorẽ appendiculorū & longam distantiã earum à centro terræ,
illa appẽdicula insensibiliter in inferioribus distant à lineis æquedistan
tibus lineæ directionis, iam insensibiliter inæqualiter pondera secundū
situm quæ iudicãtur esse æqualia, eo ꝙ neutrū sensibiliter descenderet.

PROPOSITIO QVARTA.

Quodlibet pondus in quamcūꝫ partem discedat
secundū situm fit leuius.

falsa

Manifestum est hoc per suppositionẽ quartam. Aliud commentū.
Cum sunt pondera b c, dico, ꝙ si eleuetur b usꝫ ad d, ibi erit minus gra
ue q̃ꝫ in situ æqualitatis. Capiatur enim sub d arcus d g, & sub b arcus b
f sibi æquales. Capiaturꝗ supra b arcus b g æqualis arcui b f. Cum er
go per probata, in regula huius d g portio, minus capit de directo q̃ꝫ b
f, igitur secundū situm erit magis graue pondus in b q̃ꝫ in d per quartã
suppositionẽ huius. Eodem modo probandum est, c esse grauius in situ
æqualita



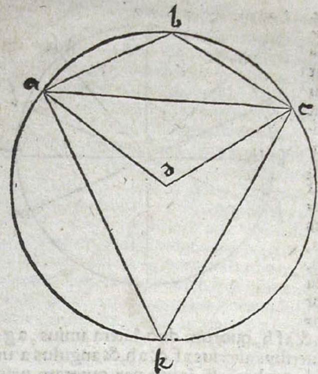
æqualitatis q̃ in k puncto. Ca-
ptis igitur portionibus æqua-
libus c m & k l, namq; k l mi-
nus capit de directo, q̃ c m, ut
patet in secunda huius. Qđ au
tē b g & b f æqualiter capiant
de directo, probatur. Nā pro-
tractis cordis b g & b f, & pro-
tractis semidiametris g a & f a,
erunt duo trianguli g a b & f
a b, per octauam primi Eucli-
dis, quorum angulus a unius
erit æqualis angulo a alterius,
eo ꝙ b f & b g sunt æquales,
per uicesimāoctauam tertij Eu
clidis. Protrahantur igitur cor
da g f, quæ secet b a in h pun-
cto, erunt duo trianguli a g h & a f h, quorum duo latera unius, a g &
a h, erunt æqualia duobus lateribus alterius a f & a h, & angulus a uni
us æqualis angulo a alterius, ut probatū est, igitur per quartam primi
Euclidis basi g h æqualis est ei, ꝙ g b capit de directo, igitur g b & b f æ-
qualiter capiunt de directo, quod fuit probandum.

PROPOSITIO QVINTA.

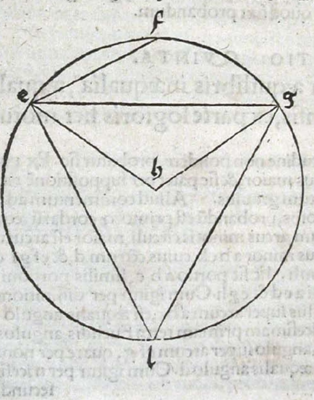
Si fuerint brachia æquilibris inæqualia, æquali- bus pōderibus appensis, ex parte lōgioris fiet motus.

Brachia inæqualia longitudine non pondere, probatur sic. Ex parte longioris describitur circulus maior, & sic patet per suppositionē tertiā ꝙ pondus est secundum situm grauius. Aliud commentum ad declarationem illius conclusionis, probandū est primo, ꝙ cordarū æqualium circulorum inæqualium, arcus minoris circuli, maior est arcui maioris circuli. Sit itaq; circulus minor a b c k, cuius cētrum d, & e f g l circulus maior, cuius centrum h. Et sit portio a b c, similis portioni e f g, & constituantur trianguli a c d & e g h. Cum igitur per diffinitionem similium portionum angulus super arcum a b c, est æqualis angulo super arcum e g f, ergo per uicesimam primam tertij Euclidis, angulus super arcum a b c, est æqualis angulo super arcum e f g, quare per nonam tertij Euclidis, angulus h est æqualis angulo d. Cum igitur per tricesimā

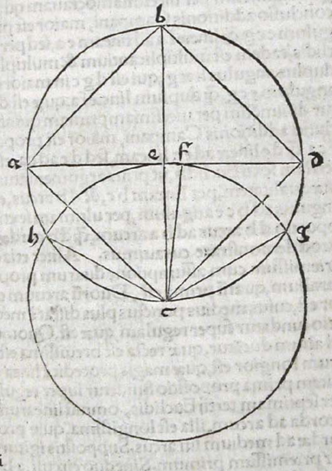
secundam



ſecundã primi Euclidis duo anguli a et c, ſunt æquales duobus angulis e et g. Sed per quintam primi Euclidis, angulus a eſt æqualis angulo c, & angulus e eſt æqualis angulo g, & ſic quilibet illorum quatuor, eſt cuilibet alteri æquiualẽs propter ſimilitudinẽ triangulorũ, et per quartam ſexti Euclidis, erit e g corda ad a c cordã, ſicut e h ſemidiameter ad d a diametrũ. Itẽ ſicut ãgulus h ad quatuor angulos rectos, ita e f portio ad totam circumferẽtiam per ultimã ſexti Euclidis, eo ꝙ quatuor recti ſuper h, occupant totam illam ſuperficiem, ut poteſt elici ex decimatertia primi Euclidis. Igit̃ d angulus, qui eſt æqualis h, ſe habet ad quatuor rectos, ſicut e f g angulus ad totam illam circumferentiam, per idẽ d ſe habet ad quatuor rectos, ſicut a b c ad totam illam circumferentiã per illud ꝙ prius. Igitur ſicut a b c arcus ad ſuam circumferentiam, ita e f g arcus ad ſuam circumferentiam, igitur permutatim per decimamſextam Euclidis. Sicut a b c arcus, ad e f g arcum, ita a b c circumferentia ad b f g circumfetentiam, ſed ſicut circũferentia ad circũferentiam



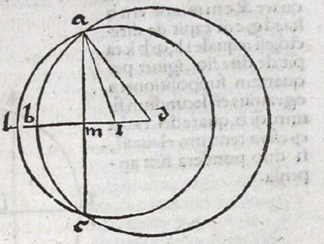
per quintam Archimedis
rentiam, ita semidiameter ad semidiametrum
de curuis superficiebus. Igitur sicut a b c arcus ad e f g arcum, ita a d ad
e h, & eadem est proportio a c lineæ ad e g lineam, ut prius fuit probatũ.
Sunt igitur a d ad e h, & a c ad e g, & a b c ad e f g secundum proportio-
nem unam. Cum a d sit minor e h, erit a b c minor e f g, & a c minor e g,
protrahe ergo e ad f cordam æqualem cordæ e c, Est igitur e f g linea ad
e f lineam, sicut e f g arcus ad a b c arcum, sed maior est proportio e f g ar
cus ad e f arcum, q̄ꝫ e g lineæ ad e f lineam, ut probat Ptolemæus primo
Almagesti capitulo quinto, et ut patet cõclusiõe prima Almagesti Alb.
Igitur maior est proportio e f g ad e f, q̄ꝫ e f g ad a b c, quare per octauam
quinti Euclidis arcus e f minor est arcui a b c, sed illis subtenduntur cor-
dæ æquales, patet igitur
quod uolumus. Si autem
propositionem Ptolemæi
probare uolumus, uideli
cet, ꝙ maior est proportio
arcuum, q̄ꝫ cordarum, de-
scribam circulũ, super quẽ
sunt a b & b d cordæ inæ-
quales, quarum breuior
sit a b, longior b d, dico er
go, ꝙ proportio b d cor-
dæ, ad a b cordam minor
est proportioni b d arcus
ad b a arcum. Diuido ẽm
angulũ a b d in duo æqua-
lia per lineam b e c, et pro-
traho lineas a e d & c a & c
d, quia igitur angulus a b c
est æqualis angulo c b d,
erit a c linea æqualis c d li-
neæ, per uicesimamquin-
tam & per uicesimãoctaua-
uam tertij Euclidis. Et d e li
nea se habet ad e a lineam,
sicut d b ad b a per tertiã
sexti, sed d b est maior b a,
igitur d e est maior e a. Ducat igit e f ad punctũ medium ad f, quæ per
octauam primi erit perpendicularis super eam. Cum igitur a c linea lon
gior sit e c linea per decimamoctauam primi Euclidis, Et cum e angu-
lus sit obtusus, eo ꝙ extrinsecus est ad f rectum, & c e longior est e f, per



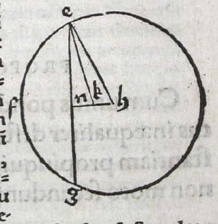
eandē decimāoctauā, eo ꝙ e f c ſit maior angulus c e f trianguli per trice
ſimāſecundā primi Euclidis, igiť circulus deſcriptus ſup c centrū, ſecun-
dū quantitatē c e ſecabit c a, & tranſibit ultra e f, fiat igiť portio circuli g e
h, & producā e f uſqꝫ ad h. Cū ergo maior ſit ꝓportio c h e ſectoris ad c e
g ſectorē, per octauā quinti Euclidis, et per eandē maior eſt ꝓportio c f e
trianguli ad c e g ſectorē, q̄ꝫ c e a triangulum, igitur à fortiori maior eſt
proportio c h e ſectoris ad c e g ſectorem, q̄ꝫ c e g trianguli ad c e a trian=
gulum, ſed c e f trianguli ad c e a triangulum, eſt ſicut f a lineæ ad lineam
e a per primam ſexti Euclidis. Et proportio ſectorum eſt, ſicut proportio
h c e anguli ad e c g angulum, per ultimam ſexti Euclidis, igitur maior
eſt proportio anguli h c e ad angulum e c g, q̄ꝫ lineæ c f ad lineam a e, Er-
go coniunctim per uiceſimamoctauam quinti Euclidis, quæ eſt quinta
concluſio additionis Campani, maior eſt proportio anguli h c e ad an-
gulum e c g, q̄ꝫ lineæ f a ad lineam e a, ſed per decimamquintā quinti Eu-
clidis, eadem eſt multiplicantium & multipliciorum proportio, Igitur
duplus angulus h et g, qui eſt d g c in maiori proportione ſe habebit ad
angulum e c g, q̄ꝫ duplum lineæ f a, quæ eſt d a, ſe habet ad lineam c a, Igi
tur diſiunctim per uiceſimam primam quinti Euclidis, quæ eſt quarta cō
cluſio additionis Campani, maior eſt proportio anguli d c e ad angulū
e c a, q̄ꝫ d e lineæ ad e a lineam, ſed d e ad c a, eſt ſicut b d ad b a cordā, per
tertiam ſexti Euclidis, ut prius argumentatum eſt, eo ꝙ b diuiditur per
inæqualitatem, per lineam b c, & d b arcus, eſt ad b a arcum, ſicut d c b
angulus ad b c e angulum, per ultimam ſexti Euclidis. Igitur maior eſt
ꝓportio d b arcus ad b a arcum, q̄ꝫ d b cordæ ad b a cordam, & hoc eſt
quod demonſtrare curauimus. Aliter etiam probari poteſt primum
præmiſſum cum aſſumptione duarum propoſitionum aliqualiter na-
turalium, quarū prima eſt, Duorū arcuum cordarum æqualiū, ille ma
ior eſt, cuius medius punctus plus diſtat à medio ſuæ cordæ. Illa propo-
ſitio fundatur ſuper regulam, quæ eſt, Quotquot lineæ ab uno puncto
ad alium ducātur, quæ recta eſt breuiſſima eſt, earum & arcualium line
arum longior eſt, quæ magis procedit à linea directe protracta. Quare
autem prima propoſitio fundetur ſuper regulam, conſtare poteſt, Ergo
per ſeptimam tertij Euclidis, omniū linearum rectarum protractarum
à corda ad arcum, illa eſt longiſſima, quæ protrahitur à medio puncto
cordæ ad medium ſui arcus. Suppoſitis igitur propoſitionibus, proba-
tur præmiſſum primum. Sint duo circuli, a b c maior, cuius cētrum d,
& ſit e f g minor, cuius centrum h, & ſint a c et g e cordæ æquales, dico er
go, ꝙ arcus a b c minor eſt arcui e f g, duco enim d m & h n lineas ad pun
cta media cordarum a c & e g, quæ per octauam primi Euclidis, eſſent
perpendiculares ſuper iſtas, erit ergo d m longior h n. Nam ſi foret ſibi
æqualis cum a m, & e n ſunt æquales per quartā primi Euclidis, a d & e h

ſemi-

ſemidiametri æquales, quod
falſum eſt, eo ꝙ circuli ſunt in
æquales, nec m d eſt minor h
n, nam ſi ſic reſecta h n in pun
cto k ad æqualitatẽ m d, erit
a d per quartam primi Eucli-
dis e k æqualis. Igitur angu-
lus e k h fit obtuſus per ſextã
primi Euclidis, eo ꝙ eſt ex=
trinſecus ad n rectum, & per
triceſimã ſecundã primi ma-
ior angulo h, quare ergo per
decimamoctauã primi e h li-
nea, maior eſt linea e k, ergo
& maior linea a d, quod fal=
ſum eſt, relinquitur igitur, ꝙ
m d maior eſt linea n h. Reſe-

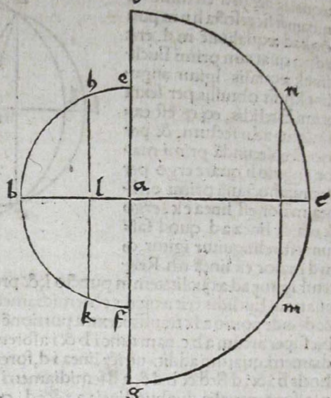


cetur igitur ad æqualitatem in puncto i, & protrahatur linea a i, quæ per
quartam Euclidis erit æqualis e h ſemidiametro. Ponatur igitur i centrũ
& deinde portio a l c tranſibit extra portionẽ a b c, ut probo, nã nõ tranſi
bit ſuper arcum a b c, nam tunc i b & i a forent æquales, eo ꝙ forent ſemi
diametri, quarum addita utriq; linea i d, foret linea b d, æqualis duobus
lineis b i & i d. Sed & b d & a d ſemidiametri ſunt æquales, igitur a d li-
nea, foret æqualis duabus lineis a i & i d, ꝙ
eſt contra uiceſimam primi Euclidis. Non igi
tur tranſibit arcus a l c ſuper arcum a b c, nec
tranſibit infra eum, quia ſi ſic, tunc i l & i a
ſint æquales, foret i b maior i a, & per conſe=
quens b d foret maior eiſdem, quod falſum
eſt, & contra uiceſimam primi Euclidis. Relin
quitur ergo, ꝙ arcus a l c tranſibit extra arcũ
a b c, igitur linea m l longior erit linea m b,
quare per primam propoſitionẽ præaſſum=
ptam, arcus a l c, maior erit arcu a b c, quod fu
it probandum. Iſtis igitur præ miſſis acce-



dam ad probationem concluſionis. Sit igitur regula a b c, & ſit a c lon=
gior q̃ a b, dico, ꝙ æqualibus appenſis ponderibus, quæ ſint b & c, fiet
declinatio ex parte c. Fiant enim ſuper centrum duo ſemicirculi d c g &
e b f, & protrahatur linea d e & f g, capiantur tunc circa a b æquales ar-
cus b h & b k, & protrahatur corda h l k. Item capiantur circa c arcus æ-
quales c n & c m, ita, ꝙ corda n m, ſit æqualis cordæ h l k. Erit igitur per

præmiſſam probatiõem m n arcus, minor arcu h k quare & c m minor erit b k, ſed ꝙ c m capit de dire- cto, eſt æquale l k, ꝙ b k capit de directo, igitur per quartam ſuppoſitionem c grauius eſt ſecundum ſitum, q̃ b, quare deſcendet ꝙ eſt in centrum eleuatũ, ſi duo pondera ſint appenſa.

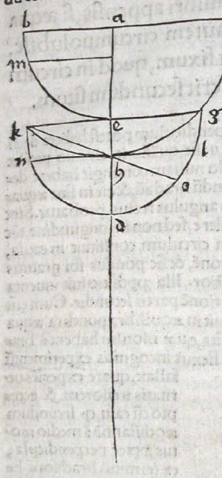


PROPOSITIO SEXTA.

Cum unius ponderis ſint appenſa, & a centro motus inæqualiter diſtent, & ſi remotum ſecundum diſtantiam propinquius acceſſerit ad directionem, alio non moto ſecundum ſitum, illo leuius fiet.

Centrum motus dicitur hic punctus in brachio libræ, circa quẽ brachia libræ uertuntur. Si igitur unum pondus ponderat in brachio, plus diſtante à centro motus illo alio dependente in alio brachio, & ſint æque grauia, ſi tunc remotius appropinquat ad diſtantiam, uel ad directionẽ, moto appenſili ad ſitum æqualem, quod prius in remotiori parte fuerit æque graue, nunc eſt leuius, quia tunc à ſeipſo, q̃ prius eſt leuius, quia obli-

obliquior est descensus. Est enim semicirculus minor, q̃ tunc fuit. Aliud commentum. Sit ut prius regula b a c, a c longior, q̃ a b, sitq; linea directionis a e d, circumducanturq; quarta c a circa centrum a, circumducatur etiam portio circuli c g h k, donec linea k g æque distans lineæ h b c, sit duplũ lineæ b a, erunt tunc per tertiam Euclidis b a & k e & g e æquales. Dico ergo si b & c sint posita æqualia, & c ponatur in situ g, quiescente b, q̃ c in situ g sit leuius secundum sitũ, q̃ b in suo situ. Statuatur enim circa cẽtrum e semicirculus g d k, in quo fiat arcus g l, capiens h e de directo, cui arcui sit b m æqualis, ductaq; linea l h n, erit arcus g h maior arcu g l, quod probabitur. Per tracta em̃ linea k h o, erũt g h & g o arcus similes p diffinitionem arcuum similium, propter hoc, q̃ angulus h k g constitutus super arcum h k g, & completur circulus, est idem cum seipso constituto super arcum o d k g, si compleretur circulus. Cum enim angulus k oppositus cordæ h g, sit idem angulus qui & opponitur cordæ o g, erit per uicesimamprimam tertij Euclidis angulus constitutus super arcum g h æqualis angulo consistente super arcum g o, quare arcus g h & g o sunt arcus similes. Cum igitur g h sit arcus maioris circuli q̃ g o, erit probata in præmissa conclusione g h maior g o, ergo g h erit maior q̃ g l, sed g h & g l æqualiter capiunt de directo, eo q̃ ex utraq; capit h e. Igitur põdus descendens per g h, obliquius descendet q̃ descendens per g l, & per consequens q̃ pondus descendens per b m. Cum igitur c pondus in puncto g descendat per arcum g h, patet per quartam suppositionẽ, q̃ c pondus in puncto g leuius est secundum situm, q̃ b in suo situ, & hoc est q̃d ostendere curabamus.

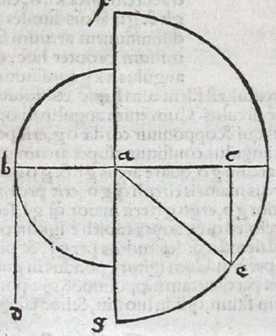


PROPOSITIO SEPTIMA.

Aequis ponderibus in æquilibri appensis, si æqualia sint appensibilia, alterum autem circumuolubile, & alterũ secundũ angulũ rectũ fixum, quod in circumuolubile appenditur, grauius erit secundum situm.

falsa
imo sequitur oppositum
[illegible] principia [illegible]
sit [illegible]

Circumuolubile dicitur, quando perpendiculum potest habere declinationem plus largam, q̃ brachia libræ, ut fit, quãdo in circulo pendet scd̃m angulũ rectũ fixum, dicitur, quando nullam contingit habere declinationem perpendiculorum, nisi secundũ brachiũ, & est in situ æqualitatis inter brachium & perpendiculum angulus rectus, probatur. Sint appensa æqualia, ut uult positio, in pondere, sed non in longitudine, tũc illud quod est circumuolubile, maiorem circulum constituit in causa, quia plus declinat propter circumuolutionẽ, & sic pondus ibi grauius est secundũ situm, cũ eius descensus sit rectior. Illa ꝓpositio fuit inuenta de quodam experimento facto ad probationẽ partis secundæ. Cum enĩ aliquis uoluerit experiri, an ita esset, posuit in æquilibra pondera æqualia, cuius appendentia erunt filo composita, quæ motum habent à brachijs alienũ, etiam ꝓpter perpendiculorũ flexus incognitis experimentũ fallax, quare experiẽs ueritatis irrisorem, & accepto cũ casu, q̃ secundum æqdistantiã à medio motus ꝓpter perpendicula, ex terminis brachiorũ lineæ sic describunt, utrũq; intelligit, q̃d prius negauit, q̃ est, quia ꝓpter mutationes brachiorũ alij nõ erunt flexus, & ex hoc nõ conclusit secundũ rectos angulos idẽ cõgruere, cũ motus brachiorũ similiter contingit. Aliud cõmentũ. Sit regula b a c, cuius centrũ a, & appendicula b d circumuolubile, & c e fixum, pondera quoq; appensa



appensa e & d. Dico igitur, ꝙ d pondus grauius est secundum situm q̃
e, Trãseat enim hypotenusa a e, secundũ cuius quantitatem describatur
semicirculus e f g circa centrum, & ita, ꝙ diameter f a g sit perpendicula-
ris super regulam b a c, Ex quo igit angulus cõtinue manebit rectus. Ma
nifestum est ꝙ e in descendendo, describit arcum e g, quare æque graue
est secundũ situm, sicut foret, si appẽdatur super hypotenusam a e, quia
tunc esset idem transitus, sed e a in ho situ leuius est, q̃ b per præmissa, ꝓ
pter hoc, quod e, tantum distat à linea directionis sicut b, eo ꝙ b distat tan
tum sicut c, quia c e & a g sunt lineæ æquedistantes, & d e æque graue in
d situ, sicut foret in b termino regulæ, ut patet per probationẽ tertiæ hu-
ius, igitur e minus graue est secundum situm q̃ d, q̊d fuit probandum.

hoc

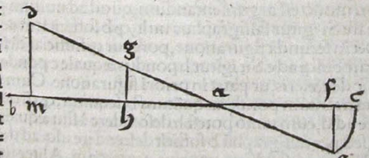
PROPOSITIO OCTAVA.

Si fuerint brachia libræ proportionalia ponderibus appensorum, ita, ut in breuiori grauius appendatur, æque grauia erunt secundum situm.

propositio quem uero . demonstrationem sequentium uero nulla ex necessitate concludit

Si pondus grauius tantum ualet in termino breuiori, quantum brachium libræ longius in suo loco, & similiter pondus minus in breuiori, tunc dico, sic ualebunt secundum situm, quando non essent sic secundũ naturam, necessariò erunt pondera secundum situm æqualia, quia pondus & brachiũ hic ualet per oppositum totum reliquũ, quia propter neutrum pondus declinat, sicut patet in propositione huius prima. Aliud commentũ. Sit ut prius regula b a c, cuius centrum a, & sint appensa b & c, sitq; proportio b ad c, tanq̃ c a ad b a. Dico, ꝙ non faciet motum in aliquã partem regula recta, ascendat primo b & descendat c, ita ut d a e sit quasi regula, & d quasi pondus c, sint d m & e f perpẽdiculares super b c, palam est igitur per uicesimamnonam & decimamquintam primi Euclidis, ꝙ trianguli a d m et a e f sunt similes. Quare per q̃r tam sexti Euclidis, sicut d a ad a e, ita d m ad e f. Sed sicut d a ad a e, ita c pondus ad d pondus, Igitur sicut d m ad e f, ita c pondus ad d pondus, Sit igitur g a æqualis a e

de quasi pondere



c
c

& eri-

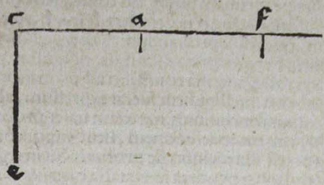
& erigatur perpendicularis ſuper b a, & ſit hoc unum põdus æquale põ
deri c. Cum igitur g a & a e ſunt æquales, conſtat per quartam ſexti Eucli
dis, ꝙ g h & f e ſint æquales, ſed & c & h pondera ſunt æqualia, igitur ſi=
cut d m ad h g, ita h pondus ad b pondus. Arguatur igitur ſic. Si b & h
aſcenderent à ſitu æqualitatis ad lineam directionis, d a linea fieret æqua
lis lineæ, quam b acquireret de directo, eo ꝙ d a eſt ſemidiameter circuli
cuius circumferentiam b deſcribit. Item, h a foret æquale ei, quod h a ac=
quireret de directo, eo ꝙ h a eſt ſemidiameter circuli deſcripti per f h, igit
ſi h & b forent pondera æqualia, ſimiliter b foret grauius ſecundũ ſitum
ꝗ̃ h ſecundum proportionem d a ad h a per primam huius, ſed per quar
tam ſexti Euclidis, ſicut d a ad h a, ita d m ad g m. Cum igitur ſi h & b fo
rent æqualia, b foret grauius ſecundum ſitum ꝗ̃ h ſecundum ſitum pro
portionem d a m ad h g. Cum igitur in eadem proportione eſt h graui=
us, ꝗ̃ b, ut prius fuit argumentatum, palàm eſt h & b in ſitibus æqualita
tis æqualiter ponderare. Nam quanto b foret grauius ſecundum ſitum,
ꝗ̃ h, ſi forent æqualia ſimpliciter, tanto h eſt grauius ſimpliciter, Ergo
quantum b promouetur propter ſitum, tantũ h promouetur, eo ꝙ gra=
uius eſt ſimpliciter ꝗ̃ b, ergo comparando ſingula ſingulis, tantũ pon=
derat b in ſuo puncto æqualitatis, quantum ponderat h in ſuo pondere
in puncto lineæ æqualitatis, igitur quodlibet quod ſufficit leuare b a ſi=
tu æqualitatis ad punctum in quo nunc eſt, tunc idem ſufficeret leuare h
in quo nunc eſt h, Igitur per falſigraphũ c pondus ſufficit leuare b a uſq;
ad d, idem c ſufficeret leuare h ad punctum in quo iam eſt, ſed hoc conſe=
quens eſt falſum, & contra ſecundam huius, eo ꝙ c pondus & h ponebã=
tur eſſe æqualia. Aliter poteſt argumentari ſecundum communiter
loquentes. Sicut h pondus ad b pondus, ita permutatim aſcenſus b põ
deris, qui eſt d m, ſe habet ad g h aſcenſum h ponderis, per ſecundã partẽ
primæ huius, Ergo quod ſufficit leuare b h ſecundum quantitatem d m,
ſufficit leuare h ſecundum quantitatem g h, eo ꝙ d m & g h æqualiter ſe
habent ad motus contrarios alternatim. Conſequens eſt falſum, ut prius
eſt argumentatum, ergo pondus c non ſufficit leuare b uſq; ad d, & eo=
dem modo eſt argumentandum, quod ad nullum punctũ ſufficit eum
leuare. Si igitur falſigraphus uult, ꝙ b ſufficit leuare c, & non econtra, ut
patet in ſecunda figuratione, ponatur ꝙ ſufficiat deſcendere uſq; ad b, &
leuare c uſq; ad e. Sit igitur h pondus æquale c ponderi, & h a æquale h e,
& ſic de cæteris, ut patet in priori figuratiõe. Cum igitur per prius ar=
guta h tantum ponderat quãtum b, ſequitur, ꝙ cũ quãto b poteſt deſcen=
dere ad d, cum tanto poteſt h deſcendere à ſitu æqualitatis ad ſitũ in quo
eſt, ſed per falſigraphũ b ſufficit deſcendere uſq; ad d, eleuando c uſq; ad
e, cõſequens eſt falſum, ut prius per tertiam. Aliter ſic. Sicut h ad b, ita m
d ad h g per prius arguta, Igitur quantum b poteſt eleuare in ſitu ſuo, tan
tum

tum poteſt h leuare in ſitu ſuo per primam huius, ſed conſequens proba
tur eſſe falſum, igitur nec c ſufficit eleuare b, nec econtra, Igitur b a c æque
grauia ſunt ſecundum ſitum, quod fuit probandum. Iſtæ ergo ſunt pro
poſitiones, quæ conueniunt in ſua cum probationibus, ex quibus palã
eſt, propter allegationem, qua allegatur iſta concluſio ad probationem
illius, ꝙ illa prima cõcluſio habet intelligi, ſicut fuerat expreſſum, aliter
enim non ualeret probatio illius concluſionis, nec etiam ualet probatio
ſua ibi, & ideo intelligendo primam concluſionem, ſicut exponebatur
ibidẽ, facillime per omnia poteſt iſta cõcluſio ſic probari. Sit in regula
b a c, cuius centrum a, ſuſpendantur pondera inæqualia c maius b mi-
nus. Sitq́ꝫ proportio b a d ſecundum proportionem c a brachij ad b a
brachium. Sit igitur d pondus æquale b ponderi, & ſit d a linea æqualis
a c lineæ, Et arguatur ſic, b pondus plus ponderat q̃ꝫ d pondus ſecundũ
proportionem b a ad d a per primam huius, ſic c pondus plus ponderat
q̃ꝫ d pondus ſecundum eandem proportionem, eo ꝙ b d pondera ſunt
æqualia, & d a & a c brachia æqualia, Igitur per nonam quinti Euclidis
b & c in ſuis ſitibus æqualiter ponderat, quod eſt propoſitum.

PROPOSITIO NONA.

Si duo oblonga unius groſſiciei per totum ſimilia & pondere & quantitate æqualia, appendantur, ita, ut alterum erigatur, & alterum orthogonaliter dependeat, ita etiam, ut termini dependentis, & medij alterius, eadem ſit à centro diſtantia, ſecundum hunc ſitũ æque grauia fient.

Vnum pondus ſecet brachium tranſuerſum, & aliud pondus dependeat deſcenſu uerſo, & ſit terminus illius inæquali diſtantia à centro motus cum medio alterius, quia ſicut illius extremum plus à centro diſtat, ita iſtius medium. Probatur ſic, Grauitas naturalis eſt æqualis utrobiqꝫ propoſitum & uiolentum, ſimiliter, quia ſemicirculi ſunt æquales, ergo æque grauia ſecundum ſitum ſunt appenſa. Aliud commentũ. Sit a b c regula, cuius centrum a, & erigatur pondus oblongum b d, cuius medium f ſecundum ſitum uere ut æquediſtẽt orizonti, dependeatq́ꝫ orthogonaliter pondus oblongum c e, ſintq́ꝫ a f & a c æquales, Dico ꝙ illa pondera appenſa ſunt æque grauia ſecundum ſitum. Ad cuius euidentiam probo primo, ꝙ ſi ex parte b fieret motus, ut ſi a d ſuſpendant



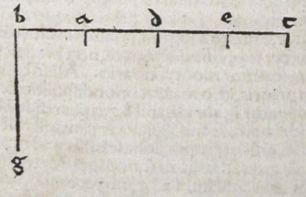
in d & b duo pondera æqualia g & h, cõtrariũq; in c, aut duo æqualia g & h, quæ sunt k l, in quorũ sitibus g h a k l æqualiter ponderabũt, Nã k se habet ad g secundum proportionem c a ad a d per primã huius, eo q; k tantũ ponderat in c, quantum ponderat in f, propter hoc, quod f a & a c sunt æqualia. Item l se habet ad h secundum proportionem c a ad a b per primam huius, ut prius, ergo k l se habet ad g h, sicut duplum a c se habet ad aggregatũ ex a d & a b, propter hoc, quod f d & b a simul sumpta, sunt æqualia a c, eo q; d f & f b sunt æqualia, igitur k l & g h in istis sitibus æque grauia sunt, quod promisi probare. Et eadem ratione quælibet duæ partes b d ponderis æquales, & æqualiter ab a f ex utraq; parte distantes, æqualiter ponderant cum duabus partibus sibi æqualibus in c termino. Sed omnes partes æquales c e ponderis æqualiter ponderant per tertiam huius. Et quot sunt partes in c e, tot sunt partes illis æquales in d b, igitur c e & e b in suis sitibus æqualiter ponderabunt, & hoc est quod ostendere & finaliter probare uolebamus. Sed nota, q; oportet c e pondus esse circumuolubile in termino, & non fixum, quia aliter nõ omnes partes sui æquales, æqualiter ponderarent, imò pars superior plus ponderaret inferiori sibi æquali, ut patet ex prima huius. Si cum sit circumuolubile, tunc per tertiam huius omnes partes æquales æqualiter ponderant, ut assumitur in probatione conclusionis huius. Hic explicit secũdum aliquos liber Euclidis de ponderibus.

PROPOSITIO DECIMA.

Si canonium fuerit symmetrũ magnitudine, & substantiæ eiusdem, diuidaturq; in duas partes inæquales, & suspendatur in termino minoris portionis pondus, quod faciat canonium paralellum epipedo orizontis, proportio ponderis illius, ad superabundantiam ponderis maioris portionis canonij ad minorẽ,

est si-eſt ſicut proportio totius canonij ad duplum longitudinis minoris portionis.

Canonium eſt idem quod brachiū libræ, quia eſt regula, Symmetrū eſt ꝓportionale .i. brachiū ſit æquale brachio, zona et magnitudine eiuſdem in quantitate & pondere, & paralellū .i. æquediſtans, epipedo .i. ſuperficiei, probatur ſic. Sit æquilibra æquelonga, & omnia æqualia, & in omni parte æque groſſum, ſit utrunq; & æque graue. Sit ergo longitudo uniuſcuiuſq; ſex palmarū, & tollantur poſt hoc quatuor palmi de uno, Manifeſtum itaq;, quoniam brachium longius, eſt grauius triplici grauitate, ſicut etiam longius grauius dicitur naturaliter, quia breuius tantum duos palmos, ſicut fit, pro ponderoſitate cuiuſq; appendatur pondus ſex ad terminum breuioris partis. Arguitur ſic, Illud pondus facit canonium pararellum epipedo orizontis, ſicut patet, quia cum linea recta perpendicularis erecta fuerit à ſuperiori plano orizontis, ad canonium conſtituit angulos rectos, manifeſtum eſt propoſitione prima per Euclidem, canonium ſæpe paralellum empipedo, ſi altera pars eſſet grauior altera, alia eam ſequeretur, ſicut aliud canonium motu contrario, patet ſuppoſitione ſexta, ergo æque graues ſunt partes alternarum ſecundum ſitum, qd' ſi ſic eſt, tunc additio addaẽ ponderi, tunc minor erit canonij inclinatio, Sicut iſta probaẽ geometrice, ita poſſunt omnes ꝓbari p̄miſſæ per ꝓportionẽ illarū linearū, & angulorū ſuorū cōſtructorū. Aliud cōmentum. Sit canoniū .i. regula b a c eiuſdem groſſiciei undiq;, & eiuſdem compoſitiōis, et ita quælibet duæ partes eiuſdẽ, æquales ſint æque graues ſimpliciter, ſumaturq; a d æqualis a b eſt igitur d c, cuius mediū ſit e exceſſus a b c brachij, ſupra brachiū a b ſuſpendeẽ, igiẽ pōdus in b termino, ita ꝙ faciat b a c regulā æquediſtare orizonti, tūc dico, ꝙ g pondus ſe habet ad d c pondus, ſicut b c linea ad b d lineam. Cum enim amotis g & d c ponderibus, b d foret æquediſtans orizonti, ſed per ultimam concluſiōem præmiſſam d c, ſi ſic dicatur, tantū pōderat quantū ponderaret, ſi ſuſpenderetur in e puncto medio, Igitur per cōuerſam octauæ p̄miſſarum g pondus eſt ad d c pondus ſcd'm proportionẽ e a brachij



ad a b brachium,ſed ſicut e a ad a b,ita b c ad b d,quoniã b c eſt duplum
ad e a,& ꝓpter hoc,q̊d b d eſt duplum ad a d,& d c duplũ ad d e,igitur ſi
cut g ad d e,ita b c ad b d,quod fuit probandũ. Et uerum,quia in præ
miſſis non probatur conuerſa octauæ concluſionis, ideo ſic ꝓbetur. In
regula b a c,cuius longius brachiũ ſit a c,appendant̃ pondera b & c, ita
ꝙ æque diſtent orizonti,dico ꝙ c pondus ſic ſe habet ad b pondus,ſicut
a b ad a c.Sin aũt,ſit prima maior proportio c ad b,q̃ꝫ a b ad a c,tunc re=
ſecetur aliquid de c,ita ꝙ reſiduũ ſit d,quod ſe habet ad b a , ſicut b a ad
a c,igitur per octauã præmiſſam d & b æqualiter ponderabunt in illis ſi
tibus,igit̃ d tantũ ponderat ſicut c,quod eſt ſuum totũ,conſequens eſt im
poſſibile.Item ſi minor ſit proportio c ad b,q̃ꝫ b a ad c a,addatur d ad c,
ita ꝙ d c ſit ad b ſicut b a ad c a,igit̃ per octauã præmiſſarũ d c a b æque
grauia ſunt ſecundũ ſitum,ſed c & b ſunt æque grauia in iſtis ſitibus,igit̃
c tantũ ponderat,quantũ c d,conſequens eſt falſum,ergo etc.Igitur ſi c &
b ſint æque grauia ſecundũ ſitum,proportio c ad b eſt ſicut b a ad c a,q̊d
fuit ꝓbandũ,ſic igitur patet conuerſa octauæ concluſionis præmiſſarũ.

PROPOSITIO VNDECIMA.

Si fuerit proportio ponderis in termino minoris portionis ſuſpenſi ad ſuperabundantiã ponderis maioris portionis ad minorẽ,ſicut proportio totius longitudinis canonij ad duplã longitudinẽ minoris portionis,erit canoniũ paralellũ empipedo orizontis.

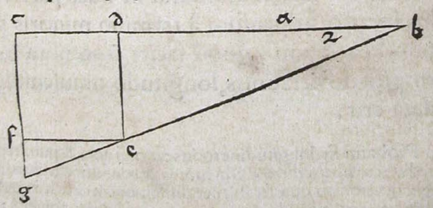
Commentũ prius probatũ eſt,ꝙ æquediſtantiã canonij à ſuperficie o
rizontis,oportet eſſe pondus iam dictũ,ex quibus ſequitur conuerſa ſci=
licet,ꝙ talis æquediſtantia ſemper ſit tali pondere,quia ſi nõ ſit æquedi=
ſtantia,ſequitur,ꝙ quæ æquẽtur,pondere nõ æquuntur.Prius em̃ oſten
debatur,brachio longiori pondus in ſitu coæquari, uel correſpondere,
igitur per ſuppoſitionẽ ſextam,neq; brachiũ pondus,neq; pondus bra
chium ſequitur motu contrario. Aliud commentũ ſequitur,hæc eſt cõ
uerſa prioris,ideo maneat prior diſpoſitio,& fiat motus primo ex parte
g , auferatur igitur aliquid à g,cuius reſiduũ ſit f,quod facit canoniũ eſſe
æquidiſtans orizonti,igitur per præmiſſa f ſe habet ad d c,ſicut c b ad b
d,ſed in eadem proportione ſe habet g ad b c,igitur f,quod eſt pars g, eſt
æquale g,quod falſum eſt,non igitur fiet motus ex parte g.*Si* ex parte d
c fiet motus,addatur f ad g,ita ꝙ totum faciant canoniũ æquediſtare ori
zonti,erit tunc per præmiſſam f g ad d c,ſicut c b ad b d,ſed eadẽ eſt pro
portio

portio g ad d c, igitur f g & g sunt æqualia, consequens falsum, igitur ex una parte fiat motus.

PROPOSITIO DVODECIMA.

Ex ijs manifestum est, quoniā si fuerit canoniū sim metrū magnitudine, & zona eiusdē notū lōgitudine & pondere, & diuidat̄ in duas partes inæquales datas, tunc possibile est nobis inuenire pondus, quod cum suspensum fuerit à termino minoris portionis, faciet canonium paralellum empipedo orizontis.

Illa probatio satis patet ex prædictis. Sit canoniū b a c eiusdē grossiciei, & eiusdem cōpositionis, sitq́ꝫ utrunq́ꝫ brachiū notum, ut sit b a longitudinis duorum palmorū, & a c longitudinis octo palmorū, & sit pondus totius canonij scilicet decem libræ, dico quod notum erit aliud pondus, quod suspensum in b termino, faciet canoniū æquedistans orizonti. Protraham em̄ lineam d e orthogonalē super b c, & æqualem lineæ d c, & protraham lineā b e hypotenusam. producam b e ultra in continuum & directū, donec concurrant in puncto g cum linea c g, quæ sit æquedistans lineæ d e, erunt igitur per uicesimamnonā primi Euclidis trianguli b d e & b c g similes, quare per quartā sexti Euclidis, sicut g c ad d e, & per consequens ad d c sibi æquale, ita c b ad d b, igitur per præmissum c g suspēsum in termino b, faciet canoniū esse æquidistans orizonti. Qualiter aūt cognoscemus c g, constat ex uicesimaprima septimi Euclidis, ex quo em̄ ibi sunt quatuor proportionalia, quorū tantū unum est ignotum, uidelicet c g, multiplicemus c d q̃d est superabundantia c a super b a per c b canoniū, & multiplices sex palmos p̄ de-



 cem, &

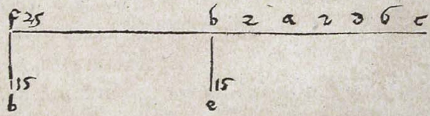
cem,& resultant sexaginta,quæ diuidemus per d b,id est per duplum mi
noris brachij,quod est quatuor palmæ,& numerus quotiēs est quinde-
cim palmarū,igitur canoniū,quod est quindecim palmarū, est æqualis
grossiciei cum b c,& consimilis cōpositionis,suspensum in b,faciet cano
nium in b c æquedistans orizonti. Arguatur tunc ultra,quod sicut decē
palmi ad quindecim palmos,ita uiginti libræ ad triginta libras, igitur
c g ponderaret triginta libras,uel sic deueniemus ad libras c g. Sicut c g
pondus ad c d pondus,hoc est ad duodecim libras,ita c b libra, quæ est
decem palmarū ad a b libram,quatuor palmarum. Multiplices igitur
duodecim,quod est secundū,per decem,q̊d est tertiū, & resultant centum
uiginti,quæ diuidamus per quatuor,quod est quartū, & numerus quo-
tiens est triginta. Ergo ut prius,pondus c g,quod est suspensum in b,faci
et canoniū æquedistans orizonti,cōtinet triginta libras,aliter potest em̄
producta linea e f æquidistante lineæ d c,sitq; d e f c quadratum per uice
simamtertiā primi Euclidis. Arguatur tunc,d & f anguli,sunt anguli re
cti,& angulus g est æqualis angulo d e b per uicesimamnonā primi Eu-
clidis,ergo d e b & e f g trianguli sunt similes,ergo per quartam sexti Eu
clidis sicut b d ad a d e f uel ad c d sibi æquale,ita d c ad f g. Multiplica igi
tur d c superabundantiā per seipsum,scilicet duodecim libras,per duode
cim,& resultabit centum quadraginta quatuor,quæ diuidas per octo li
bras,scilicet per b d,& numerus quotiens erunt decem & octo libræ,q̊d
est pondus f g,addantur igitur decem & octo ad duodecim, quod est pō
dus c d,& resultabunt triginta,quod est pondus c g, eo q̨ c f & c d sunt
partes æquales.

PROPOSITIO TREDECIMA.

Si fuerit canonium datum longitudine, spissitudi
ne,& grauitate,& diuidatur in duas partes inæqua-
les,fueritq; suspensum à termino minoris portionis
pondus datum , quod faciet canonium paralellum
empipedo orizontis,longitudo uniuscuiusq; portio
data erit.

Probatur sic,longitudine totius canonij nota,& pondere noto,Pone
pedem circini in centro medij motus,& constitue circulum super mino-
rem portionem,quæ secabit per diffinitionem circuli æqualem de bra-
chio longiori,parti autē reliquæ æquatur portio ablata à termino ubi
pendet

pendet pondus, quia ex hac exceditur brachium brachio, unde sequitur
quæsitum. Aliud commentum. Sit enim canonium paralellum ori=
zonti, cuius longitudinis brachium sit a c, sitq́ꝫ totum canonium datum
& sit a d æquale a b, & suspendatur in termino ad terminum b pondus
e. Dico ꝙ longitudo a b erit data, & per consequens longitudo c a etiam
erit data. Dirigatur enim canonium b f æqualis grossiciei, & eiusdem
compositionis cum canonio b c, ita ꝙ b c sit primum canonium unum,
& sit b f æqualis ponderis cum eo pondere, Verum, quia ad hoc ꝙ b f
sic dirigatur, oportet ꝙ longitudo sua fuerit nota, ideo ad illam sic deue
nies. Sicut d c pondus notum ad e pondus notum, & per consequens ad
b f notum, ita e b longitudo nota ad b f longitudinem, & productum
diuide per d c pondus, & numerus quotiens ostendit tibi longitudinem
b f. Cum igitur præmissa b f se habet ad d c, sicut b c ad b d, igitur per=
mutatim per decimamsextam quinti Euclidis, sicut f c ad b c, ita c b ad d
b, igitur coniunctim per decimamoctauam quinti Euclidis, sicut f c ad
b c, ita c b ad d b, igitur b c est medium proportionale inter f c & b d.
Multiplica igitur longitudinem b c per seipsam, & productum diuide
per longitudinem f c, quæ nota est, eo ꝙ tam f b q̃ꝫ b c sunt notæ, & nu=
merus quotiens per uicesimamprimam septimi Euclidis, est longitudo
b d, cuius medietas, longitudo b a, quæ subtrahitur à longitudine b c, &
remanet longitudo a c, nunc ergo est utrunqꝫ brachium notum, quod
erat probandum. Explicit.



Excussum Norimbergæ per [redacted],
Anno domini M. D. XXXIII.

Part 3: Facsimile of Benedetti's Chapter on Mechanics

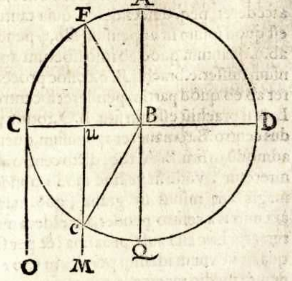
DE MECHANICIS.

SCRIPSERVNT multi multa, & quidem ſcitiſſimè, de mechanicis, at cum natura vſusq; aliquid ſemper vel nouum, vel latens in apertum emittere ſoleant, nec ingenui aut grati ſit animi, poſteris inuidere, ſi quid ei contigerit comperiſſe prius tenebris inuolutum: cum tam multa ipſe ex aliorum diligentia ſit conſequutus. Paucula quædã futura, vt reor, non ingrata his qui in hiſce mechanicis verſantur, nuſquam ante hac tentata, aut ſatis exactè explicata in medium proferre volui: quo vel iuuandi deſiderium, vel ſaltem non ocioſi ingenioli argumentum aliquod exhiberem: atque vel hoc vno modo me inter humanos vixiſſe teſtatum relinquerem.

De differentia ſitus brachiorum libræ.

CAP. I.

OMNE pondus poſitum in extremitate alicuius brachij libræ maiorem, aut minorem grauitatem habet, pro diuerſa ratione ſitus ipſius brachij. ſit exempli gratia. B. centrum, aut, quod diuidit brachia alicuius libræ; &. A. B. Q. verticalis linea, aut, vt rectius dicam, axis orizontis, &. B. C. vnum brachium dictæ libræ, & in. C. ſit pondus, &. C. O. linea inclinationis, ſeu itineris. C. verſus centrum mundi, cum qua. B. C. angulum rectum conſtituat in puncto. C. Exiſtente igitur in huiuſmodi ſitu brachio. B. C. dico pondus. C. grauius futurum, quam in alio quolibet ſitu. quia ſupra centrum. B. omninò non quieſcet, quemadmodum in quouis alio ſitu faceret. Ad quod intelligendum, ſit dictum brachium, in ſitu. B. F. cum eodem pondere in puncto. F. & linea itineris ſeu inclinationis dicti ponderis ſit. F. u. M. per quam lineam dictum pondus progredi non poteſt, niſi brachium. B. F. breuius redderetur. Vnde clarum erit quòd pondus. F. aliquantulum ſupra centrum. B. mediante brachio. B. F. nititur. Eſt quidem verum, quòd pondus. C. nec ipſum etiam per lineam. C. O. proficiſcetur, quia iter extremitatis brachij eſt circularis, &. C. O. in vno quodã puncto eſt contingens. Sit hoc iter. A. C. Q. Oportet nunc præſupponere pondus extremitatis brachij debere tanto magis cẽtro. B. inniti, quanto magis linea ſuæ inclinationis (ponamus. F. u. M.) propinqua erit dicto centro. B. quod ſequenti cap. probabo, vt exempli gratia, ſit. F. ſuper. u. punctum medij ex æquo inter. C. et. B. quapropter. u. B. æqualis erit. u. C. vnde ſe-



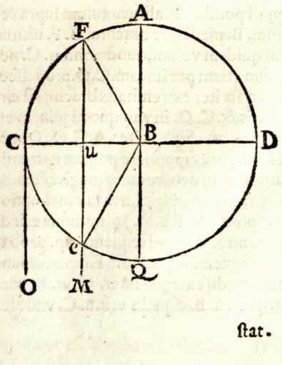
quetur

quetur dictum pondus grauius futurum pro parte. F. C. quam pro ea, quæ est. A. F. & minus supra centrum. B. pro dicta parte. F. C. quam pro parte. A. F. quieturum; & dictum brachium quanto magis orizontale erit à situ. B. F. tantò minus supra dictum centrum. B. quiescet, & hac ratione grauius quoque erit, & quanto magis vicinum erit ipsi. A. à dicto. F. tantò magis super centrum. B. quoque quiescet, vnde tātò quoque leuius existet. Idem dico de omni situ brachij per girum inferiorem. C. Q. vbi pondus pendebit à centro. B. dictum centrum attrahendo, quemadmodum superius illud impellebat. Hæc verò omnia cap. sequenti melius percipientur.

De proportione ponderis extremitatis brachij libræ in diuerso situ ab orizontali.

CAP. II.

PROPORTIO ponderis in. C. ad idem pondus in F. erit quemadmodum totius brachij. B. C. ad partem. B. u. positam inter centrum & lineam. F. u. M. inclinationis, quam pondus ab extremitate. F. liberum versus mundi centrū conficeret. Quod vt facilius intelligamus imaginemur alterū brachium libræ. B. D. & in extremo. D. locatum aliquod pondus minus pondere. C. vt. B. u. pars. B. C. minor est. B. D. clarè cognoscetur ex. 6. lib. primi de ponderibus Archimedis, quòd si in puncto. u. collocatum erit pondus ipsius. C. libra nihil penitus à situ orizontali dimouebitur. Sed perinde est quòd pondus. F. æquale. C. sit in extremo. F. in situ brachij. B. F. quā vt sit in puncto. u. in situ ipsius. B. u. orizontali. Ad cuius rei euidentiam imaginemur filū. F. u. perpendiculare, & in cuius extremo. u. pendere pondus, quod erat in. F. vnde clarum erit quòd eundem effectum gignet, ac si fuisset in. F. quod, vt iam diximus remanens affixum puncto. u. brachij. B. u. tantò minus graue est situ ipsius. C. quantò. u. B. minus est ipso. B. C. Idem assero si brachium esset in situ. e. B. quod facilè cognoscere poterimus, si imaginemur filum appensum ipsi. u. brachij. B. C. & vsque ad. e. perpendicularē, in quo extremo appensū esset pondus æquale ponderi. C. & liberū ab. e. brachij. B. e. vnde libra orizontalis manebit. Sed si brachium. B. e. consolidatum fuisset in tali situ cum orizontali. B. D. & appēso pōdere. C. in. e. libero à filo, nec ascēderet, neq; descenderet. quia tantum est quod ipsum sit appensum filo, ꝙ pendet ab. u. quantum quòd ab ipso liberum appēnsum fuisset. e. brachij. B. e. & hoc procederet ab eo quòd partim penderet à centro. B. & si brachiū esset in situ. B. Q. totum pōdus centro. B. remaneret appensum, quemadmodū in situ. B. A. totū dicto centro anniteretur. vnde fit vt hoc modo pondus magis aut minus sit graue, quò magis aut minus à centro pendet, aut eidem nititur: atq; hæc est causa proxima, & per se, qua fit vt vnum idemq; pondus in vno eodemq; medio magis aut minus graue existat.



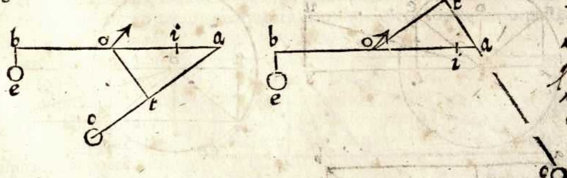
nõ est neq; p[ro]xima neq; p[er] se; nam [pond]us in f brachij nõ est equegrave ut pondus in u brachij bu; pondus in e brachij be est equegrave ut pondus u brachij bu. Unde tota hec demonstratio falsa est.

ſtat. Et quamuis appellem latus.B.C.orizontale, ſupponens illud angulum rectum cum. C.O. facere, vnde angulus.C.B.Q.fit vt minor ſit recto, ob quantitatem vnius anguli ęqualis ei, quem duæ.C.O.et.B.Q.in centro regionis elemẽtaris conſtituũt, hoc tamen nihil refert, cum dictus angulus inſenſibilis ſit magnitudinis. Ab iſtis autem rationibus elicere poſſumus, quòd ſi punctus.u.erit ex æquo medius inter centrum.B.& extremum.C. pondus.F.aut.M.pendebit, aut nitetur pro medietate dicto centro.B.& ſi dictum.u.erit propius.B.quam puncto.C.pendebit ab ipſo, aut nitetur ipſi amplius quã ex medietate, & ſi magis verſus. C. minus quã ex medietate nitet.

Quòd quantitas cuiuſlibet ponderis, aut uirtus mouens reſpectu alterius quantitatis cognoſcatur beneficio perpendicularium ductarum à centro libræ ad lineam inclinationis.

CAP. III.

EX ijs, quæ à nobis hucuſque ſunt dicta, facilè intelligi poteſt, ꝙ quantitas. B. u. quæ ferè perpendicularis eſt à centro. B.ad lineam. F.u. inclinationis, ea eſt, quæ nos ducit in cognitionem quantitatis virtutis ipſius. F.in huiuſmodi ſitu, conſtituens videlicet linea. F.u.cum brachio. F.B.angulum acutum.B. F.u. Vt hoc tamen melius intelligamus, imaginemur libram.b.o.a.fixam in centro.o. ad. cuius etrema ſint appenſa duo pondera, aut duæ virtutes mouentes.e.et.c.ita tamen ꝙ linea inclinationis.e.ideſt.b.e.faciat angulum rectum cum.o.b.in puncto.b. linea verò inclinationis.c.ideſt.a.c.faciat angulum acutum, aut obtuſum cum.o.a.in puncto.a.Imaginemur ergo lineam.o.t.perpendicularem lineæ.c.a. inclinationis, vnde. o. t. minor erit.o.a.ex.18. primi Euclidis. ſecetur deinde imaginatione o.a. in puncto. i. ita ut o.i.æqualis.ſit.o.t.& puncto.i.appenſum ſit pondus æquale ipſi.c.cuius inclinationis linea parallela ſit lineæ inclinationis ponderis.e.ſupponendo tamen pondus aut virtutem.c.ea ratione maiorem eſſe ea, quæ eſt.e.qua.b.o.maior eſt.o. t. abſque dubio ex.6.lib.primi Archi.de ponderibus.b.o.i.non mouebitur ſitu, ſed ſi loco.o.i. imaginabimur.o.t.conſolidatam cum.o.b.& per lineam.t.c.attractam virtute. c. ſimiliter quoque continget ut b.o.t; communi quadam ſcientia, non moueatur ſitu. Eſt ergo quod propoſuimus verum quantitatem alicuius ponderis reſpectu ad eam, quæ eſt alterius debere deprænhendi à perpendicularibus, quæ à centro libræ ad lineas inclinationis exiliunt. Hinc autem innoteſcit facillimè, quantum vigoris, & vis pondus, aut virtus.c.ad angulum rectum cum.o.a.minimè trahens, amittat. Hinc quoque corollarium quoddam ſequetur, quò d quantò propinquius erit centrum.o.libræ centro regionis elementaris, tantò quo que minus erit graue.

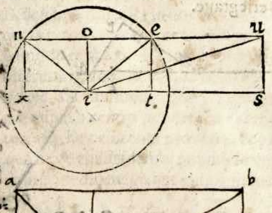


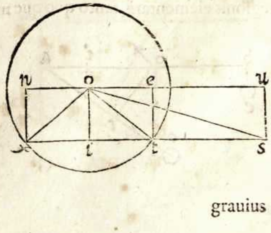
CAP.

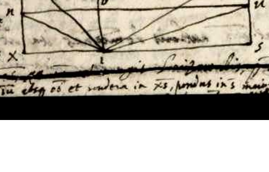
Quemadmodum ex ſupradictis cauſis omnes ſtaterarum & uectium cauſæ dependeant.

CAP. IIII.

VIs brachij longioris alicuius ſtateræ, aut vectis, maior breuioris, ab ijs, quæ in ſuperioribus capitibus diximus, ideſt ꝙ nitatur pendeatuè magis aut minus à centro pondus in extremitate brachij maioris poſitum, oboritur. Quamobrem illud à nobis primò eſt cognoſcendum, ſtateras, aut vectes, puras mathematicas lineas non eſſe, ſed naturales, hincque exiſtere corpora cum materia coniuncta. Nunc igitur imaginemur.n.s.eam ſuperficiem eſſe, quæ ſecundum longitudinem axem ſtateræ ſcindit.& ſupponamus ipſius centrum eſſe primum in.i.& maius brachium eſſe .i. u: minus autem. i. n. & lineam verticalem. i. o. quæ tanta ſit, quanta eſt ſpiſſitudo, aut craſſities ipſius ſtateræ à ſuperiori latere ad inferius, ad faciliorem intelligentiam, ſupponendo.n.s.parallelogrāmam. Poſitis igitur duobus ponderibus æqualibus in extremitatibus brachiorum, experientia innoteſcit, ꝙ pondus ad. u. s. appenſum, viol entiam faciet ponderi appenſo ad.n.x.ſed nos volumus inueſtigare cauſā huius effectus, quæ à nemine vnquam literarum monumentis, ꝙ ſciam, conſignata fuit. Iam diximus ſtateram, aut vectem materialem eſſe &.n.s.eius ſuperficiem mediam, ſupponendo.i.eſſe centrum quo nititur dicta ſtatera aut vectis; Cum hoc ergo ita ſe habeat, ſint.u.s.et.n.x.lineæ inclinationum ponderum, & imaginemur, ꝙ dicta pondera pendeant à punctis.u.et.n.vt reuera pendent, etiam ſi appenſa eſſent ſub.s.et. x. quia punctum.u.& punctum.n.ita coniuncta ſunt cum.s. et.x.ut qui vnū trahit alterum quoque trahat. Imaginemur quoque duas lineas. i. u: i.n. et.i.e.quę i.e.faciat angulum.o.i.e.æqualem angulo.o.i.n.Hinc clarè nobis patebit, ſi quis ipſi e.pondus ipſius.u.(ꝙ æquale eſt ponderi.n.) appenderet, id eandem planè vim haberet, quam pondus ipſius.n.habet, & ſtateram neque ſurſum, neque deorſum mouerет, quia ambo pondera ad centrum.i.mediantibus lineis.e.i.et.n.i. ex ęquo annitenrentur, ſed dicto pondere poſito in.u.linea.u. i. per quam pondus centro annititur, magis orizontalis quam.e.i.fit, & linea.u.s.inclinationis longius diſtans à centro. i. quām linea.e.t.vnde huiuſmodi pondus magis quoque liberum à centro.i.reſultat. magiſque ponderoſum, quam cum erat in.e.ratione eorum, quæ primo & ſecundo capitibus diximus, & ob hanc cauſam ſuperat pondus poſitum in.n. Sed ſi centrum fuerit.in.o. imaginabimur duas lineas.o.s.et.o.x.& ſupponemus quòd pondera poſita ſint in. s. et. x.vnde exiſtente magis orizontali linea.o.s.quam erit.o.x.& linea u.s.inclinationis longius diſtante à centro.o.quàm linea.e.t.eius pondus erit quoq;







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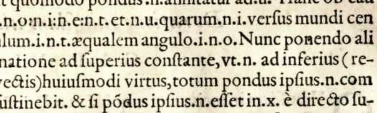
grauius, quia tantò minus pendebit à centro. o.& ratiocinando, vt ſuperius diximus, inueniemus eundem effectum verum eſſe. In ſtateris, rectè & propriè appellari poteſt.x.i.s.aut.n.o.u.orizontalis, ſed in omni vectium ſpecie, hoc tãtum per quandam ſimilitudinem dicetur. Idem contemplari licet ſupponendo centrum in medio inter.o. et.i.quod vnuſquiſque ex ſe abſque alterius auxilio facile præſtare poterit.

De quibuſdam rebus animaduerſione dignis.

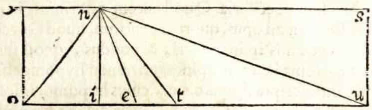
CAP. V.

NOn omittenda mihi vidẽtur quædam, quæ ad tractationẽ vectium admodum ſunt neceſſaria. Quod autem quærimus, in eo conſiſtit, quòd aliqui vectes adhibeantur ad opus, quorum centrum, quod Græci hypomochliõ appellant vnum eſt ex extremis ipſius vectis, & pondus, quod ſurſum eleuari debet, inter ipſamet extrema iacet, propinquum tamen hypomochlio, vt exempli gratia, ſi vectis eſſet infraſcripta figura.o.s.u.x.cuius hypomochlion eſſet in puncto.o. & pondus in puncto.n.clarum erit, ꝙ cum eleuari debeat.n.oportebit quoque opera manus eleuari.u. Nunc conſiderandum eſt quomodo pondus.n.annitatur ad.u. Hanc ob cauſam imaginabimur rectas lineas.n.o:n.i:n.e:n.t. et.n.u.quarum.n.i.verſus mundi centrum ſit poſita, et.n.t.faciat angulum.i.n.t.æqualem angulo.i.n.o. Nunc ponendo aliquam virtutem in.i.æquali inclinatione ad ſuperius conſtante, vt.n. ad inferius (remota tamen grauitate materiæ vectis) huiuſmodi virtus, totum pondus ipſius.n.communi quadam ſcientiæ notione ſuſtinebit. & ſi põdus ipſius.n.eſſet in.x. è directo ſuper.o.totum pondus ſuper hypomochlio ſe haberet, & tanta virtus ipſius hypomochlij ſufficeret ad reſiſtendum pro ſuſtinendo, quanta eſt grauitas ipſius ponderis, ſed ipſum iterum ponamus in.n.ibi clarum erit, quòd ſi alia virtus à parte inferiori ad ſuperiorem vectis non opponitur, excepto tamen hypomochlio, oportebit virtute cuiuſdam partis ponderis.n. (abſque conſideratione tamen, vt iam dixi, ponderis materiæ vectis) vt vectis à parte.s.u.deprimatur, & dixi vnius cuiuſdam partis ponderis.n.quia alia eiuſdẽ ponderis pars annititur ipſi hypomochlio. o. mediãte linea o.n.quæ angulos rectos cum.o.x.non facit. Si autem à puncto.t.opponet ſeſe huiuſmodi reſiſtentia, vt vectis non deprimatur, clarum erit communi ſcientia, ꝙ virtus ponderis.n.diuiſa erit per medium æqualiter, cuius vna medietas ſuper. o.quieſcet, & alia ſuper.t.mediantibus duabus lineis.n.o.et.n.t. Imaginemur nunc reſiſtentiam t.ablatam eſſe, poſitamq; in.e.clarum quoque erit, ꝙ maior pars ponderis. n. ipſi. e. annitetur beneficio lineæ.n.e.quàm ipſi. o.cum linea.n.i. inclinationis ipſi.e. ſit propinquior quam.o.quia omnis reſiſtentia aut in. i.aut in. e. aut in. t. aut in. u. eſt loco centri, quemadmodum eſt.o.& alter alterius opera iuuatur. Si verò eadem reſiſtentia poſita erit in.u.clarum quoque erit, ꝙ minor pars ponderis.n.annitetur ipſi.u.quã ipſi.o.cum dicta.n.i.à centro.u.longius quam à centro.o.diſtet, & proportio partis ponderis.n.in.o.ad proportionem partis ponderis.n.in u.non erit ſecũdum proportionem angulorum.u.n.i.et o.n.i.ſed ſecundum proportionem. u.i.ad.i.o.quod clarè compræhendi poteſt ab



T huius

huius effectus conuerſo, ideſt,vt quemadmodum nunc ſupponuntur.o.et.u.eſſe duo centra quibus ſuſtinet pondus.e. ipſius.n. imaginemur. n. eſſe quoddam centrum à quo pendeant duo pondera. o. et. u.ſic inuicem proportionata,ut ſunt.u.i.et.i. o. certe horum ponderum cauſa ſtatera. o.s.quam vectem appellabamus à nulla parte inclinabitur. Redeuntes nunc ad propoſitum, dicemus q̃ annitente pondere ipſius. n.minus ad.u.quam ad.o.ideſt ad.t.minori vi opus erit in. u. quàm in.t. ad attollendum pondus ipſius.n. & ſic per conſequens quantò longius erit punctum.u. ab.t.tantò minori quoque vi egebit, & conſequenter quando vis,aut reſiſtentia in.u.ita proportionata erit illi,quæ eſt ipſius.o.vt eſt.o.i.ad.i.u.vectis non mouebitur. Sed quando erit proportio maior,reſiſtentiæ ipſius.u.ad eam, quæ eſt ipſius. o.ea,quæ eſt. o. i. ad. i. u. tunc vectis à parte ipſius.u.s. eleuabitur, ſi vero proportio minor eſſet quàm.o.i. ad. i. u. tunc vectis ab eadem parte deprimetur.



De ratione cuiuſdam uis adauctæ.

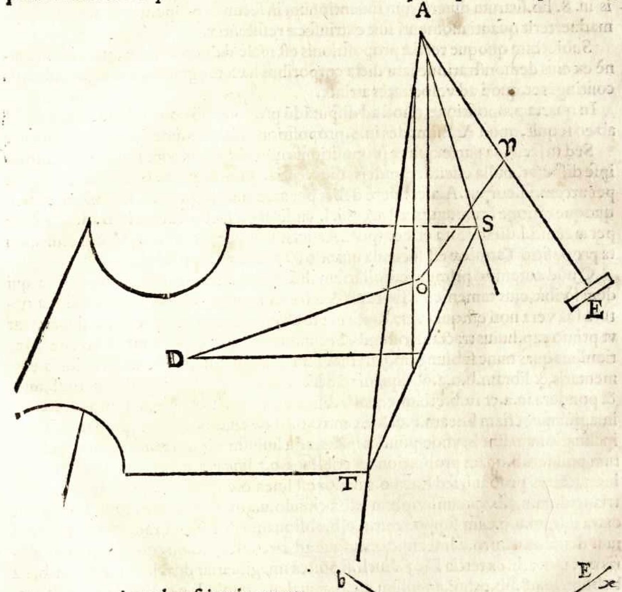
CAP. VI.

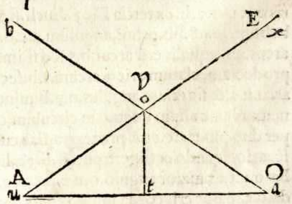
QVibuſdam in locis vtuntur quidã quodã inſtrumẽto piſtorio ad ſubigẽdã paſtam,vnius tantum hominis ui adhibita, quæ quidem machina cum mihi digna contemplatione eſſe videatur, eius aliquam rationem proponere volui,pro cuius deſcriptione imaginemur planum, in quo ſedet ille,qui voluit paſtam, & in quo ipſa paſta eſt repoſita.T. S. D.& triangulum.T. A. S. immobile perpendicularemque ſuperficiei dicti plani,angulo autem.A.coniunctum lignum.A.E.vt ſemidiametrum mobilem,& æqualem perpendiculari ipſius trianguli,unde. A. loco centri erit et. D. O.ſit ſemidiameter, qui paſtam contundit,& ab eius extremo.O. (quod. O. quando. D. O.orizontalis eſt,in baſi dicti trianguli reperitur) veniat lignum. O. V. quod cum.A.V.ſit æquale perpendiculari imaginatæ ab angulo. A.baſi.T.S. denodatũ tñ utvulgo dicĩ ſeu flexile in.O.& in.V.vt elleuare atq; deprimere ſemidiametrum.D.O.poſſit,et.V.O.ſit æqualis.A.V.et.V.medium ſit inter.A.et.E.vnde.A.V. cum.O.V.æquales erunt.A.E.ſunt deinde duo ligna perpẽdicularia ab. A.ad baſim fixa, & immobilia inter ſe adeò diſtantia,vt inter ipſa pertrãſeãt.O.V.et.D.O.ſupra & infra, ne deuiet ſemidiametrum.D.O.In extremitate deinde ipſius.E.ſit lignum quoddam tenue, vt digitus polex,ad angulos rectos cum.A.E.quod ab aliquo,qui antedictam machinam ſtet, manibus teneatur, qui quidem homo idipſum lignum, ideſt ſemidiametrum. A.E.à ſuperficie trianguli dicti,ad ſe trahendo, & deinde verſus eundem triangulum impellendo, vim quandam maximam mediante ſemidiametro. D.O.ſuper paſtam excitat.

Pro cuius rei contemplatiõe volo vt ſecundam hanc ſubſcriptam figuram. b. a. u.x.imaginemur, in qua.u. exprimat.A.primæ figuræ,&.a.denotet.O.&.o.V. &. x. E.imaginemur etiam.u.a.baſem trianguli.a.u.o.cui.o.t.perpendicularis dictæ baſi. u.a.addatur.Hucuſq; igitur.u.o.æqualis erit.o.x.& ipſi.o.a.imaginemur etiam. a. o. vſque ad.b.ita productam vt.o.b.æqualis ſit.o.a.ponamus etiam pondus in.a.impel-

lere

lere verſus.u.vnde linea eius inclinationis ſit ſemper.a.u.ſupponamus etiam. a. o. b. eſſe librã, aut ſtateram, aut vectem, &c. o. eius centrum, vnde vis, aut virtus ipſius.a. proportionalis erit ipſi. o. t. reſpectu virtutis, aut vis imaginatæ in. b. inclinationis perpendicularis ipſi.b.a.quæ quidem virtus, aut vis in.b. proportionalis erit ipſi. b. o.ex tertio capite huius tractatus; Si ergo fuiſſet poſita in. b. virtus quædam ad angulum rectum, trahens lineam.b.o.tam proportionatam virtuti perpendiculari ipſius.a.quam eſt.o.t.proportionata ipſi.o.b.ſtatera.b.o.a.non moueretur, ſed quæuis portio maior in.b.ſuperaret.a.cum autem fuerit.o.x.æqualis ipſi.o.b.idẽ planè eue-





niet, communi quadam ſcientia, ponendo virtutem.b.in.x.Quantitas ergo virtutis in.x. quæ ſuperare debet reſiſtentiam in.a.quæ ipſi.u.contraponitur, debet habere aliquantulum maioris proportionis ad reſiſtentiam, quæ in. a. angulum rectum efficeret cum. a.o. ea, quæ eſt. o. t. ad.o.x.

De quibusdam erroribus Nicolai Tartaleæ circa pondera corporum & eorum motus, quorum aliqui desumpti fuerunt à Iordano scriptore quodam antiquo.

CAP. VII.

CVm magis amici veritatis esse debeamus quàm cuiusquam hominis, quemadmodum Aristo. scribit, detegam hoc loco quosdam errores Nicolai Tartaleę de ponderibus corporum, & velocitatibus motuum localium. Et primum decipitur is in. 8. lib. suarum diuersarum inuentionum in secunda propositione, cum non animaduerterit quanti momenti sint extrinsecæ resistentiæ.

Subiectum quoque tertiæ propositionis est malè demonstratum, quia idem planè ex eius demonstratione iam dicta corporibus hætereogeneis, aut figura diuersis contingeret, quod ad velocitates attinet.

In quarta propositione, quod ad disputãdũ proponit nõ concludit melius. autẽ id ab eo sequit, quod Archimedes in. 6. propositione lib. primi de põderibus p̃bauit.

Sed in secunda parte quintę propositionis non uidet ꝙ uigore situs eo modo, quo ipse disputat, nulla elicitur ponderis differentia. quia si corpus. B. descendere debet per arcum. i. l. corpus. A. ascendere debet per arcum. u. s. æqualem, & similem. eadem quoque ratione situatum, vt est arcus. i. l. vnde vt est facilè corpori. B. descendere per arcum. i. l. difficile ita erit corpori. A. ascendere per arcum. u. s. Hęc autem quinta propositio Tartaleæ est secuuda quæstio à Iordano proposita.

Quòd autem ad primum corollarium dictæ propositionis attinet, verum ille quidem scribit, eius tamen effectus causa & à Iordano prius, & ab ipso postea citata, natura sua vera non est. quia vera causa per se ab eo oritur, ꝙ à centro libræ dependeat vt primo cap. huius tractatus ostendi. Secundum verò corollarium falsum esse, ijs rationibus quas nunc subiungam, patebit. Imaginemur. u. pro centro regionis elementaris, & libram. b. o. a. obliquam respectu ad. u. & brachiis æqualibus constãtem, & pondera in. a. et in. b. etiam æqualia. lineæ autem inclinationum sint. a. u. et. b. u. imaginemur etiam lineam. o. u. & à centro. o. libræ duas. o. t. et. o. e. perpendiculares inclinationum lineis; vnde pondus ipsius. a. in huiusmodi situ tam erit proportionatum ponderi. b. quam proportionata erit linea. o. t. lineæ. o. e. ex eo ꝙ tertio cap. huius tractatus probaui, sed linea. o. t. maior est linea. o. e. quod sic probo. Imaginemur triangulum. u. a. b. circunscriptum esse à circulo. u. a. n. b. cuius. c. sit centrum, ꝙ erit extra lineam. u. o. cum supponatur. a. o. b. obliquam esse respectu ad. u. o. Imaginemur deinde à centro. c. lineam. c. o. s. vsque ad circunferentiam, quæ perpendicularis erit ipsi. a. b. ex tertia lib. 3. Eucli. si posteà imaginemur duas lineas. c. a. et. c. b. habebimus ex. 8. lib. primi, angulum. a. c. o. æqualem angulo. b. c. o. Vnde ex. 25. lib. 3. arcus. a. s. æqualis erit arcui. b. s. sed si imaginabimur. u. o. ad circunferentiam vsque productam, clarum erit ꝙ arcum. s. b. secaret in puncto. n. vnde arcus. n. b. minor erit arcu. n. a. & sic etiam angulus. n. u. b. minor erit angulo. n. u. a. ex ultima lib. 6. Imaginemur nunc alium quendam circulum, cuius. o. u. sit diameter, cuius circunferentia per duo puncta. e. et. t. prætergradiat̃, cum in ipsis sint anguli recti, quod quilibet ex se ratiocinando colligere potest, si. 30. lib. 3. in mentem reuocauerit. Sed cum angulus. o. u. t. sit maior angulo. o. u. e. arcus. o. t. maior erit arcu. o. e. ex vltima. 6. vnde corda. o. t. maior erit corda ipsius. o. e. ex conuerso. 27. lib. 3. quod est propositum. Pondus igitur ipsius. a. in huiusmodi situ, pondere ipsius. b. grauius erit. Quod è directo ijs repugnat quæ Tartalea in 2. parte quinræ propositionis edixerit, & per consequens 2. corollarij falsitatem ostendit, vt eam quoque, quæ in 6. propositione latet. quia cũ

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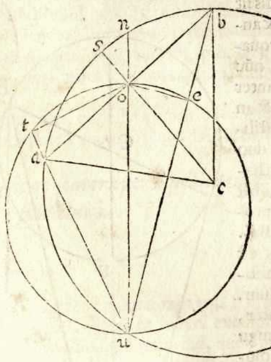
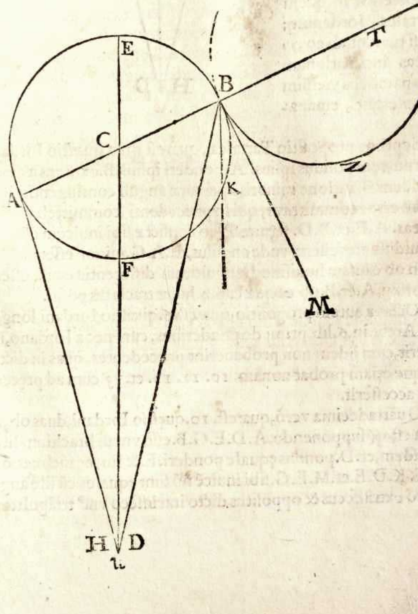


proportio põderis.a.ad pondus ipsius.b.eadem sit cum ea quę est.o.t.ad.o. e.sub cognitionẽ nostram cadere potest, primum cognoscendo angulos obliquitatis librę, idest angulos.b.o.u.et. a. o. u.quia oportet semper supponere situm aliquem notum. Si nobis deinde cognita erit proportio ipsius. o.u.ad.o.b.et. ad. o. a. assequemur cognitionem anguli.b.et.o.a. u. & per consequens ipsius.o.a.t. eius residui, vnde postea beneficio angulorum.e.et.t. rectorum & laterum.o.b.et.o.a.cognitorum in cognitionem.o. t. et.o.e. facile deueniemus.

CAP. VIII.

QVod autem idem Tartalea in.6.propositione, & Iordanus in secunda parte. secundæ propositionis scribunt,maximum quoque errorem in se continet. Dicunt enim angulũ h. a. f. differentem ab angulo.d. b. f. alia ratione non esse quàm per angulum contactus duorũ circulorũ, vt in sua figura scribit Tartalea; id quod falsissimum est. Quã ob causam in subscripta figura sit libra. B. A. & eius centrum.C et. u. centrũ regionis elementaris,et. A.u.et.B. u.lineæ inclinationũ. Imaginemur deinde lineam.B.K. parallelã ipsi.A. u. quæ gyrum. B.F. A. in puncto. K. communi scientiæ prę cepto scindet,& habebimus angulum.K. B. Z. æqualem angulo. H. A. F. idest.u. A. F. (quia.H. u.et.D. unũ sunt) cum ex.29. libr. primi Euclidis angulus.



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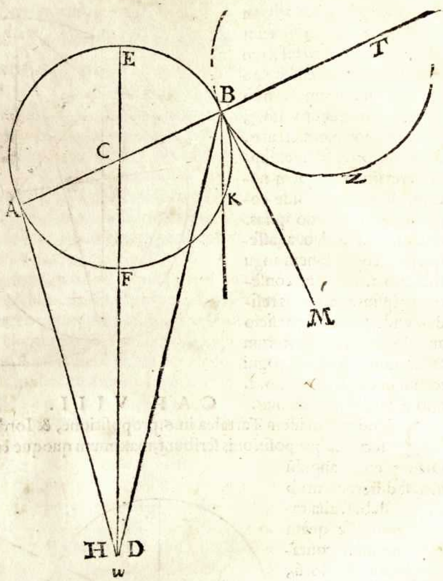
lus.u.A. C. æqualis ſit angulo.K. B. T. & angulus. C. A. F. æqualis angulo.T.B.Z. nũc comparatio eſt inter angulum.D.B.F.& angulum.K.B.Z. miſtilineos,qui quidem duo anguli, cõmunem habent angulum miſtilineum.K.B.F.quapropter ſi angulus.K.B.Z. miſtilineus maior eſt angulo.D.B. F. miſtilineo per angulum. K.B.Z.contingentiæ, circulorum ergo angulus miſtilineus communis.K.B.F. æqualis erit miſtilineo, angulo. D.B. F. pars videlicet ſui toto. Omnis autem error in quem Tartalea, Iordanusq; lapſi fuerunt ab eo, ꝙ lineas inclinationum pro parallelis viciſſim ſumpſerunt, emanauit.



Septima propoſitio Tartaleæ, quæ eſt q̃nta quæſtio Iordani mihi videt̃ excipienda riſu,cum pondus ipſius. A.ponderi ipſius.B.exiſtens æquale, grauius ſit pondere eiuſdem.B. ratione minoris aperturæ anguli contingentiæ in.A.quam in. B. in quo idem error committitur, qui in præcedenti committebatur, cum ſcilicet ipſe putet lineas. A.E.et.B.D. figuræ ab eo confictæ ſibi inuicem eſſe parallelas, quæ etiam ſi æquidiſtantes eſſent(vnde angulus. E.A.G.minor eſſet angulo.D.B.F.) non eam tamen ob cauſam huiuſmodi angulorum differentia cauſa eſſet differentiæ grauitatũ ipſorum.A.et.B.ob ea quæ cap.4.huius tractatus poſui.

Octaua autem propoſitio,quæ eſt.6.quæſtio Iordani longè melius demonſtratur ab Archi.in.6.lib.primi de ponderibus, cum nec à Iordano,nec à Tartalæa probata fuerit,cum ijdem non probauerint præcedentes, quas in dicta.8.Tartalęa citat, qui neque etiam probat nonam. 10. 11. 12. et.13.cum ad pręcedentes probandas minimè acceſſerit.

Quartadecima verò,quæ eſt. 10.quęſtio Iordani,duas ob cauſas eſt falſa, quarum vna eſt,ꝙ(ſupponendo.A.D.E.G.B.eſſe vnum brachium librę,et.A.punctum cẽtri eiuſdem,et.D.pondus ęquale ponderi.E.& lineas inclinationum.D.K.et.E. M.) anguli. K.D.E.et.M.E.G.ſibi inuicẽ nõ ſunt ęquales;cũ ille angulus ſit intrinſecus,hic verò extrinſecus & oppoſitus dicto intrinſeco vniꝰ triãguli terminati à.D.E. à.D.K. et.E.

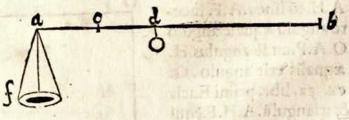
et. E. M. lineis productis vsque ad centrum regionis elementaris,vnde dictus angulus. M.E.G.maior est alio,ex.16 lib.primi Eucli.Qua ratione fit,vt hanc ob causam E.grauius sit ipso.D.cum minus dependeat à centro.A.vt primo cap.huius tractatus iam dixi. Alia quoque est ratio,qua dictum. E. grauius fit ipso. D. quę quidem est maior distantia à centro.A. libræ, per similes rationes capit. 4. huius tractatus citatas.

Decimaquinta quoq; nil penitus valet, quę est.11.quęstio Iordani,cuius Authoris opusculum opera Traiani Bibliopolę Venetijs è tenebris in lucem emersit.

Quòd summa ratione stateræ per æqualia interualla sint diuisæ.

CAP. IX.

Magna cum ratione diuidūtur stateræ per interualla ęqualia, in libras, aut in vncias, aut quoquo alio modo. Nam sit statera exempli gratia. a. b. & punctum, ꝙ eam sustinet sit. c.& vas illud,ꝙ continet id, quod ponderari debet f.Imaginemur nunc quod pondus brachij.c.b.ab una parte,& pondus brachij.c.a.cū eo, ꝙ est dicti vasis.f.ab altera parte,sint causę,quibus statera.a.b. c. stet orizontalis.cui sic orizontali manenti imaginemur ad punctum. a. adiunctum esse pondus, veluti vnius librę. & ad punctum.d.tam distanti à.c.ut est.a.ab ipso.c. aliud quoque pondus vnius libræ additū esse,vnde cōi quadā scientia statera,non mouebitur situ. qa existentibus duobus hisce ponderibus æqualibus,altero in.d. & altero in.a.remota cum essent.d.b.et.f.absque dubio.a.d.non mutaret situm,sed. d.b. et, f. in situ, in quo reperiuntur,à centro paribus viribus prędita sunt. Addendo igitur.d.b. ipsi. d. et.f.ipsi.a:summa earum, æqualibus quoque viribus constabunt. ex communi sententia, quæ habet si ęqualibus addas ęqualia, tota quoque fient ęqualia. Si verò ponderi ipsius. a. aliud adderetur eidem ęquale, haberemus in. a.duplum pondus ei ꝙ est ipsius.d.sed volentes vt solum cum pondere ipsius. d.statera stet orizontalis,si dictum pondus ipsius.d.longè distabit à centro.c.per duplum ipsius.c.a.idest ipsius.c.d. id ꝙ volumus assequemur, beneficio supradictarum rationum,adiuti opera sextę lib.primi de pōderibus Archimedis. Et si quis aliud quoq; pondus adiungeret ipsi.a.æquale illi priori, ad efficiēdum,vt statera semper orizontalis maneret,oporteret, vt pōdus ipsius.d.ab.c.longè distaret, ita vt huiusmodi distantia tripla esset primæ, & sic per quosdam quasi gradus interualla redderentur æqualia.



Quòd

Quòd linea circularis non habeat concauum cum conuexo coniunctum, & quod Ariſto. circa proportiones motuum aberrauerit.

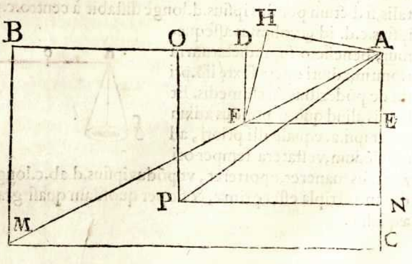
CAP. X.

ARiſtoteles in principio quæſtionum Mechanicarum ait lineam, quæ terminat circulum videtur conuexum habere coniunctum cum concauo, quod falſum eſt: quia huiuſmodi linea partes nullas ſecundum latitudinem habet, (vt ipſe etiam confirmat) ſed eſt idem conuexum circuli: linea verò quæ terminus eſt ſuperficiei ambientis, & amplectentis circulum eſt eadem concauitas dictæ ſuperficiei eundem circulum ambientis, quæ nullam conuexitatem habet. & hæ duæ ſunt lineæ, quarum vna diuerſa eſt ab alia, neque altera alterius, quod ad conuexum, & ad concauum attinet.

Sed illud, quod Ariſtoteles ſcribit de duplici reſpectu motus vnius puncti ſecundum vnam datam proportionem, non ſufficit, ille enim ſic ait.

Sit proportio ſecundum quam latum fertur, quam habet. A.B. ad. A. C. et. A. quidem feratur verſus. B: A.B. verò ſubterferatur verſus. M. C. latum autem ſit. A, quidē ad. D. vbi autem eſt. A. B. verſus. E. Quoniam igitur lationis erat proportio, quam. A.B. habet ad. A. C. neceſſe eſt &. A.D. ad. A.E. hanc habere rationem. Simile igitur eſt proportione paruum quadr ilaterum maiori. Quamobrem etc.

Cui reſpondeo, punctum. A. quod mouetur in linea. A.M. ab. A. verſus. M. vſque ad. F. non moueri ab aliqua proportione determinata magis quàm ab alia: vnde nō ſolum poſſumus imaginari dictum punctum. A. moueri ab. A. vſque ad. F. eiuſdem velocitatis ſub alia quadam proportione, ſed etiam ſub alia, quæ iam datæ contraria ſit, vt eſt proportio ipſius. A.C. ad. A.B. imaginātes moueri. A. verſus. C. et. A. C. verſus. B.M. delatam. Dico etiam idem. A. moueri vſque ad. F. ſecundum proportionem ipſius. A.O. ad. A.N. Quamobrem imaginemur à puncto. F. lineam. F. H. cum linea. F.A. efficere angulum æqualem angulo. O. P.A. & à puncto. A. lineā A.H. cū linea. A.F. facere angulū æqualē angulō O.A.P. unde angulus. H. æqualis erit angulo. O. ex. 32. libr. primi Eucl. & triangulū. A.H.F. ęquiangulum erit triangulo. A.O.P. Quam ob cauſā eadē proportio erit ipſi⁹ A.H. ad. F.H. quę ē ipſius A. O. ad. O.P. punctum



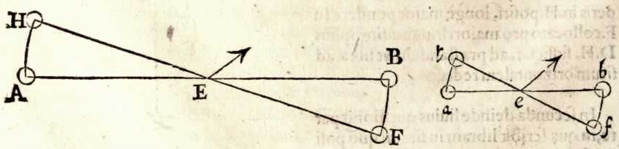
igitur. A. vſque ad. F. mouetur ſecundum proportionem etiam ipſius. A.O. ad. O. P. Huiuſmodi igitur conſideratio, ab Ariſtotele facta, nullius eſt momenti.

Quòd

Quod Aristo. in prima mechanicarum quæstionum eius quod inquirit, ueram causam non attulerit.

CAP. XI.

QVærens Aristoteles vnde fiat, vt eæ libræ, quæ brachia habent alijs longiora, sint exactiores cæteris, ait hoc euenire ratione maioris velocitatis extremorum earundem. Quod verum non est; quia hîc effectus nil aliud est, quam clarius proponere ob omnium oculos obliquitatem brachiorum à linea orizontali, & ostendere etiam facilius à dicto orizontali situ exire brachia iam dicta. Quæ quidem per se neque à velocitate, neque à tarditate motus, sed à ratione vectis, & à maiori interuallo inter secundum situm extremorum à primo proficiscuntur. Vt exempli gratia, imaginemur magnam libram. A. B. orizontalem, cuius centrum sit. E. et pondus. B. maius sit pondere ipsius. A. vnde conceditur, quòd ob hanc rationem dicta libra situm mutabit, qui secundus situs sit in. H. F. Imaginemur etiam paruã quãdam libram. a. e. b. orizontalem, quæ pondera habeat. a. et. b. æqualia duobus ponderibus alterius libræ & secundus situs sit in. h. f. ita tamen vt anguli circa. e. æquales sint ijs, qui sunt circa. E. ideſt. b. e. f. sit ęqualis. B. E. F. Nunc dico situm. H. F. exactiorẽ futurum & clariorem situ. h. e. f. ratione interualli. B. F. maioris, interuallo. b. f. quod. B. F. in eadem proportione maior est ipso. b. f. in qua. B. E. maius est. b. e. quod autem interuallum. B. F. breuiori, aut longiori temporis spacio quam. b. f. sit factum, nil planè refert. Ratione vectis deinde, dico ꝙ si supponemus duas libras pares æqualesq; in omni alio respectu, præter quàm in brachiorum longitudine, pondus. B. maiorem vim habebit ad deprimendum brachium. E. B. quàm pondus. b. quia libræ materiales, cum sustineantur ab. E. e. & non à puncto mathematico, sed à linea, aut superficie naturali in materia existente. vnde aliqua resistentia ipsi motui brachiorum oritur, & hanc ob causam, supponendo hanc resistentiam æqualem tam in. E. quàm in. e. clarum erit ob ea, quæ in cap. 4. huius tractatus ostendi. B. cum minus dependeat ab. E. aut minus quoque eidem. E. annitatur, ponderosum magis futurum, quam. b. & hac de causa mouebit ad partem inferiorem, maiori cum agilitate, brachium. E. B. multo magis etiam illud ipsum deprimet, ideſt maiorem etiam angulum. B. E. F. quàm erit angulus. b. e. f. faciet.



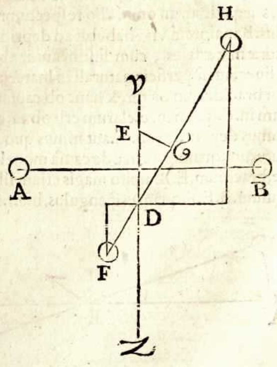
De uera causa secundæ, & tertiæ quæstionis mechanicæ ab Aristotele non perspecta.

CAP. XII.

ARistoteles in secunda quæstionum mechanicarum quærens illius rationem sic scribit.

Cur siquidem sursum fuerit spartum quando deorsum lato pondere quispiam id admouet rursus ascendit libra: si autem deorsum constitutum fuerit non ascendit, sed manet? an quia sursum quidem sparto existente plus libræ extra perpendiculum sit (spartum enim est perpendiculum) quare necesse est deorsum ferri id, quod plus est, quare & cætera.

Sed vera causa, vnde fiat, vt si spartum fuerit sursum, & brachium vnum ipsius libræ deprimendo, & idem liberum deinde permittendo, ad situm orizontalem redeat, non solum est maior quantitas ponderis brachiorum quæ iam prætergressa est vltra verticalem lineam, sed etiam est longitudo brachij eleuati, quæ vltra verticalem lineam reperitur, vnde eius extremì pondus redditur grauius in proportione, quam in hoc exemplo proponam, sit. A. B. libra in situ orizontali, cuius spartum sit. E. super ipsam. & deprimentes brachium ipsius. A. vsque ad. F. eius situs sit in. F. H. vnde medium pũctum. G. prætergressum erit lineam verticalem. V. Z. versus. B. quæ. V. Z. secabit brachium. F. G. in puncto. D. vnde. D. H. longius erit ipso. F. D. Nunc nobis supponendum est id, quod verissimum existit, dictam scilicet libram in situ. F. H. etiã si sustineatur à puncto. E. idem tamen futurum ac si sustentaretur in puncto. D. vnde sequitur, quod pondus appensum ex ipsa. H. ita grauius reddatur, ipso. F. in eadem proportione, quæ maior est. D. H. ipso. D. F. ob rationes quas in primis huius tractatus capitibus posui, vt etiam si. D. H. quod materiale esse supponitur, nullam planè grauitatem haberet, solus tamẽ excessus vis ponderis in. H. positi, longè maior pondere in F. collocato pro maiori longitudine ipsius D. H. sufficiat. ad præstandum vt libra ad situm orizontalem redeat.



In secunda deinde huius quęstionis parte, in qua scribit libram in situ, in quo posita est, firmam manere, toto cęlo aberrat, quia necessariũ est, vt omninò cadat, eòusq; quò spartum sursum remaneat: ablato tamen omni impedimento, quod nulla eget probatione, cum natura sua clarissimè pateat.

Causa, deinde, vera tertiæ quæstionis non est ea, quam Aristoteles ponit, sed huiusmodi effectus ab eo, quod capitibus. 4. et. 5. huius tractatus proposui originem habet.

Quod

Quòd Aristotelis ratio in 6. quæstione posita non sit admittenda.

CAP. XIII.

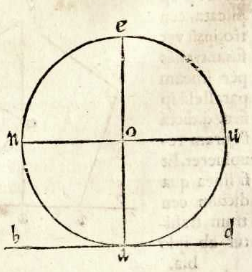
VOlens Aristoteles rationem proponere, vnde fiat, vt nauis velocius moueatur cum antennam altiorem quàm cum depræssiorem habet, id ad vectis rationem refert, quod verum nõ est. Huiusmodi enim ratione nauis tardius potius, quàm velocius ferri deberet, quia quantò altius est velum, vi venti impulsum, tãtò magis proram ipsius nauis in aquam demergit. Sed huiusmodi effectus à maiori potius quantitate venti quam recipit, quàm ab alia aliqua causa oritur, quia ventus liberius vehementiusq; in altiore parte, quàm in depræssione vagatur & perflat.

Quòd rationes ab Aristotele de octaua quæstione confictæ sufficientes non sint.

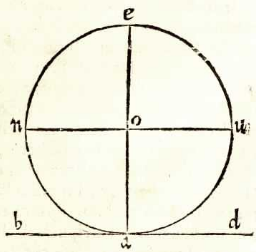
CAP. XIIII.

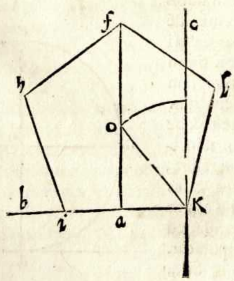
RAtiones etiam ab Aristotele propositæ pro indaganda octauæ quæstionis veritate, in qua quærit vnde fiat, vt corpora rotundæ figuræ, ad voluendũ sint faciliora reliquis, quarum reuolutionum corporum tres species assignat, quarũ vna est, vt rotarum curruũ; altera vt rotarum puteorum, aut trochlearum, quibus hauritur aqua; & tertia, vt paruorum vasorum a figulis fabricatorum, sufficiẽtes nõ sunt.

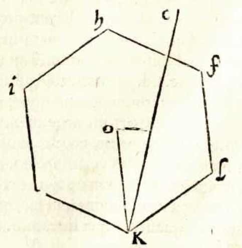
Incipiens autem à prima dico dubium non esse, quin tangente corpore aliquo rotundo aliquod planum mediante solo quodam puncto contingat, quemadmodum probat Theodosius in.3.lib.primi & Vitellio in.71.lib.primi, & ducẽdo per centrũ sphæræ lineam vsque ad punctum contactus, ipsa erit perpendicularis plano contingenti sphęram dictam, vt probat idẽ Thęodosius in.4.lib.primi Alhazẽ in.25.quarti, & Vitellio in.7.primi. Verum etiam est omnem inclinationem ponderosam huiusmodi corporis homogęnei totam hanc lineam æqualiter omni ex parte circundare; cuius quidem rei exemplum in carta describere possumus mediante figura circulari hîc subscripta.a.n.e.u.contigua lineæ rectæ.b.d.in puncto.a.vnde.e.o.a.perpendicularis erit ipsi.b.d.ex.17.lib.3.Eucli.& tantũ ponderis habebimus à parte.a.u.e.quantum ab ipsa.a.n.e. Nunc igitur si imaginabimur ductum esse centrum versus.u. per lineam.o.u. parallelam ipsi. a. d. clarum nobis erit, ꝙ absq; vlla difficultate aut resistentia idẽ ducemus, quia huiusmodi centrum ab inferiori parte ad superiorem, nunquam mutabit situm respectu distãtiæ seu interualli, quę inter ipsum lineamque.a.d. intercedit, ꝙ quidem centrum in se colligit totum pondus figurę.a.n.e.u.& beneficio lineæ.e.o.a.illud ipsum puncto. a. in linea.b.a.d.committit, productum.a. nil refert, vt magis, aut minus versus ipsum. d. aut versus b.dirigatur; ita vt cũ non oporteat vt huius figuræ põdus, vna vice, magis eleuetur, quàm alia, sed semper ęqualiter super lineam.b.a.d.quiescat.



Sitq; ſemper diuiſum à linea.a.o.e.per medium, ſequitur communi quodam conceptu, nullam nobis difficultatem oborituram, dictum centrum ad quam voluerimus partem ducendo, quemadmodum à qualibet alia figura,quæ perfectè rotunda non eſſet,emergeret; Vt exēpli gratia,ſi imaginabimur pentagonum.K.i. h.f.l.quieſcere ſuꝑ eandē lineā.a.b. K.ita ut primū totū latus. i.K.in linea.b.K.extēdat, ducēdo poſteà centrum.o. (ponamus.) verſus.l.dubium non eſt,quin oporteat,vt dictum centrum.o.à linea.b.d.eleuetur,ab eademq; magis diſtet, voluens ſe per arcū vnum circuli,q̄ ꝓ ſuo ſemidiametro habeat.o.K.quę maior eſt ipſa.o.a.ex.18. li. primi Eucli. vnde ſi à puncto.K.imaginabimur lineam. K. c. reſpicientem centrum regionis elementaris,dubium non eſt,quin ſi velimus transferre cētrum hoc à priori ſitu vſq; ad dictam lineam,oporteat addere pondus parti ipſius.l.quæ à linea.K. c.fuit ſecta, aut aliquid de ipſo pondere partis centri detrahere. quod quibuſuis modis fiat, arduum certè eſt ad efficiendum;neque hoc etiam accidit figuræ perfectè rotundæ, cum cētrum ꝙ perfectè in medio ipſius ponderis eſt,reperiatur ſemper in linea perpendiculari ipſi plano, in quo animaduertendum eſt, ꝙ etiam ſi ipſum planum appellem;pro plano tamen perfecto intelligi nolo,ſed pro ſuperficie perfectè ſphęrica circa centrum à corporibus grauibus expetitum; nam ratione magnæ amplitudinis huiuſmodi ſuperficiei,nullam differentiam notatu dignam à perfecto aliquo plano exigui interualli ad curuitatem eiuſdem ſuperficiei imaginari poterimus. Sed ut redeamus ad ſermonem de reuolutione figuræ rotundæ ſuſceptum, clarū igitur erit quamlibet minimam vim(vt ita dicam)quę trahat,aut impellat centrum.o.verſus.u. huiuſmodi figuram reuoluturam,cuius media pars ad trahendum, aut impellendum punctum.e.ſufficiere; Imaginemur autem ꝙ linea.n.o. u. eſſet libra quędā in figura perfectè rotunda.a.n.e.u.poſita,& vis, quę trahere centrum deberet, diuiſa eſſet per medium, cuius medietas appenſa eſſet extremitati. u. diametri.n.o.u.clarū erit, ꝙ abſque vlla difficultate reuolueret figuram ſuper lineam.b.a.d.verſus. d. quia huius vis,aut pondus nullū contra pondus haberet vltra centrum.o.uerſus. n. ꝙ centrum. o.perpetuo quieſcit ſuꝑ. a.in linea. e. o. a. per medium diuidente ſemper totum pondus figurę ſuppoſitę. Tantò facilius ergo tota dicta vis applicata centro,ipſū verſus.u.trahēs per lineam parallelā ipſi.a. d.dictā figuram reuolueret.Et ſi linea qua dictum centrum trahitur ab ipſo b.a.

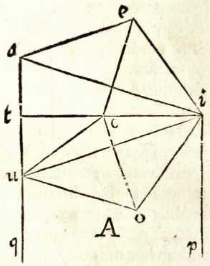


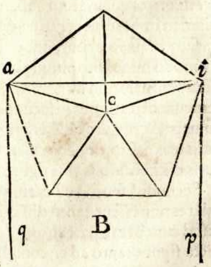


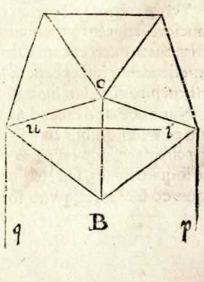


b.a.d.non æquediſtaret,ſed ſurſum traheret ſuper.u.aut ſubter, aliquid de ſua vi virtuteq́; amitteret,& tantò plus, quantò inclinata magis eſſet verſus.a.o.e. & tandem cum eſſet vnita cum.a.o.e.aut ad ſuperius,aut ad inferius quantalibet ui, etiam ſi infinita,figuram extra ſitum primæ lineæ.a.o.e.non moueret,ſed ſi ſurſum traheret ſe iungeret eam à linea.b.a.d.non ob id tamen efficeret,ut centrum.o. exiret extra primam lineam.a.o.e.

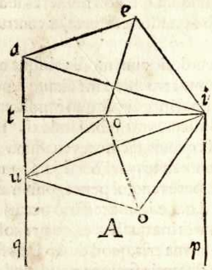
Secunda verò ſpecies,tribus reuolutionum modis,abſque axis mutatione conſtare poteſt,ideſt modo,quo reuoluuntur trochleæ mediante fune, & quo reuoluuntur aliquæ rotæ,in quibus aliquod animal incedit; & quo reuoluuntur illæ,quæ in hominis manu circunuoluuntur medio alicuius manubrij inflexi. Hi omnes modi cum circulari figura magis,quã cum alia quauis,faciliores euadunt. Et primò ſi priorem modum conſiderabimus,vt mediante fune quælibet figura, quæ circularis non ſit, voluatur,ſupponamus exemplo debere reuolui pentagonum æquiangulum.a.e.i.o.u.circa centrum.c.mediante fune.q.u.a.e.i.p.neceſſariò occurrent(in hac figura angulorum,laterumq́; diſparium) plures inæqualitates,quæ reuolutionem eiuſdem figuræ irregularem efficient; quarum vna erit,quod duæ partes funis,ideſt.u.q. et.i.p. non erunt in vna eademq́; inter ſe diſtantia ſemper,quod facile intellectu erit,ſi imaginabimur ductas eſſe lineas.a.i: u. i: et. i. c. t. ſi funis duo pondera habebit alterum altero maius,ſuis extremis appenſa,vnde debeat figura virtute ponderis maioris circunuolui: dictæ duæ partes. u.q.et.i.p.eiuſdem funis,mũdi centrum,dum firmæ manebunt,reſpicient;ſed permittentes pondera libera;maius,efficiens vt circunuoluatur figura; efficiet,vt aliquando vnum ex lateribus,eiuſdem figuræ mundi quoq; centrum reſpiciet, vt in figura.A.ſicq́; etiam linea.i. c.t.(pro exẽplo)erit menſura diſtantiæ funium inter ipſas, & deinde circũuoluendo etiam diſtabunt inter ſe per lineã.i.a.aut. i. u. vt in figura.B. ĩnotuit exẽplo, & ſic etiam aliquando erunt magis diſtãtes, quàm linea t. i. & minus quàm.i. a:nunquam tamen minus quam.t.i.neque magis quã i.a.aut.i.u. quæ ſunt æquales; Quæ quidem varietas, in hanc,& in illam partem impellet partes pendentes funis, vnde æqualiter non trahent. Idem dico, ſi extrema.q.et.p.eſſent quoque ſemper in vna eadẽq́; diſtantia; neque à corpore põderoſo eſſent attracta, quia aliæ partes ipſius.u.q.et.i.p.ex ſupradictis rationibus vnam eademq́; diſtantiam nõ ſemper ſeruarẽt. vnde fieret vt cum diuerſis angulis tam.i. p. quã.u.q. traherẽt ſemidiametros.c.i:c.e:c.a:c.u.et.c.o.quia nõ ſemper traherent ope ſeu virtute anguli æqualis ipſi. c.i.p.Hæc autem inæqualitas communis eſt omnibus figu-

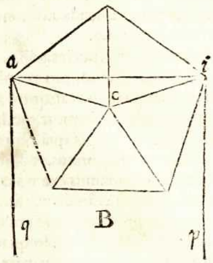




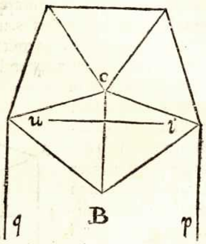


figuris rectilineis tam paris, quàm disparis numeri. Sed aliam quandam maiorem inęqualitatem habent hæ figuræ numeri disparis,quæ est,quòd quãdo linea. t.i.tam .u .q .quàm ipsi. i.p. ppẽdicularis fuerit, idest quãdo.t.i. cum dictis partibus funis angulos rectos constituerit, tũc ratione lõgitudinis ipsius . c. i. maioris quam . t. c.(quia cum sit.c.i.ęqualis ipsi.c.a.et.c.a. maior ipsa. c. t : c . i. etiam maior sit ipsa. c.t .) pondus aut vis ipsius.p.superabit eã quæ est ipsius. q. sed quando.t. erit in opposita parte,et.i.in ea, quæ est ipsius.t:q. eãdem ob causam superabit.p.& sic mo tum faciet irregularem,& nõ vniformem; & ob id etiam perarduum, præter ictus,quos infligunt anguli in partem pendentem ascendẽtem funis,quãdo vnum ex lateribus vnitur cum fune.





Aliam inęqualitatem habent figuræ pares, quæ etiam in imparibus cernitur,etsi aliquantulum diuersa;quæ ab eo oritur, quod funes sit modò magis,modo minus propinquę centro;quæ inæqualis distantia,maiorem minoremq; vim super dictum centrum ob rationes in secunda parte cap. decimi huius tractatus propositas, gignit. Nulla autem ex ijs inæqualitatibus circulari figuræ contingit. Illud verò , quod de pentagonis figuris dixi,omnibus aliis figuris disparibus accommodari potest.



Secundus modus est earum rotarum,in quibus aliquod animal incedit,quæ si circulares non essent,tantò difficilius voluerentur,quantò pauciores angulos haberent. quod cum per se pateat,non demonstrabo. Si ergo quantò plures angulos habebit dicta figura,tantò ad circunuoluendum hoc modo agilior erit. Circularis igitur figura,quæ ex infinitis angulis efficitur,omnium agillima erit.

Tertius modus est earum rotarum, quæ manubrium habent, quæ etiam quantò pauciores angulos habebunt,tanto quoq; difficiliores reddentur, tam ratione inimicitiæ:quam exercet cum vacuo natura,quàm violẽtię,quam anguli aëri faciunt,eum expellendo,vt ipsi occupent locum, quem ipse aër implebat . Quod nullo modo potest euenire circulari figuræ.

Nunc nobis ad dicendum restat de specie reuolutionis rotarum , quæ parallelæ sunt orizonti,quibus accidit posse volui primo tertioq; modo secundę speciei,& ob id si circulares non erunt,eadem subibunt incommoda,de quibus in secunda illa specie loquuti sumus. sed circulares rotæ huius tertiæ speciei ad reuoluendum erunt reliquis eò faciliores,ꝗ vno solũ polo nituntur ; Quod alijs nequaquam conceditur.

Super

Super hac tertia ſpecie formari poteſt problema, vnde fiat, vt quieſcens huiuſmodi rota parallela orizonti ſuper vnum punctum,& quantò fieri poteſt exiſtens ęqualis,ſi eam circunuoluamus maiore qua poterimus ui, & eãdem poſtea dimittentes non perpetuò circunuoluatur.

Hoc quidem,quatuor fit ob cauſas.quarum prima eſt,quia huiuſmodi motus,eius rotæ non ſit naturalis. ſecunda eſt, quia etiamſi rota ſuper punctum mathematicum quieſceret,oporteret tamen vt ſuperius alterũ haberet polum, qui ipſam orizontalẽ teneret, qui quidem munimento aliquo corporeo indigeret; vnde fricatio quędam conſequeretur, ex qua reſiſtentia prodiret.

Tertia eſt, quia aër contiguus eam perpetuò aſtringit, hocq; modo eius motui reſiſtit.

Quarta eſt,quia quęlibet pars corporeã,quę à ſe mouetur,impetu eidem à qualibet extrinſeca virtute mouente imprę̃ſſo,habet naturalem inclinationem ad rectum iter, non autem curuum,vnde ſi à dicta rota particula aliqua ſuę circunferentiæ diſiũgeretur,abſque dubio per aliquod temporis ſpatium pars ſeparata recto itinere ferretur per aërem, vt exemplo à fundis, quibus iaciuntur lapides,ſumpto,cognoſcere poſſumus, in quibus, impetus motus imprę̃ſſus naturali quadam propenſione rectum iter peragit, cum euibratus lapis,per lineam rectam contiguam giro, quem primo faciebat,in puncto,in quo dimiſſus fuit,rectum iter inſtituat, vt rationi conſentaneum eſt.

Eadem, quoque ratione fit, vt quantò maior eſt aliqua rota,tantò maiorem quoque impetum,& impreſſionem motus eius circunferentiæ partes recipiant, vnde ſę pe euenit,vt dum eam ſiſtere volumus,id cũ labore & cum difficultate agamus; quia quantò maior eſt diameter vnius circuli,tantò minus curua eſt eiuſdem circunferentia,& tantò propius accedit angulum eiuſdem circunferentiæ ad quantitatem duorum angulorum rectorum rectilineorum,ideſt circunferentia ad rectitudinem linearem. Vnde earundem partium dictæ circunferentiæ motus ad inclinationem ſibi à natura tributam, quæ eſt incedendi per lineam rectam,magis accedit.

Quod Ariſtotelis ratio nonæ quæſtionis admittenda non ſit.

CAP. XV.

VEra ratio nonæ quęſtionis à ſecunda parte decimi cap.huius tractatus, & non aliunde,accerſiri debet.

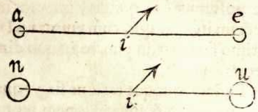
Quod Ariſtotelis rationes de decima quæſtione ſint reyciendæ.

CAP. XVI.

ARiſtotelis rationes,vnde fiat, vt facilius moueantur libræ vacuæ, quàm plenè ad propoſitam diſputationem non pertinent; quia ſemper ineunda eſt ratio proportionis virtutis mouentis ſuper mobile;quod ipſe non fecit.

Sit

Sit exempli gratia libra. a. i. e. quæ in vtraque extremitate vnciam vnam solum ponderis obtineat, & sit libra.n.i.u.æqualis priori,quæ pro singula extremitate vnã ponderis libram habeat. Aristoteles admiratur, quòd addendo ipsi.e.mediam pon deris vnciam,brachium.i.e.velocius cadat,quàm adijciẽdo ipsã mediã vnciã ipsi. u. brachij.i.u. Quod à duabus causis proficiscitur, quarum prior est,magna differentia proportionis vnius libræ ad medietatem vnius vnciæ,ad proportionem vnius vnciæ ad ipsam medietatem, quia si pondus adiectum extremo.u.dimidiæ esset libræ, & cum eadem tarditate brachium moueret,optimo iure in admirationem posset Aristoteles duci. Sed hoc fieri non posset,quia ipsum deprimeret cum eadem quasi velocitate,qua media vncia brachium.i.e. Dixi autem quasi, quia nonnihil discriminis intercederet,quod proficiscitur à secunda ratione. Et hæc, resistentia est, quæ oritur à sparto, quia quantò maius pondus continet libra,tantò magis præmit spartum in loco,in quo sustinetur;vnde maior resistentia in circunuolutione eiusdẽ sparti,in loco,in quo quiescit,exoritur,quia ipsum est corpus materiale. Si quis autem vellet,vt brachium.i.u.eadem agilitate,qua.i.e.descenderet,oporteret, vt proportio dimidiæ libræ adiectæ ponderi ipsius.u. quod est vnius libræ, vim suam haberet, quæ excederet resistentiam sui sparti (medio brachiorum maiorum ijs qui sunt. a. i. e.) ita proportionatam, vt proportionata est vis dimidiæ vnciæ ipsi. e.iunctæ,resistentiæ sui sparti. Huiusmodi rationes cum rotis grauioribus leuioribusq;,& ijs,quę à corporibus quibuslibet grauibus impelluntur,accommodatæ fuerint,titubantem intellectum confirmabunt.



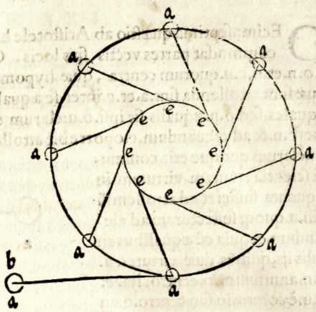
De uera causa. 12.*quæstionis mechanicæ.*

CAP. XVII.

Vera ratio, cur multò longius corpus aliquod graue impellatur funda, quam manu,inde oritur, quòd circunuoluendo fundam, maior impræssio impetus motus fit in corpore graui, quàm fieret manu, quod corpus liberatum deinde cum fuerit à funda, natura duce,iter suũ à puncto,à quo prosilijt, per lineam contiguam giro,quem postremò faciebat,suscipit. Dubitandumq; non est, quin dicta funda maior impetus motus dicto corpori imprimi possit,cũ ex multis circumactibus, maior semper impetus dicto corpori accedat. Manus autem eiusdem corporis motus, dum illud ipsum circunuoluitur (pace Aristotelis dixerim) centrum non est, neque funis est semidiameter. Immo manus quam maximè fieri potest in orbem cietur; qui quidem motus in orbem,vt circumagatur etiam ipsum corpus, cogit, quod quidem corpus, naturali quadam inclinatione, exiguo quodam impetu iam incępto, vellet recta iter peragere,vt in subscripta figura patet, in qua.e.significat manum.a. corpus.a.b.lineam rectam tangentem girum.a.a.a.a.quando corpus liberum remanet. Verum quidem est,imprętium illum impetum,continuò paulatim decrescere vnde statim inclinatio grauitatis eiusdem corporis subingreditur, quæ sese miscens cum impręssione facta per vim,non permittit,vt linea.a.b.longo tempore recta per

maneat,

maneat, sed citò fiat curua, cum dictum corpus.a. duabus virtutibus moueatur, quarum vna est, violentia impræssa, & alia natura, contra opinionem Tartaleæ, qui negat corpus aliquod motibus violento & naturali simul & semel moueri posse. Neq; est silētio prætereūdus hac in re q̄dā notatu dign⁹ effectus, qui eiusmodi est, ꝙ quantò magis crescit impetus in corpore.a.causatus ab augumento velocitatis giri ipsius.e.tātò magis oportet, vt sentiat se trahi manus à dicto corpore a.mediante fune, quia quantò maior impetus motus ipsi.a.est impressus, tantò magis dictum corpus. a. ad rectum iter peragendum inclinatur, vnde vt recta incedat, tantò maiore quoque vi trahit.



De decimatertia quæstione.

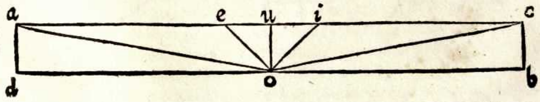
CAP. XVIII.

DEcimatertia quæstio ad vectem omnino est referenda. Imaginari debemus axem cylindrici iugi, hypomochlion esse. Quod restat, illud ipsum totum dependet à.4.quintoq; cap.huius tractatus. Vna tamen differentia inter hanc machinam, vectemq; reperitur, quæ est, ꝙ iugum aliquam resistentiam pro coniunctione calcata in loco, in quo voluitur, magis quàm hypomochlion vecti efficiat.

De decimaquarta quæstione.

CAP. XIX.

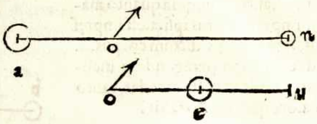
RAtiones etiam decimæquartæ quæstionis dependent ab ijs, quæ sunt vectis, vt exempli gratia sit lignum.a.b.c.d. frangendum in medio, annitendo genibus in punctum.o. clarissimè tunc videbimus, ꝙ tenentes manus longè à medio, in locis a. et. c. facilius minoriq; cum labore illum frangemus, quàm si easdem vicinas medio eiusdem ligni in locis.e.et.i.poneremus. Cuius rei rationes eædē sunt cū ijs, quæ primis huius tractatus capitibus propositæ fuerunt. Imaginemur lineas rectas ductas à puncto.o.ad loca.a.e.i.et.c.hinc manifestè perspiciemus eorum, quæ iam diximus ratione, ꝙ loca.e.et.i.mediantibus duabus lineis.e.o.et.i.o.magis annitentur.o. centro, quàm loco.a.et.c.duarū linearū.a.o.et.c.o.beneficio; vnde vim quoq; maiorem habebūt in.a.et.c. quàm in e.et.i.



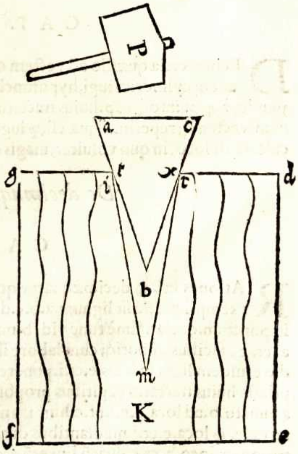
De uera ratione. 17. quæstionis.

CAP. XX.

DEcimaseptima quæstio ab Aristotele haud benè percepta fuit, quia is non accommodat partes vectis suis locis. Quamobrem imaginemur duos vectes. a.o.n.et.o.e.u.quorum centra, quæ hypomochlia appellantur sint. o. & pondera, quæ sunt attollenda sint.a.et.e.inter se æqualia,& distantię sint.a.o.et.e.o.sibi inuicē æquales,sed.o.n.æqualis sit ipsi.o.u:clarum erit,ꝙ ad eleuandum.a.oportebit deprimere.n.& ad eleuandum.e.oportebit attollere.u.Et quia omnia supponuntur æqualia,clarum quoque erit,communi scientia, tantam virtutem in n.quanta sufficiet ad attollendū a.in.u.quoq; suffecturam ad eleuandum.e. quia cū æqualibus angulis ijs,quibus duæ virtutes. a. et.n.anniţuntur.o.centro, ita. e. et.u.è contrario suo centro.o.annituntur. & omnes rationes pro vecte.a.o.n.quarto quintoq; huius tractatus capitibus citatæ,vecti.o.e. u. vt satis superq; dixi in dicto capit. 5.conuenire possunt.



Nunc sit aliqua pars ligni cindenda secundum venulas suas.d.e.f.g. & sit cuneus a.b.c.qui vi mallei. P. vsque ad.t.x.penetrarit. Hinc clarum erit, quòd apertura i.m.r.ligni, postquam infigitur cuneus secundum venas,longior erit parte.x.b.t.cunei, quæ ingressa est. Oportet nunc imaginari duos vectes similes supradictæ. u. e. o. in hunc modum, vt puncta.i.r. ligni sint loco.u.extremi ipsi⁹ vectis, et.t.x.loco virtutis applicatæ ipsi. u. & resistentia circa punctum.m.loco ponderis.e.vectis.o.e. u. dicti, & pars. K. quasi immediata post.m. versus extremitatem.f.e.ligni,sit loco hypomochlij.o. Hinc fiet vt quanto longiores erunt lineæ.i.m.K.et.r.m.K.tantò quoque facilius virtutes.t.x.impellent.i.r.

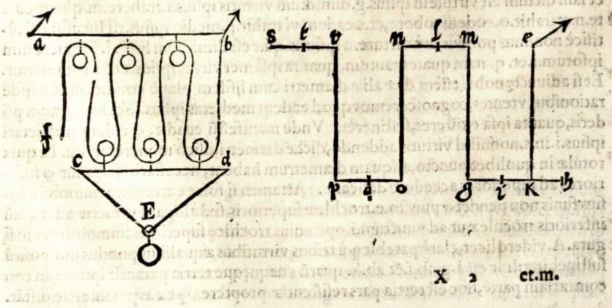


De

De uera & intrinſeca cauſa trochlearum.

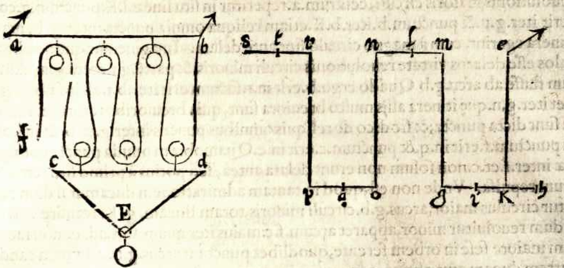
CAP. XXI.

PRo intelligenda vera,& intrinſeca ratione, vnde fiat ut multitudo rotularum in trochleis cauſa ſit,ut exigua vis ſurſum moueat,aut attollat pódera magna. Imaginemur duas hîc ſubſcriptas trochlæas explicatas tranſuerſaliter in hunc modum, ideſt ſit paruũ tignũ.a.b.fixum & parallelũ orizonti. cui ſint rotulæ appenſe ab inferiori parte ad ſuperiorem huicq; è regione oppoſit⁹ ſit aliud tignũ.c.d.quod moueri poſſit ab imo ad ſumum,ſuper quod totidem ſint rotulæ aut radij, cũ annexa poſtea fuerit funis puncto. b. fixo, eam faciendo pertranſire per rotulas tam à parte ſuperiore,quam ab inferiore; & appenſum deinde cum erit paruo illi tigno. c. d. mobili pondus.E.ducendo poſtmodum extremum.f.funis tranſeuntis per rotulas,idem plane fiet quod à trochleis ſimul unitis fieri ſolet. Cuius quidem effectus ratio ſub noſtram cognitionem cadet facilius in huiuſmodi figura. Imaginemur ſeparatim ſtateram.g.h.cuius cẽtrum ſit.K.ita ſitum,ut brachium.g.k.ſit duplum ad brachium.K.h.ſupponendo igitur in puncto.g.pondus,aut virtutem mouentem unius libræ, & in h.duarum librarum,abſq; dubio hæ duæ uirtutes in huiuſmodi diſtantijs à centro ęquales inuicẽ erũt,ob rationes prioribus capitibus iam allatas,& ſtatera orizontalis manebit. Vnde clarum erit,ꝙ quæuis etiam exigua virtus adiuncta ipſi. g. mouebit ſtateram extra orizontalem ſitum. Nunc ſi puncto.i.ex æquo medio inter.g. et. K. applicata erit virtus ipſius.h.non amplius conſiderato brachio.K.h.inclinante uirtute ipſius.i.eandem partem verſus, in quam inclinabat,quando erat in.h.ſed uirtus ipſius.g.inclinet contrario modo, diuerſoq; ab eo,quo inclinabat prius;clarum quoq; erit,communi conceptu,& ob ea,quæ cap.5.huius tractatus ſunt dicta. g. h. ſemper in eodem ſitu abſque motu manſuram,hancq; ſtateram appellabimus mobilem, & primam. Imaginemur nunc à puncto.e.fixo deſcendere funem.e.K. quæ fulciat punctum.K.extremum diametri.g.K.quam intelligo pro diametro vnius ex rotulis inferioribus trochleæ; & ſit.n.l.m.diameter vnius ex rotulis ſuperioribus alterius parui tigni defixi à parte inclinationis ipſius.g.& parallela diametro. g. K. cuius diametri centrum fixum ſit.l.& ſit coniunctum.g.punctum, à fune cum puncto.m.quæ tã perpendicularis ſit primo diametro.g.i.K.quàm ſecundo.n.m.ideſt ita vt anguli.n.m.g.

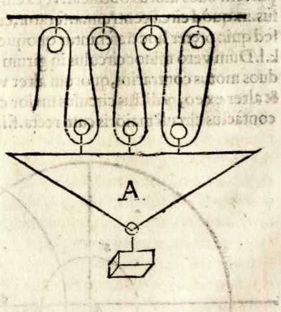


et.m.g.k.ſint recti. Imaginemur quoq; virtutem ipſius.g.applicatam eſſe extremo. n.cum inclinatione tamen contraria,ideſt ad inferiorem partem,quæ quidem virtus communi quodam conceptu eandem poſſidebit vim ſuſtentandi immobilem diametrum.g.i.k. quam habebat, qñ erat in.g.cum inclinatione ad ſuperiorem partem, & ſic etiam diameter.n.l.m.non magis ab una,quàm ab alia parte declinabit, quia cum quædam virtus in.n.reperiatur æqualis medietati uirtutis ipſius.i. quæ uirtus ipſius.i.uim habet deprimendi ipſum.g.ideſt.m.pro dimidia ſui ipſius parte, ſequitur. n.m.debere immobilem permanere. Nunc ſi alia diameter rotulæ mobilis erit deſumpta,quæ ſit.p.q.o.cuius centrum ſit.q.in ſitu parallelo ipſi.n.l.m.& ſic collocata, vt coniungendo.o.cum.n.anguli.m.n.o.et.n.o.p.ſint recti: ſi imaginati fuerimus trãſlatum eſſe pondus ipſi⁹.n.in.o.cũ eadẽ inclinatione ad depræſſiorem partem,illud ipſum,ac ſi eſſet in.n. communi conceptu,ſine alicuius diametri mutatione præſtabit. Et ſi centrum.q.fixum eſſet,& extremo.p.appoſitum fuiſſet pondus ipſius.o.cum inclinatione ad ſuperiorem partem,idem etiam planè pręſtaret,etiam ſi nullum ullius diametri ſitum,communi ſcientia,mutaret, cum extremum. m. deorſum ſit ductum à.g.uirtute dimidiæ partis ipſius.i.& ab alia huic ſimili. m. quoque deorſum ſit tractum ab . o : quod quidem.o.deorſum eſt alteratum,ob inclinationem ad ſuperius à uirtute poſita in.p.ſupponendo centrum.q.fixum. Sed ſi loco centri fixi,imaginabimur in.q.pondus aliquod æquale ipſi. i. quod duplum erit in uirtute ad eam, quæ eſt ipſius.p.& ipſius quoque. g: ſequetur etiã eadem immobilitas horum trium diametrorum. Quia cum ſit huiuſmodi pondus ſeu virtus in.q. cum inclinatione contraria virtuti in.p.quæ æquipollet dimidiæ parti ipſius.q.& ſic ei quæ eſt ipſius.o. ſimiliter quia.o.tractum eſt ſupra ab.n.virtute ipſius.g.quod.m.deorſum trudit; idcirco quanta erit vis quam habebit virtus in.q.ferendi deorſum diametrum.p. o. tanta quoque virtutes ipſorum.p.et.o.æquales,& æqualiter diſtantes à.q.ipſum ad ſuperiorem partem inclinabunt. Quamobrem nec aſcendet, nec deſcendet, nec locum mutabit. Supponamus nunc quartum diametrum rotulæ.s.t.r. quæ ſit ſecunda rotularum fixarum,parallela ipſi.p.o.& in eo ſitu,quo coniungendo extrema.r.p.anguli o.p.r.et.p.r.s.ſint recti,& imaginemur virtutem ipſius.p.reperiri in.s.cum inclinatione tamen contraria,ideſt deorſum verſus,ex his idẽ quoque planè ſequetur, ideſt ꝙ nulla harũ quatuor diametrorum mouebitur. quia eundem effectũ cũ inclinatione deorſum verſus efficeret dicta virtus in.s.quem in.p.cum inclinatione ſurſum verſus. et iam dictum eſt virtutem ipſius.g.dimidium virtutis ipſius.i.trahere.m.quæ mediãte.n.attrahit.o.eodem robore,et.s.eadem vi trahit.p.medio ipſius.r.Hucuſque ſciẽtificè nouimus pondus,aut virtutem ipſius.s.quæ eſt dimidium ipſi⁹.i. ſuſtinere uim ipſorum.i.et. q. nam quater tantum, quanta ipſamet virtus ipſius.s.eſſe conſpicitur. Et ſi adiunctę nobis eſſent duæ aliæ diametri cum ijſdem planè conditionibus ijſdẽ rationibus vtentes,cognoſceremus quod eadem medietas ipſius.i.ſexies tantum põderis,quanta ipſa exiſteret,ſeſtineret. Vnde manifeſtũ euadit, ꝙ eidem medietati ipſius.i.in.s.nonnihil virtutis addendo,dictæ diametri,illicò mouerẽtur ſitu. Et quia rotulæ in quolibet puncto,aliquam diametrum habent, neceſſariò ſequitur ꝙ inferiores ad ſuperiores accedere debeant. Attamen ſi forte extremum immobile ipſius funis non pendet à puncto.e.trochleæ ſuperioris,ſed alligatum fuerit ad mediũ inferioris trochleæ ut ad punctum.i.ope unius trochleę ſuperioris immobilis vt in figura.A.videre licet,clarè patebit ꝙ à tribus virtutibus æqualibus pondus in.i.poſitũ ſuſtinebitur:hoc eſt à.g.ab.i.& ab.k. quarũ vnaquęque tertia pars erit ipſius.i.in contrariam partẽ,hoc eſt tertia pars reſiſtentiæ.propterea ꝙ ex æquo inter ſe diſtãt.

g. i.et. K: Quà propter augebitur virtus per numeros impares, hoc modo; Nam. g. eſſet tertia pars reſiſtentię, quemadmodum prius media erat. Idem infero de.m.n. o.p.r.et.s.Sed cum oporteat pondus.q.tantum eſſe vt ſufficiāt reſiſtentiæ in. o. et.p. ipſum ſuſtinere, idcirco ipſum pondus.q.ſubſeſquialter erit pōderi in.i.poſiti.Quapropter.s.quinta pars erit ponderum.i.et.q.Deinde ſi adhuc duo diametri vnus inferior,alter verò ſuperior additi fuerint cum pondere æquali.q. ad medium diametri inferioris,tunc pondus.s.erit ſeptima pars trium ponderum.i.q.& tertij additi,ex



ſupradictis rationibus. Et quia virtus ſuſtinens totale pondus trochleæ inferiori appenſum in tot diuiditur partes æquales, quot ſunt diametri orbiculorum trochleæ inferioris,quando extremum immobile funis alligatum fuerit trochleę ſuperiori, vt puta in puncto.e.cum verò alligatum fuerit trochleæ inferiori,virtus primi diametri.g.i.K.trochleæ inferioris ſemper ſeſquialtera erit vnicuique aliorum diametrorū; ideò virtus reſiſtentię alterius extremi mobilis funis,puta.s.ſubmultiplex erit totalis ponderis,eo modo quo diximus, cuius virtus,ſeu grauitas diuiditur ſeu diſtrubuitur diametris inferioris trochleæ vt dictum eſt.

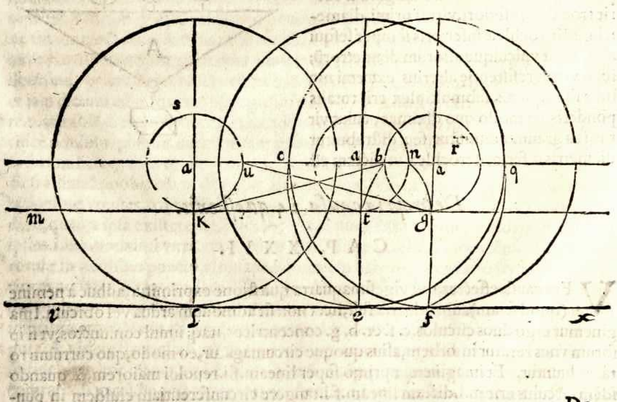


De propria cauſa. 24. quæſtionis.

CAP. XXII.

VEra cauſa effectus,qui vigeſimaquarta quæſtione exprimitur,adhuc à nemine (quod ſciam)animaduerſa fuit,licet non ſit admodum ardua vel obſcura.Imaginemur ergo duos circulos.c. f. et. b. g. concentricos,itaq; ſimul coniunctos,vt ſi ipſorum vnus feratur in orbem,alius quoque circumagatur,eo modo,quo curruum rotæ voluuntur. Et imaginemur primò ſuper lineam.f.i.reuolui maiorem,& quando idem circulus erit in.l.dictam lineam.f.i.tangere circunferentiam eiuſdem in puncto.c.

cto.c.vnde linea.g.m.mediante.K.continget circunferentiam circuli minoris in puncto.b:et.K.g.ex.34.primi Eucli.æqualis erit ipsi.f.l.quia ex.17.tertii, anguli.f. et. g. sunt æquales,vnde ex.28.primi.f.l.et.g.K.sunt parallelæ.& sic erunt.k.l.cum. f.g. ex eadem supradicta.Ratio autem,qua arcus.g.b.transierit lineam.g. K. maiorem ipsa, est, quia dum mouetur,quodlibet punctum ipsius.g.b.virtute reuolutionis ipsius.f.c. omne punctum eiusdem arcus.g.b.vlterius versus.K.quam si moueretur virtute reuolutionis ipsius.g.b.super lineam.g.m.defertur. vt exempli gratia, quando virtute reuolutionis maioris circuli, centrum.a.reperitur in situ lineæ.l.K.punctum.g.confecerit iter.g.u.& punctum.b.iter.b.K.etiam reliqua omnia puncta inter.g. b. magna itinera egerint, cum à magno circulo sint ante delata. Imaginemur quoque hos circulos esse delatos virtute reuolutionis circuli minoris,& partẽ.g.t.rectè.g.m.dimensam fuisse ab arcu.g.b. Quãdo ergo.b.erit in.t.factum erit iter.b.t.ab ipso.b.et.g.faciet iter.g.n.quę itinera alijs multò breuiora sunt, quia breuioribus cruribus reuoluta sunt dicta puncta;& sic dico de reliquis omnibus punctis inter.g.et.b. & in hoc casu punctum.f.erit in.q.& punctum.c.erit in.e.Quamobrem omnia puncta cõtingentiæ inter.f.et.c.non solum non erunt delata anteà, sed potius à primo situ retrorsum erunt repulsa. Vnde non est,quòd in tantam admirationem ducamur si dum reuoluitur circulus maior,arcus.g.b.circuli minoris,totam lineam. g. K. transire videtur, & dum reuoluitur minor,apparet arcum.f.c:maius iter quam ab.f.ad. e. non facere, cum maiore sese in orbem ferente,quodlibet punctum arcus.g.b. ad vnam eandẽq; partem duos motus obtineat.vt exempli gratia punctum.b.non solum mouetur versus.m.quòd circa centrum.a.feratur,cum ipsum etiam centrum moueatur versus.m. sed quia pręter hoc deferantur quoque à circulo maiori versus.m. vsque ad lineam. k.l.Dum verò minor circulus in girum ducitur,habet quodlibet punctum arcus. f.c. duos motus contrarios,quorum alter versus.i. virtute reuolutionis circuli minoris, & alter ex eo, ꝙ dictus circulus maior circa centrum.a. voluatur, vnde omne punctũ contactus circuli maioris cum recta.f.i.retrorsum pellitur versus.x.



De uera causa.30.quæstionis.

CAP. XXIIII.

VEra ratio, cur homo dum sedet (non tamen Turcarum more) si velit sese in pedes erigere, calcaneos retrahit, vt efficiat angulum acutum, cum fęmoribus coxis à parte inferiori, & ventrem inclinat, ad constituendum etiam angulum acutum in superiori parte, ea est; vt totius corporis pondus, ex ęquo, ideſt ab oppositis partibus circundet lineam rectam, quæ transit per locum, in quo conquiescunt pedes versus mundi centrum. ideſt, ut edatur ęquilibrium ponderis ipsius corporis circum lineam illam, quę sub pedibus inseruit pro sparto. Vnde aperiendo, deinde dictos duos angulos circa dictam lineã, absque vlla difficultate erigitur corpus, & absque periculo in alterutram partem cadendi.

De ratione.35.& ultimæ quæstionis.

CAP. XXV.

VEra ratio, quare, quę reperiuntur in vorticibus aquarum, semper versus medium ipsarum vertiginum vniuntur, inde promanat, quod media vertiginum semper depressiora sunt. vnde quòd dicta corpora ad medium accedant, nihil aliud est, quàm ipsa corpora suo pondere grauitateque descendere, figura enim vorticibus est quasi conica, & concaua cum angulo deorsum, & gyro basis sursum. Atque hæc vera est huius effectus causa, & non ea quam Aristoteles ponit, à quo aliarum omnium quæstionum, quas ego omisi rationes sunt benè propositæ.

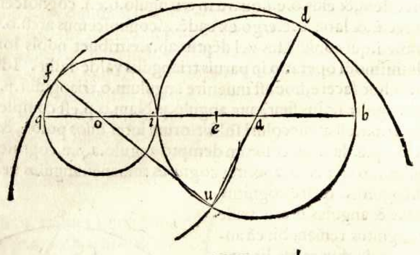
DISPV-

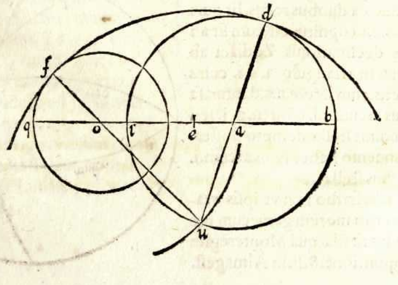
Qualiter circulus deſignari poßit alios duos circulos propoſitos includens.

CLARISS. PETRO PIZZAMANO.

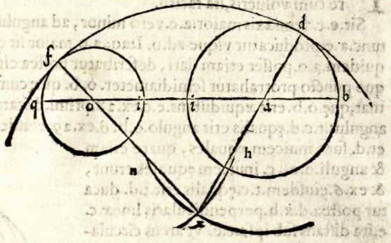
SVperioribus diebus per tuas literas à me quæſiuiſti, vt modum tibi ſcribere vellem, quo circulus deſignari poſſit circunſcribens alios duos propoſitos circulos. Qua in re vt tibi ſatisfaciam quod maximè cupio ita rem accipe.

Propoſiti circuli ſint, aut inter ſe contigui, aut interſecantes vel ſeparati. Eſto primũ contiguos eſſe, qui ſint.d.b.et.f.q.quorũ.d.b.maior ſit et.f.q. minor, eorũ vero centra ſint.a et.o.punctũ autem cõtingentię ſit.i. Nũc ꝓtrahať.b.a.o.q.per cẽtra eorum ab vna circunferentia ad aliam, quę quidem linea tranſibit per punctum. i. ex 11.tertij Eucli.deinde à diametro maiori abſcindatur. i. e. ad æqualitatem minoris ſemidiametri, quo facto ſumatur diſtantia inter.e.et.b.circino mediante factoq; centro.o.ſcindatur, alio circini pede, circunferentia maioris circuli in puncto. u. à quo ſi mente concipiemus duas lineas.u.a.d.et.u.o.f.tranſeuntes per eorum centra.a. et. o. vſque ad circunferentias in punctis.d.et.f. ipſę erũt inuicem ęquales, eo quod.e.i.sũpta fuit æqualis.o.f.et.o.u.æqualis.e.b. quare. u. f.æqualis erit.b.i.ſed.u.d.etiã æqualis.b.i.ergo.u.d.æqualis erit u.f.& circulus, cuius u.d.vel.u.f.erit ſemidiameter, contiguus erit ipſis propoſitis circulis ex conuerſo.11.iam dictæ. Idem dico pro circulis ſe inuicem ſecantibus. Sed



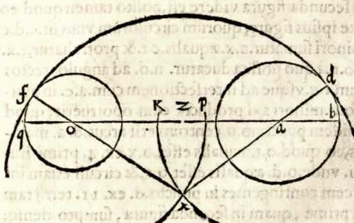


Sed si circuli propositi seiuncti fuerint, sumatur.b.i.diameter maioris, qui fiat semidiameter vnius circuli circa centrum. o.& hic circulus vocetur. h. x. coniungatur deinde semidiameter.o. i. minoris circuli cum semidiametro.a.i. circuli maioris,& ex huiusmodi composita linea, fiat vnus semidiameter.a.x.circuli.x.n.concentrici cum maiori,& à puncto.x.intersectionis horum circulorum (posito quod se inuicem intersecent) ducantur per eorum centra.x.a.et.x.o.vsque ad ipsorum circunferencias in punctis.d.et.f.duę lineæ, vnde habebimus. x. d. æqualem.x. f. eo quod tam in x.d.quam in. x. f. reperiuntur diametri,& semidiametri amborum circulorum,facto denique centro.x. vnius circuli,cuius semidiameter ęqualis sit vni earum.x. d. vel. x. f. solutum erit problema, dicta ratione.



Si verò distantia duorum propositorum circulorum tanta fuerit, quod secundi circuli nequeant se inuicem tangere, vel secare,tunc alia via incedendum erit, quę talis est & generalis. Diuidatur tota.q.b.per æqualia in puncto.z.circa quod signētur duo puncta ab ipso ęquidistantia.K.et.p.distantia vero.a.K.facta sit semidiameter esse vnius circuli. K.x. circa centrum. a . distantia autem . o . p . semidiameter alterius circuli . p . x. circa centrum . o . qui quidem circuli se inuicem secent in puncto. x. à quo cum ductę fuerint. x.a.d.et.x.o.f.per centra dictorum circulorum,ipse erunt inuicē ęquales, eo qđ cum.b.K.æqualis sit.q.p.igitur.x.d.et.q.p.erunt inuicem ęquales, sed.f.x.æqualis est q.p.quare.x.f.æqualis erit.x.d.tunc si.x.centrum fuerit vnius circuli, cuius semidiamer sit vna dictarum, problema solutum erit.

Talis etiam solutio commoda erit ad inueniendum dictum circulum cuiusuis magnitudinis, dato tamen ꝙ eius diameter, maior sit.b.z. cum in nostra potestate sit accipere puncta.K.et.p.proxima vel remota ab ipso.z.ad libitum. Vnde absque vlla diuisione ipsius. q. b. per medium, satis erit signare puncta.K. et. p. duabus distantijs mediantibus. b . K. et. q. p. inuicem æqualibus, & etiam propositis.



In ribus casib, diuisa bq bifariam, quod quidem puncto fiat centrum, circulus descriptus p bq transiens semper datos circulos in punctis bq contingent.

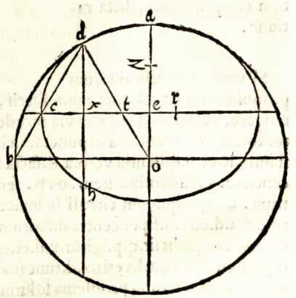
Figuram

Figuram superficialem ellipsi similem, ex datis axibus circino mediante delineari posse.

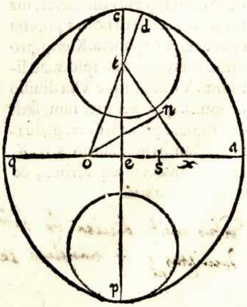
AD EVNDEM.

FIguram superficialem ellipsi similem, ex datis axibus, circino mediante delineare cum volueris, ita facito.

Sit.e.c. semiaxis maior.a.e.verò minor, ad angulum rectum inuicem coniuncti, tunc.a.e.producatur vsque ad.o. Itaq;.a.o.maior sit quam distantia inter.o.et.c.quę quidem.a.o.posset etiam dari, describatur postea circulus.a.d.b.circa centrum.o. à quo puncto protrahatur semidiameter. o. b. quæ cum.a. o. angulum rectum constituat, quę.o.b.erit æquidistans.e.c.ex.28.primi, ducatur postea.b.c.d.et.o.t. d. vnde angulus.t.c.d.ęqualis erit angulo.o.b. d.ex.29.eiusdem.ex quinta autem anguli. b. et.d. sunt inuicem æquales, quare etiam & anguli.d.et. c. inuicem ęquales erunt, & ex.6.eiusdem.t.c.ęqualis erit.t.d. ducatur postea.d.x.h.perpendicularis lineæ.c. e.ita distans sub ipsa.c.e. vt arcus circularis circa.t. delineatus ex semidiametro. t. d. aptus sit eam secare, sumpto postea. r. tam distante ab.e. vt. t. reperitur ab ipso e.et.z.ab.e.vt.o.ab eodem, ducendo postea duos alios arcus magnitudinis priorū circa centra.r. et. z. habebimus propositum.



Sed cum quis voluerit prius arcus minorum circulorum delineare circa maiorem axem, fiant cuiusuis magnitudinis, vt in secunda figura videre est, posito tamen quod eorum diameter, minor sit minore axe ipsius figurę, quorum circulorum vnus sit.c.d.circa.t.eius centrum, deinde in axe minori sumatur.a.x.æqualis.c.t.& protrahatur.t.x.quę per ęqualia diuidatur in puncto.n.à quo postea ducatur. n.o. ad angulos rectos cum.t.x.vsque ad intersectionem cum.a.e. in puncto.o.minori axi producta cum oportuerit, quod quidem punctum.o.centrum erit arcus.d. a. maioris, eo quod.o.t.æqualis esset.o.x.ex. 4. primi Eucli. vnde.o.d. æqualis esset.o.a.& circuli etiam inuicem contingentes in puncto.d. ex. 11. tertij tam in prima, quam in secunda figura, sumpto deniq; puncto.s.tam remoto ab. e. quam. o. reperitur ab eodem, ipsum, centrum erit alterius arcus oppositi, possemus etiam absq; diuisione ipsius, t.x.constituere angulum.x.t.o.æqualē angulo.t.x.o.vnde ex 6.primi haberemus.o.t.æqualem.o.x.



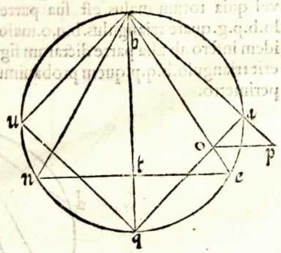
De

negotio cordarum & arcuum possumus geometricè demonstrare quod valde desideras.

Quapropter sit circulus.b.a.e.q.in quo sit triangulũ æquilaterum.b.e.n.& quadratum. b.a. q.u. cuius periferiam probabo longiorem esse periferia trianguli. Sit enim diameter circuli.b.q.qui etiam erit diameter quadrati, vt à te scire potes. Sit etiam punctũ.b.commune tam anguli quadrati quam trianguli. vnde sequitur quod dictus diameter secabit latus.n.e.trianguli ad rectos & per æqualia in.t. Nam cum arcus.b.e.æqualis sit arcui.b.n.ex.27.tertij, remanet vt arcus.q.e.equalis sit arcui. q.n.vnde angulus.q.b.e.æqualis erit angulo.q.b.n.ex. 26. eiusdem. quare ex. 4. primi anguli ad.t.erunt recti,et.n.t.æqualis erit ipsi.t.e. vt diximus.

Deinde.b.e.et.q.a.seinuicem secãt in puncto. o.vt ex se clarum patet, ducatur postea.q.e.vnde habebimus angulum.b.e.q.rectum ex.30.tertij, quare ex. 18.primi.q.o. longior erit ipsa.q.e.et.q.e.longior erit ipsa.e.t. quare.q.o. longior erit ipsa.t.e.

Vt probemus postea.b.a.o.longiorem esse ipsa. b.e. producatur.b.a. ita quod.a.p.æqualis sit ipsi.a.o.ducaturq; o.p et.a.e.cum autem ex iam dicta.30. tertij angulus b a.o.rect⁹ sit, erit angulus.o.a.p.similiter rect⁹ ex.13. primi, vnde ex.5.et.32. eiusdẽ angulus.a.p.o.erit dimidium recti,& similiter, ex ijsdem, angulus.b.q.a.est dimidium recti quare angulus.a.p.o.æqualis erit angulo.a.q.b.sed angulus.a.e.b.æqualis est angulo.a.q.b.ex.20.tertij, ergo angulus.b.p.o.æqualis erit angulo.b,e.a.angulus vero a.b. e.communis est ambobus triangulis.a.b.e.et.o.b.p.quare ex. 32. primi anguli. b.a.e.et.b.o.p. reliqui ex duobus rectis æquales inuicem erunt. Quare ex quarta sexti, et.18. quinti proportio.b.o.ad.b.p.erit, vt b.a.ad.b.e.sed ex.18. primi.b. o. maior est ipsa.b.a.quare ex. 14.quinti.b.p.maior erit ipsa.b.e.sed.b.p.æquatur ipsis.b.a.cum.a.o ex hypotesi, ergo. b. a. cum.a.o.maior erit ipsa.b.e.sed.q.o.maior erat ipsa.t.e.vt superius vidimus, quare.b.a.cum.a.o.et.o.q maior est ipsa.b.e.cum.e.t. hoc est dimidium periferię ipsius quadrati, mai⁹ erit dimidio periferię ipsi⁹ triãguli propositi, quare ex 14 dicta tota periferia dicti trianguli, similiter probarem de omnibus alijs figuris regularibus eodem circulo inscriptis.



CONSIDERATIONES NONNVLLÆ IN Archimedem.

Doctissimo atque Reuerendo Domino Vincentio Mercato.

QVOD tibi aliàs dixi verum est, intellectum scilicet non omninò quiescere circa illas duas Archimedis propositiones, quæ in translatione Tartaleæ sunt sub numeris. 4. et. 5. & in impressione Basileæ sub numeris. 6. et. 7. vbi

tractat

tractat de centris libræ, seu stateræ: Aspice igitur in.4. supradicta, quod cum appensæ fuerint omnes illæ partes ponderum, partibus longitudinis ipsius.l.K.in qua volo vt à punctis.e.et.d.imagineris duas lineas.e.o.et.d.u.inuicem æquales, & ferè perpendiculares ipsi.l. K. hoc est respicientes mundi centrum; imagineris etiam. o.u. quæ sit parallela ipsi.l.k.quæ diuisa sit in puncto.i. supra.g. Hinc nulli dubium erit, cum.g.fuerit centrum totius ponderis appensi ipsi.l.K.quod.i. similiter erit centrum cum directe locatum sit supra.g.hoc est in eadem directionis linea, quod quidem non indiget aliqua demonstratione, cum per se satis pateat. Vnde ex communi conceptu.o.erit centrum ponderis appensi ipsi.l.h. et.u. erit centrum ponderis appensi.ipsi h.K. Scimus igit. i. esse cētrum duorum, hoc est ipsius.l.h. & ipsius.h.k.continuatorum per totam. l. k. Nunc ergo si consideremus.l.k.diuisam esse, hoc est disiunctam in puncto.h.inueniemus nihilominus.i.centrum esse dictorum ponderum, & quod tantum est, ipsam esse continuã, quantum diuisam in dicto puncto.h. neque ex hoc, punctum.i.erit magis vel minus centrum duorum ponderum.l.h.et.h. k.quorum vnum pendet totum ab.o.aliud verò totum ab. u. & hoc modo in longitudine. o.u.diuisa vt dictum est, habebimus propositum.

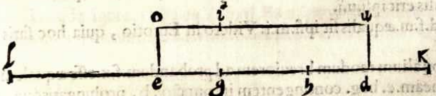
Reliquam propositionem tibi relinquo.

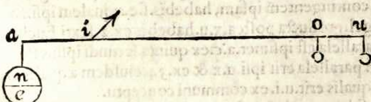
Illa verò propositio, quam tibi dixi Archimedem tacuisse in huiusmodi materia est, quod si duo pondera æquilibrant ab extremis alicuius stateræ, in certis præfixis distantijs à centro. Tunc dico si eorum vno manente alterum moueatur remotius ab ipso centro quod illud descendet, & si vicinius ipsi centro appensum fuerit ascendet. Hæc enim propositio quotidie omnibus in locis videtur, ipsam verò puto Archimedem prætermisisse ob facilitatem, cum ab antedicta ferè dependeat.

Sit exempli gratia statera.a.u.cuius centrum sit.i.& pondera.u. a. appensa, seinuicem habeant vt.i.u.et.i.a.se inuicem habent. Nunc dico quod si pondus ipsius.u. positum fuerit vicinius centro vt puta in.o. inmoto existente pondere, a. quod brachium.i.o.u.ascendet, & è conuerso, si remotius positum fuerit, descendet.

Ponat̃ ergo vt dictũ est in.o.vicinius cētro, quapropter brachium. i. o. breui⁹ erit brachio.i.u.vnde minor proportio erit ipsius.i.o.ad.i.a.quàm.i.u.ad eundem.a.i. & consequenter quam ponderis ipsius.a. (quod sit.n.e.) ad pondus ipsius.u.Quare si ex pondere.n.e.dempta fuerit.e.pars eius, ita quod reliqua pars.n.se habeat ad pondus o. vt se habet.i.o.ad.i.a. tunc statera non mouebitur; addita verò parte. e. ex communi conceptu, a.descendet vnde.o.ascenderet conuersum verò ex similibus rationibus per te concludes.

Inco





In eo quod à me petis, mittendo te ad Eutotium, tibi non satisfacerem, cum Eutotius citet sextum librum Pergei, quem nunquam vidimus, supponatq; ea, quæ nec ipse nec alius vnquam quod scimus probauit.

Desideras enim demonstrationem illius quod Archimedes dicit inter primam, & secundam propositionem secundi libri, vbi tractat de centris grauium, propterea quod illud supponit pro manifesto.

Sit enim figura hic subscripta, ferè similis parabolæ positæ in. 2. propositione dicti libri, vt in impressione Basileensi habetur, sintq; diuisæ duæ.a.b.et.b.c.per æqualia à punctis.x.et.u.protractisq;.f.x.et.u.i.ad.b.d.quæ inuicem etiam erunt parallelę ex.30.primi Eucli.vnde ipsæ etiam, diametri erunt ipsarum portionum: vt ex eo colligere est, quod in.49.primi lib. Pergei probatur. Imaginando postea ad puncta.b. f.et. i.tres contingentes, manifestum erit punctum.b.illud esse quod terminat altitudinem huiusmodi portionis, et. f.et.i. terminantia altitudines partialium, ex.5. secundi ipsius Pergei, eo quod dictæ contingentes paralellæ erunt ipsis basibus, vnde trianguli inscripti, easdem habebunt altitudines, quas portiones ipsæ, quod erit ex mente Archimedis. Et sic deinceps poteris multiplicare angulos figuræ rectilineæ in parabola, quæ designata erit vt desiderat Archimedes, qui quidem dicit, quod protractæ cum fuerint aliæ deinceps post.f.i. ipsæ inuicem ęquidistantes erũt, diuisęque peræqualia ab.d.b. quod quãuis verũ sit, tñ ab Eutotio non satis demõstratũ est, cum supponat.a.f. b.æqualem esse ipsi.b.i.c.probare volens eius diametros æquales esse absque aliqua citata ratione, quæ quidem ratio esset conuersum.4. propositionis libri de conoidalibus. Sed oporteret nos etiã videre.6.librum ipsius Pergei, & propterea tibi non satisfacerem.

Esto igitur, ut inuenta sit linea. K. cuius productum in.u.i.æquale sit quadrato ipsius.u.c.inuenta etiam sit linea.h. cuius productum cum.f. x.æquale sit quadrato ipsius.a.x.vnde ex conuerso.49. primi ipsius Pergei, proportio ipsius.K.ad.b.c.erit ut ipsius.b.c.ad.b.d.& ipsius.h.ad.a.b.vt ipsius.a.b.ad.b.d. Erit igitur ex.16.sexti Eucl. quadratum. b.c. æquale producto ipsius.K.in.b.d.& quadratum.a.b. æquale producto ipsius.h.in.b.d.& ex prima sexti, ita erit ipsius. K.ad.h.vt producti quod fit ex.K. in.b.d.ad productum ipsius.h. in.b.d.hoc est vt quadrati ipsius.b. c.ad quadratum ipsius.b.a. ex. 16.et.11.quinti, hoc est vt quadrati ipsius.u.c.ad quadratum ipsius.a.x. hoc est ut productum ipsius.k.in.u.i.ad productnm ipsius.h.in.x.f. Nunc si ipsius.k. ad.h.est vt producti ipsius.K.in.u.i.ad productum ipsius.h.in.f.x.ergo ex. 24. sexti, & communi conceptu, proportio ipsius.k.ad.h.composita erit ex ea quæ ipsius. u. i. ad.f.x.& ex ea quæ ipsius.k.ad.h. Cum ergo dempta fuerit proportio ipsius. k.ad.h. (vt simplex) à proportione ipsius.k.ad.h. (vt composita) reliquum nihil erit. Quare.f. x.æqualis erit ipsi.u.i.

Sed quod.f.m.æqualis sit ipsi.m.i. Videto in Eutotio, quia hoc satis sui natura facile est.

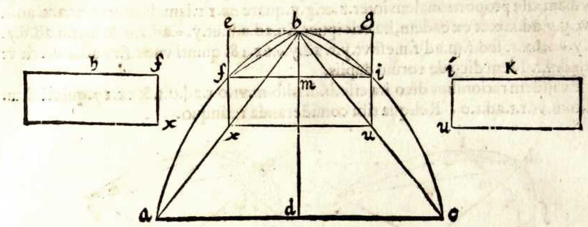
Sed accipe alium modum breuiorem ad probandum.f.x.esse æqualem ipsi.u.i.

Finge lineam.e. b. g. contingentem in puncto. b. prolungatisque diametris f. x.et.u.i.vsque ad contingentem ipsam, habebis.f.e.æqualem ipsi.f.x et.g.i.ipsi. u. i. Ex.35.primi Pergei, producta postea.x.u.habebis ex.2.sexti Eucli. x. u. parallelam ipsi.a.c.sed.e.g.parallela est ipsimet.a.c.ex quinta secundi ipsius Pergei, quare ex.30 primi Euclid.e. g. parallela erit ipsi.u.x.& ex.34.eiusdem æqualis erit.e.x.ipsi. u. g. vnde.f.x. etiam æqualis erit.u.i. ex communi conceptu.

Sed ne quid desideres probabo.f.m.æqualem esse ipsi.m.i. Iam igitur scis quod

cum

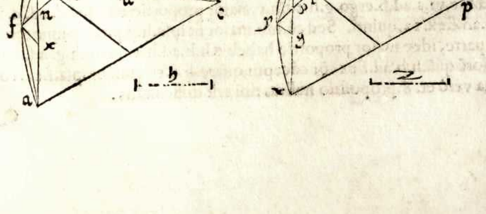
cum ſit.f.x.æqualis ipſi.u.i. vt tibi probaui, & inuicem parallelæ ideo.f. i. parallela erit ipſi.x.u. ex.33. primi Euclidis. Vnde ex.30. eiuſdem, parallela erit etiam ipſi.a. c.ſed cum.x.u. diuiſa ſit ab. d.b. per æqualia, eo quod diuidit.a.c.eodem modo, quę ipſi parallela eſt ex. 2.ſexti. Reliqua tibi conſideranda relinquo. cum verò ambæ. f. x.et.u.i. parallelæ ſint ipſi.b.d.ſequitur quod cum ex.34.primi vnaquæq;.f.m.et. m. i. æqualis ſit medietati ipſius.x.u. erunt inuicem æquales.



Minime dubitabam tibi non ſatisfacere Eutocium in. 3. propoſitione ſecundi lib.de centris Grauium Archimedis, cum citet.6. librum de elementis conicis, adde quod ſi aliud in ipſo.6. libro ab eo citato non eſſet magis ad propoſitum, quàm ea quæ ab ipſo citata ſunt, nihilominus adhuc irreſolutus maneres.

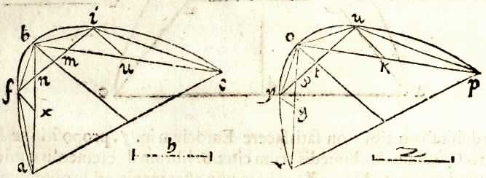
Conſidera igitur eandem ipſam figuram præcedentem; pro alia verò parabola ſimili dictæ, accipe ſecundam figuram ipſius tertiæ dictæ propoſitionis. Deinde imaginabis duo latera.o.x.et.o.p.diuiſa eſſe per æqualia in punctis.g. et. K. protractisq; diametris.g.y.et.K.u.quæ, vt in præcedenti probaui, ſunt inuicem æquales, ſcire debes quod ſimiles parabolæ inuicem aliæ non poſſunt eſſe, niſi eæ quæ diametros proportionales ſuis baſibus habeant, ſimiliterq; poſitæ, hoc eſt, ut proportio ipſius b.d.ad.a.c. ſit eadem quæ ipſius.o.r.ad.x.p.& quod anguli ad.r.ſint æquales angulis circa.d. Notentur ergo primum puncta communia ipſius.o.g.cum.y.t.& ipſius.b.x cum.f.m.characteribus.∗.et.n. Nunc igitur ſcimus.f.m. æqualem eſſe.m.i.totamq;.f. i.parallelam eſſe ipſi.a.c. Idem dico de.y.t.u.trianguliq;.x.f.n.et.g.y.∗. eſſe ſimiles triangulis.n.m.b.et.∗.t.o.quod ita probatur, nam ex.15.primi Euclid. anguli ad. n. ſunt inuicem æquales, ex. 29.verò eiuſdem anguli.f.x. n.et.n.b.m. ſimiliter æquales ita etiam.n.f.x.et.n.m.b.

Idem dico in ſecunda figura, vnde ex.4.ſexti Eucli.proportio.n.f.ad.m.n.erit eadem quę.f.x.ad.b.m.& ipſius.n.f.ad.x.f.vt.n.m.ad.m.b.ex. 16.quinti. Quare ex.11. eiuſdem



eiuſdem erit vt.a.d.ad.d.b. Idem etiam dico in ſecunda parabola, ſed ipſius.x.o.ad o.r. eſt vt.a.b. ad. b. d. ex. 6.ſexti Eucli. vnde ex.11.quinti.n.f.ad.f.x. erit vt. ω. y. ad.y.g. Sed in precedenti iam tibi dixi.a.b.mediam proportionalem eſſe inter. h. et.b.d. Sit nunc.z. pro ſecunda parabola, ita ut.h.eſt pro prima,vnde.o.x.erit media proportionalis inter.z.et.o.r.& ex.11.quinti ita erit.h.ad.a.b.vt.z.ad.x.o.& ex. 22. h.ad.a.x.ut z.ad.x.g.& quia ex.16. ſexti. a.x.media proportionalis eſt inter.h.et.f.x. cum ſupponatur productum.h.in.f.x.æquale eſſe quadrato. a. x.Idem dico.x.g. mediam eſſe proportionalem inter.z.et.g.y.quare ex.11.iam dicta,ita erit.a.x. ad.f.x.vt.y.g.ad.x.o.& ex eadem,ita erit ipſius.f.n.ad.a.b.ut.y.ω.ad.x.o.& ſic.f.n.ad.d.a. vt.y.ω.ad.x.r. ſed.f.m.ad f.n.eſt vt.y.t. ad.y.ω.ex.18. quinti vnde.f.m.ad.a.d.erit vt y.t.ad.x.r. Idem dico de eorum duplis.

Ex ijſdem rationibus dico ita eſſe.b.d.ad.b.m.vt.o r.ad.o.t.& ex.17.quinti.d.m. ad.b.m.vt.r.t.ad.t.o. Reliqua tibi conſideranda relinquo.



In reliquis verò propoſitionibus illius lib. nullo pacto poteris dubitare: Verum ne in. 4. aliquid tibi noui exurgat, te ſcire volo corollarium. 20. in libr. de quadratura parabolę docere poſſibile eſſe inſcriptionem rectilineæ,ea tamen conditione quã dicit Archimedes.

In quinta poſtea animaduertendum eſt,quod prima pars,probat tantummodo de centro trianguli,et.2.pars probat de centro pentagoni, à te ipſo deinde potes probare de centro nonanguli:& ſic de cæteris: eo quod cum probatum fuerit de centro figuræ in medio locatæ ſi conſtitutæ poſtea fuerint ſimiles figuræ in portionibus lateralibus habebitur propoſitum in infinitum.

Idem intelligendum eſt in.3. propoſitione quamuis exemplum vlterius non extendatur quam ad pentagonos.

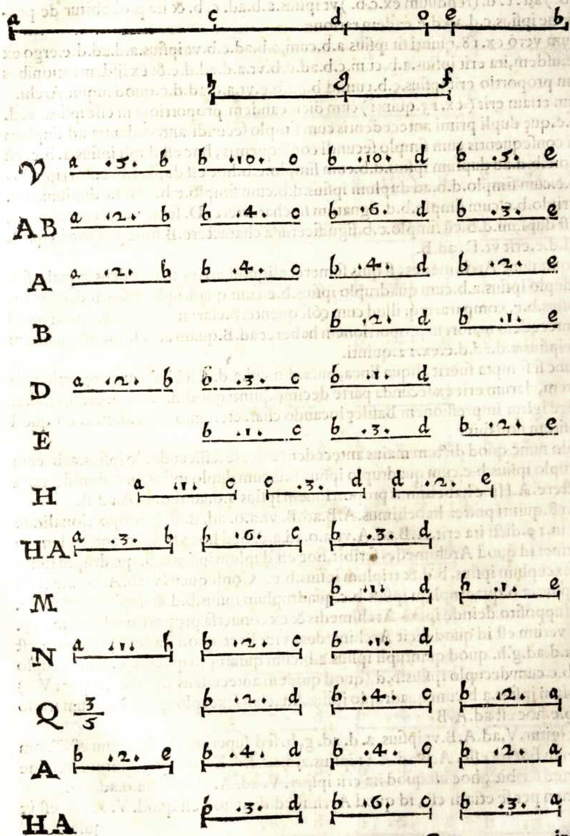
Sexta verò ꝓpoſitio tibi facilis erit,quæ nihilominus pōt demōſtrari hoc mō ſcilicet. Sint.4.quãtitates.a.b.c.d. ipſius Archimedis ſupponédo.a.pro figura rectilinea inſcripta in parabola,et.b.pro reſiduo ipſius parabolę et.c.pro triangulo.a.b.c.in medio ipſius parabolę et.d.pro triangulo. r. Nunc cum. a. maior ſit.c.prout totum maius eſt ſua parte, ideo ex.8.quinti maior proportio habebit.a.ad.b. quam. c. ad. b. Cum autem.b.minor ſit.d.ex ſuppoſito,ideo ex eadem dicta,maior proportio habebit.a.ad.b.quam.c.ad.d.cum verò centrum cuiuſuis figuræ plenæ neceſſariò ſit intra ipſam figuram, idcirco centrum reſidui ipſius parabolę intra ipſam reperietur. quod ita clarũ ꝑ ſe eſt,queẽadmodũ quoduis aliud axioma,& quia dictũ centrũ ex.8.primi de centris, neceſſariò eſt in linea.b.h.inter.b.et.h. Sit igitur.g.vnde ex eadem.8.ita erit.g.h.ad.h.e.vt.a.ad.b.ergo.g.h.ad.h.e.maior proportio erit quã.c. ad. d. hoc eſt quam.b.h.ad.f.ex.12.quinti. Sed cũ.h.b.maior ſit ipſa.h.g.prout omne totum maius eſt ſua parte, ideo maior proportio habebit.h.b.ad.h.e.quam.h.g. ad.h. e. vnde multo maioré quã.h.b.ad.f.ex cōi cōceptu,quare.h.e.erit minor ipſa.f.ex.10.quĩti.

Septima verò et. 8.propoſitio nullius tibi erit difficultatis.

Quam-

Quamuis Eutotius scribat super duas vltimas lib. secundi de centris grauiũ, nihil miror ipsum tibi non satisfacere. Accipe igitur quod ego nunc tibi mitto.

Archimedes eo in loco primũ supponit in penultima dicti libri quatuor lineas proportionales.a.b: c.b: d.b: et. e.b: supponit etiam quod proportio quæ est ipsius. e. b.ad.e.a.eadẽ sit quæ ipsius.f. g.ad tres quintas ipsius.a.d.& quod proportio com positi dupli ipsius.a.b.cum quadruplo ipsius.b.c.cum sexcuplo ipsius.b.d. cum triplo



ipſius.b.e.ad compoſitum quintupli ipſius. a.b.cum decuplo ipſius.c.b.cum decuplo ipſius.b.d.cum quintuplo ipſius. b.e.eadem ſit quæ ipſius.g.h.ad.a.d. & vult probare.f.h.eſſe duas quintas ipſius.a.b.

Cum autem dicit proportionem ipſius.a.c.ad.c.d.& ipſius.c.d.ad.d.e.eſſe vt ipſius a.b.ad.b.c.& cętera, verum dicit ex.19.quinti Eucli.eo quod cum ex hypotheſi ſit ipſius,a.b totalis ad.c.b.totalem vt ipſius.c.b. partialis (ſumptæ vt pars abſciſa ab.a. b.pro nunc) ad.d.b. partialem (abſciſam ab.c.b.) erit ex.19.dicta ipſius.a.c.(reſidui ex.a.b.) ad.c. d. (reſiduum ex.c.b.) vt ipſius.a.b.ad. c. b. & ita probabitur de proportione ipſius.c.d.ad.d.e.eadem ratione.

Cum verò ex.18.quinti ſit ipſius.a.b.cum.c.b.ad.c.b.vt ipſius.a.d.ad.d.e.ergo ex 22. eiuſdem,ita erit ipſius.a.b.cum.c.b.ad.d.b.vt.a.d.ad.d.e.& ex ijſdem rationibus eadem proportio erit ipſius.c.b.cum.d b.ad.b.e.vt.a.d.ad.d.e.quod inquit Archi. Verum etiam erit (ex.13.quinti) cum dicit eandem proportionem eſſe ipſius. a. d. ad. d.e.quę dupli primi antecedentis cum ſimplo ſecundi antecedentis ad duplum primi conſequentis cum ſimplo ſecundi conſequentis, hoc eſt dupli ipſius.a. b.c. cū ſimplo.c.b.d.ad duplum ipſius.d.b.cum ſimplo.e.b.hoc eſt dupli.a.b. cum triplo ipſius.b.c.cum ſimplo.d.b.ad duplum ipſius.d.b.cum ſimplo.e.b. Nunc duplum. a.b. cum triplo.b.c.cum ſimplo.b.d.ſignatum ſit charactere. D. ſuum verò conſequens,
M hoc eſt duplum.d.b.cū ſimplo.e.b.ſignificetur à charactere.B.hinc proportio ipſius a.d.ad.d.e.erit vt. D. ad.B.

Inquit nunc Archimedes, ſi quis ſumeret aliquod maius antecedens æquale ſcilicet duplo ipſius.a.b.cum quadruplo ipſius.b.e.cum quadruplo ipſius.b.d. cum duplo ipſius.b.e. compararetq; illud cum cōſequente.B.clarum eſſet ex.8.quinti quod tale antecedens maiorem proportionem haberet ad.B.quam ad.D.hoc eſt maiorem quàm ipſius.a.d.ad.d.e.ex.12.quinti.

Nunc ſi ſumpta fuerit aliqua linea, puta.d.o.cui.a.d.dictā habeat proportionem maiorem, clarum erit ex ſecunda parte decimę quinti quod.d.o.minor erit ipſa.d.e. Corrige igitur impreſſionem Baſileę locando characterem.o.inter.d.et. e. eo quod ibi poſitum non fuit.

Volo nunc quod dictum maius antecedens æquale ſcilicet duplo ipſius.a. b. cum quadruplo ipſius.b c.cum quadruplo ipſius.b.d.cum duplo ipſius. b. c. ſignificetur à
β charactere.A.Hinc habebimus proportionem ipſius a.d.ad.d.o.ut.A.ad.B.

Ex.18.quinti poſtea habebimus.A.B.ad.B. vt.a.o. ad. d. o. & proportionalitate
T euerſa in.19.dicti ita erit.A.B.ad.A.vt.a.o.ad.a.d. Sed hoc vltimum antecedens in ſe continet id quod Archimedes ſcribit,hoc eſt duplum ipſius.a.b. quadruplū ipſius b. c. ſexcuplum ipſius. b.d.& triplum ipſius.b.e. Conſequens verò.A. continet duplum ipſius.a.b.quadruplum ipſius.b.c.quadruplum ipſius.b.d.& duplum ipſius.b.e.

Ex ſuppoſito deinde ipſius Archimedis & ex conuerſa proportionalitate in. 19. dicta, verum eſt id quod dicit Archimedes, videlicet quod eadem proportio eſt ipſius.a.d.ad.g.h. quod quintupli ipſius.a.b.cum quintuplo ipſius. b.e.cum decuplo ipſius.b.c.cum decuplo ipſius.b.d.(quod quidem antecedens ſignificetur per. V.) ad duplum ipſius.a b.cum quadruplo ipſius.b.c.cum ſexcuplo ipſius.b.d. cum triplo ipſius.b.e.hoc eſt ad.A B

Erit igitur.V.ad.A.B.vt ipſius. a. d. ad. g. h. ſed ſuperius vbi ſignatum eſt.T.iam probatum fuit ita eſſe. A.B ad.A.vt ipſius.a.o.ad.a.d. Ergò ex.23.quinti Archimedes verum ſcribit, hoc eſt quod ita erit ipſius. V. ad. A. vt ipſius.a.o.ad.g.h.

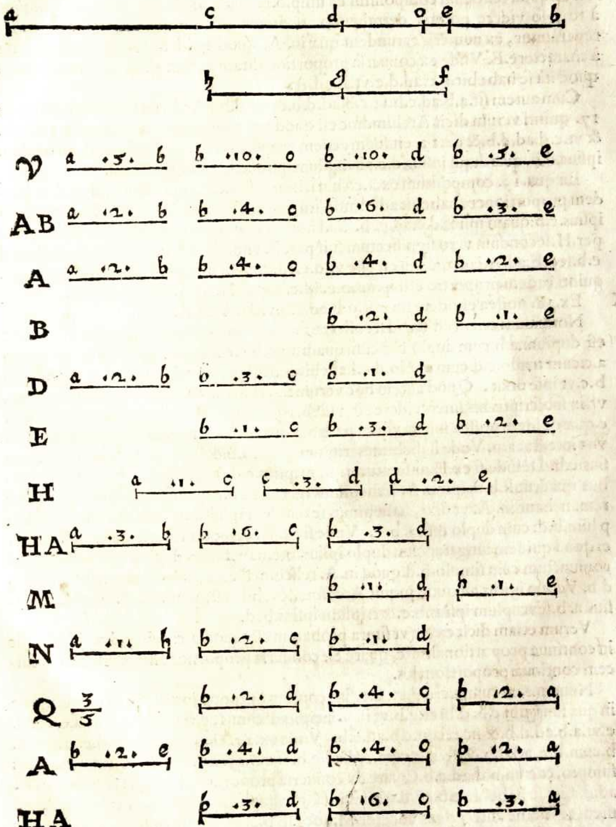
ω Clarum per ſe etiam eſt, id quod Archimed.dicit hoc eſt quod. V. ad. A. eſt vt

quinque

quinque ad duo, cum quodlibet ingredientium in composito. V. ad quodlibet ingredientium in composito. A. sit vt quinque ad duo. Quare ex. 13. quinti verum dicit. Vnde.a.o.ad.g.h.erit vt quinq; ad duo ex. 11. eiusdē vt inquit Archimedes.

Corrige impressionem vbi scriptum est, rursus quoniam.o.a.quia oportet dicere Rursus quoniam.o.d.

Archimedes igitur verum dicit, quod ipsius.o.d.ad.d.a.est vt ipsius.B.ad. A. ex



conuerſa proportionalitate in. 19. quinti,cum.a.d.ad. d. o. iam probatum fuit (vbi B.) ita eſſe ut. A.ad.B.

Sed in principio huius ſpeculationis probatum iam fuit ita eſſe ipſius. d.a.ad.d.e. vt ipſius. D.ad.B.vbi notatum eſt. M. quare ex.23.quinti, Archimedes verum dicit, quod.d.o.ad.d.e.erit vt.D.ad.A.

Sed cum.d.o.ad.d.e.ſe habeat ut.D.ad.A.erit ex conuerſa proportionalitate iam λ dicta.d.e.ad.d.o. vt.A.ad.D.per euerſam vero erit. d.e.ad.a.o.vt. A. ad ſuum reſiduum.quod reſiduum componitur ex ſimplo.b.c.cum triplo.b.cum duplo.b.o.quod à te ipſo videre poteris detrahendo numeros ipſarum quantitatum quæ in. D. reperiuntur, ex numeris earundem,quæ in. A. quod quidem reſiduum ſignificetur à charactere.E. Vnde ex conuerſa proportionalitate verum dicit Archime. hoc eſt quod ita ſe habebit.o.e.ad.d.e.vt.E.ad.A.

Cum autem ſit.a.b.ad.c.b.vt.c.b.ad.d.b.& ita.d.b.ad.e.b. ex ſuppoſito, ideo ex 17. quinti verum dicit Archim.hoc eſt quod ita erit ipſius.d.e.ad.e.b.vt.a.c.ad.c.b. & vt.c.d.ad.d.b.& ex.13. eiuſdem eadem proportio erit tripli ipſius.c.d. ad triplum ipſius.d.b. quæ dupli ipſius.d.e.ad duplum ipſius.e.b.vt inquit Archi.

Ex qua.13. compoſitum ex.a.c.cum triplo ipſius.c.d.cum duplo ipſius. d. e. eandem proportionem habebit ad compoſitũ ipſius.c.b.cum triplo ipſi⁹.d.b.cum duplo ipſius.e.b.quam ipſius.d.e.ad.e. b. Sed horum compoſitorum primum ſignificetur per.H.ſecundum verò ſignificatum fuit per. E. vnde.H. ad.E. ſe habebit vt.d. e. ad e.b.ſed.E.ad.A.iam dictum eſt eſſe vt.o.e.ad.d.e.vbi ſignatum eſt. λ. quare ex. 23. quinti eadem proportio erit ipſius.o.e.ad.e.b.quæ.H.ad.A. vt ipſe inquit.

X Ex.18. poſtea eiuſdem ita erit.o.b.ad.e.b.vt.H.A.ad.A.

Notandum etiam eſt quod ſi collectæ fuerint omnes partes compoſiti. H. A. hoc eſt duplum.a.b.cum duplo.b e.cum quadruplo.b.c.cum quadruplo.b.d.cum ſimplo a.c.cum triplo.c.d.cum duplo.d.e.habebitur triplum.a.b.triplum.b.d.& ſexcuplum b.c.vt ipſe dixit. Quod autem hoc verum ſit, cum diſtinctæ fuerint omnes partes, vt in ſubſcriptis his lineis videre eſt, videbis quod ſi ex.H.detracta fuerit ſimplex.a.c.quæ quidem poſtea iuncta vni ex partibus quadrupli.b.c.ipſius.A.reſultabit nobis vna integra. a.b. Vnde habebimus triplum ipſius. a.b.& in.A. remanebit triplum ipſius.c.b. Deinde ſi ex.H.auferatur triplum ipſius.c.d.& ipſum addatur tribus partibus quadrupli.b.d. ipſius.A. habebimus tres vices.b.c.quæ ſi iungantur tribus, quæ remanebant in. A. vt dixi, habebimus ſexcuplum ipſius.b.c. & in.A.remanebit ſimplum.b.d. cum duplo ipſius. b. e. Vnde ſi ex. H. demptum fuerit duplum ipſius. d. e. quod quidem iungatur cum duplo ipſius. b.e.habebimus duplum ipſius.b.d.quod coniunctum cum ſimplo.b.d.quod in.A. relictum fuerat, habebimus triplum ipſius d.b. Verum igitur eſt quod inquit Archimedes, hoc eſt,quod. H. A. eſt triplum ipſius.a. b.ſexcuplum ipſius.b.c.& triplum ipſius.b. d.

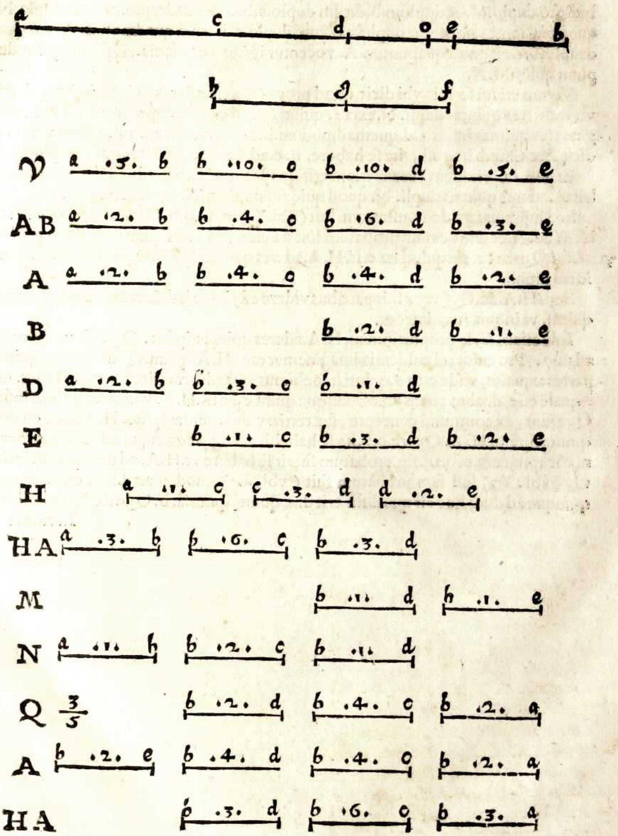
Verum etiam dicit ex eo(vt ſupra probatum eſt)quod.a.c: c.d: et.d.e. ſe habebãt in continua proportionalitate, quare ex conuerſa proportionalitate erunt ſibi inuicem continuæ proportionales.

Nunc autem cum.a.c: c.d.et. d.e.ſint continuæ proportionales in ea proportione in qua ſunt.a.b: c.b: d.b: et.e.b.vt in principio diximus, erit ex.22.quinti.a.c. ad. d.e.vt.a.b.ad.d.b.& ſic etiam.c.b.ad.e.b. Vnde ex.24. eiuſdem.a. d.ad.d.e.erit vt. a.b.cum.b.e. ad.d.b.& vt. c.b.cum.b.d.ad.e.b.& ex. 13. dicti vt.a.b. cum.b.e.bis ſumpto, & cum.b.d.ad.e.b.Quare ex conuerſa proportionalitate, vt ſe habet. e.d. ad.d.a.ita ſe habebit.e.b.cũ.d.b.ad d.b.cũ.b.c.duplicato & cũ. b. a. vt inquit Archimedes. Nunc antecedens vocetur.M.hoc eſt. b.e. cum.d.b.conſequens verò, hoc eſt.d.b.cum duplo.b.c.cum ſimplo.b.a.vocetur. N.

Animaduertendum tamen eſt quod impreſſio mendoſa eſt ubi dicit.

„ vnaquæque.c.b: b.d. & cætera,
propterea quod dicendum eſt ita
vnaquæq;.c.b:b.d.

Nunc ex.18.quinti, quemadmodum ſe habet.a.c.ad.d.a.ita ſe habebit.M.N.ad.N.



Ybi

Vbi autem ſcriptum eſt
„ ad vtrunque ſimul.b.d: d.a.cum dupla.b.c.
dicendum eſt ita,
ad vtranque ſimul.b.d.b.a.cum dupla.b.c.

Inquit deinde Archi.quod ſicut ſe habet.e.a.ad.d.a.ita ſe habebit duplum.M.N. ad duplum.N. Quod quidem verum eſt ex.13.quinti, huiuſmodi verò antecedens & conſequens, Archi.manifeſtat ex ſuis partibus, ſumendo duplum.e.b.cum duplo b.d.pro duplo.M. & duplum.b.d.cum duplo.a.b.cum quadruplo.b.c.pro duplo.N. quę ſimul iuncta æquantur duplo.e.b.cum duplo.a. b. cum quadruplo.b.d.cum quadruplo.b.c.ex quo æquabuntur. A. vocentur igitur hæc omnia.A. potius quàm duplum ipſius.M.N.

Verum etiam ſcribit,vbi dicit,quod proportio.e.a.ad tres quintas ipſius.a. d. erit vt. A.ad tres quintas dupli. N.ex.22. quinti. Sed cum ex ſuppoſito ita ſe habeat.f. g.ad tres quintas ipſius.a.d.quemadmodum.b.e.ad.e.a. erit ex. 16. quinti verum ꝙ dicit Archimed. hoc eſt, ita ſe habere. b. e.ad.f.g.vt.e.a.ad tres quintas ipſius.a.d.

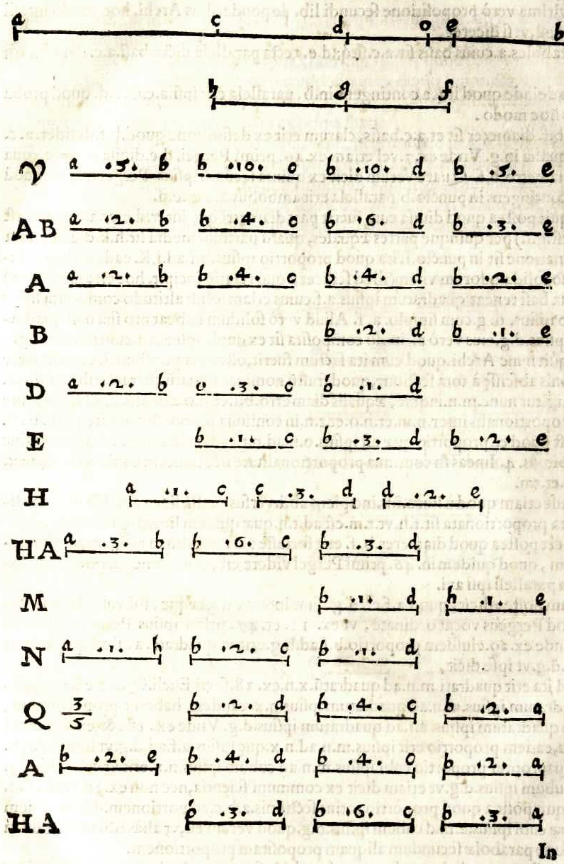
Et per. 11. eiuſdem verum etiam erit quod ſicut ſe habet.e.b.ad.f.g. ita ſe habebit.A.ad tres quintas dupli. N. quod quidem duplum.N.ſignificetur per . Q.

Sed ſuperius iam demonſtratum fuit (vbi.X.) quod.o.b.ad.b.e.ita ſe habebat vt H.A. ad. A.& nũc demum probatum fuit ita eſſe. A.ad tres quintas ipſius.Q.vt.e.b. ad.f.g.Quare ex.22.quinti ita erit. H.A.ad tres quintas ipſius.Q.vt.o. b. ad. f. g. vt idem inquit. Y

Sed. H.A.ad.Q.(vt ex ſuis partibus videre eſt) ita ſe habet vt tres ad duo ex.13. quinti, vt inquit Archimedes.

Ipſe etiam dicit proportionem. H.A.ad tres quintas ipſius. Q. eſſe vt quinque ad duo. Pro cuius rei euidentia imaginemur tam.H.A.quam.Q. diuiſa per quinq; partes æquales, vnde ex.16.quinti habebimus quamlibet quintam partẽ ipſius. Q. æqualẽ eſſe duabus tertijs vniuſcuiuſque quintæ partis.H.A. vnde tres quintæ ipſius Q. erunt, ex communi conceptu, ſex tertiæ vnius quintæ ipſius. H. A. hoc eſt duæ quintæ.ipſius. H. A. Quare.o.b.ita ſe habebit ad.f.g. vt quinque ad duo ex communi cõceptu,cum.o.b.ad.f.g.probatum fuerit ſe habere vt.H.A. ad tres quintas ipſius Q. (vbi. Y.) ſed iam probatum fuit (vbi. ⍵.) quod. o.a.ad.h.g. erat etiam vt quinque ad duo, hoc eſt quod.f.h.erit duæ quintę ipſius.a. b. Quod eſt propoſitum.

In vltima


a c d o e b
h g f
V a .5. b b .10. c b .10. d b .5. e
AB a .12. b b .4. c b .6. d b .3. e
A a .12. b b .4. c b .4. d b .2. e
B b .2. d b .1. e
D a .12. b c .3. c b .1. d
E b .1. c b .3. d b .2. e
H a .1. c c .3. d d .2. e
HA a .3. b b .6. c b .3. d
M b .1. d h .1. e
N a .1. b b .2. c b .1. d
Q 3/5 b .2. d b .4. c b .2. a
A b .2. e b .4. d b .4. c b .2. a
HA b .3. d b .6. c b .3. a

In

In vltima verò propositione secundi lib. de ponderibus Archi. hoc modo intelligendus est, vt si diceret,

Sit paraboles.a.cuius basis sit.a.c.sitq;.d.e. recta parallela dictæ basi.a.c. diameterq; b.f.

Inquit deinde quod linea contingens in.b. parallela erit ipsi.a.c.et.e.d. quod probabimus hoc modo.

Cum.b.f. diameter sit et.a.c.basis, clarum erit ex definitione quod.b.f.diuidet. a. c. per æqualia in.g. Vnde ex.7.vel etiam ex.46. primi Pergei. d.e.diuisa erit per æqualia à diametro.b.f. Quare verum dicit ex quinta secundi ipsius Pergei hoc est quod dicta contingens in puncto.b parallela erit ambobus.a.c.et.e.d.

Inquit postea quod diuisa cum fuerit pars diametri quę inter.d.e.et.a.c.posita est (hoc est.g.f.) per quinque partes æquales, quarũ partium media sit.h.k.diuisa etiam imaginatione sit in puncto.i. ita quod proportio ipsius.h.i.ad.i.K. eadem sit quæ inter duo solida quorum vnum(illud scilicet à quo relatio incipit, hoc est antecedens)
R pro sua basi teneat quadratum ipsius.a.f.cuius etiam solidi altitudo composita sit ex duplo ipsius. d. g.cum simplo. a. f. Aliud verò solidum habeat pro sua basi quadratum ipsius.d.g.eius verò altitudo composita sit ex duplo ipsius.a.f.cum simplo.d.g.

Inquit nunc Archi.quod cum ita factum fuerit,ostendet punctum. i. centrum esse portionis abscissę à tota sectione,quod frustũ nominat signatũ characteribus.a.d.e.c.

Sit igitur nunc.m.n.inquit, æqualis diametro.b.f.et.n.o.æqualis.b.g.sitq;.x.n.media proportionalis inter.n.m.et.n.o.et.t.n.in continua proportionalitate post. o. n. hoc est quod ea proportio quæ est ipsius.o.n.ad.n.t.eadem sit ipsius.x.n.ad.n.o.Hinc habebimus. 4. lineas in continua proportionalitate sibi inuicem coniunctas.m. n: x. n: o.n.et.t.n.

A Vult etiam quod à linea.i.b.incipiens ab.i.versus.g.alia linea abscissa sit, cui lineæ,ita proportionata sit.f.h.vt.t.m.est ad.t.n.quæ quidem linea signata sit.i.r.

Dicit postea quod diameter.b.f. erit fortasse axis vel aliqua reliquarum diametrorum, quod quidem in. 46. primi Pergei videre est, cum omnes diametri sint inuicem paralleli ipsi axi.

Cum postea dicit, quod.a.f.et. d.g. sunt intentæ ductæque, ibi vult idem inferre,quod Pergeus vocat ordinatè, vt ex. 11. et. 49. primi ipsius Pergei videre licet, vnde ex.20.eiusdem proportio.b.f.ad.b.g.erit vt quadrati. a. f. ad quadratum ipsius.d.g.vt ipse dicit.

α Sed ita erit quadrati.m.n.ad quadratũ.x.n.ex. 18.sexti Eucli.Quare ex.11.quinti quadratum ipsius.m.n.ad quadratum ipsius.n.x. eandem habebit proportionem, quam quadratum ipsius. a.f.ad quadratum ipsius.d.g. Vnde ex. 18. & ex communi sciẽtia,eadem proportio erit ipsius.m.n.ad.n.x.quę ipsius.a.f.ad.d.g.vt inquit Arch.

Quapropter proportio cubi ipsius.m.n.ad cubum ipsius.n.x.erit vt cubi ipsius. a. f.ad cubum ipsius.d.g.vt etiam dicit ex communi scientia,nec non ex.36.vndecimi.

Inquit postea quod proportio totius sectionis.a.b.c.ad portionem.d.b. e. eadem est quæ cubi ipsius.a.f.ad cubum ipsius.d.g.quod verum est,vt aliàs tibi monstraui in diuisione parabolæ secundum aliquam propositam proportionem.

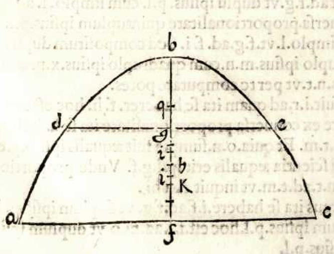
β Quando autem dicit quod proportio cubi ipsius.m.n ad cubum ipsius.n.x.eadem est quæ ipsius.m.n.ad.n.t. verum dicit ex. 36. vndecimi. Vnde ex. 11. quinti ita se habebit totalis sectio.a.b.c.ad portionem.d.b.c.vt.m.n.ad.n.t.& ex.17. eiusdem ita erit ipsius.m.t.ad.t.n.vt frusti.a.d.e.c.ad sectionem.d.b. e. quemadmodum ipse dicit. Sed quia superius,vbi.A.ipsa.f.h.(quæ est tres quintæ ipsius.f.g.)ad.i.r.ita rela-

ta fuit

ta fuit vt.m.t.ad.t.n. idcirco ex.11. quinti ita erit ipsius frusti.a.e. ad sectionem.d.b.e. vt tres quintę ipsius.f.g.ad.i.r.

Inquit deinde quod proportio corporis iam supradicti, quod pro sua basi habeat quadratum ipsius.a.f.altitudinem verò compositam ex duplo ipsius.d.g.cum simplo a.f.ad cubum ipsius.a.f.eadem erit quæ dupli ipsius.d.g.cum simplo.a.f.ad.a.f.Quod quidem verum est ex.33. vndecimi & ex prima sexti.

Sed superius (vbi.a.) iam probauimus eandem proportio nem esse inter.m.n.& n.x. quæ inter.a.f.et.d.g.ideo ex conuersa proportionalitate ita erit ipsius.x.n.ad.n.m. vt ipsius.d.g.ad.a.f.sed dupli.x.n.ad simplum.x.n.est vt dupli.d.g.ad.d.g.Quare ex.22. quinti dupli.x.n.ad.m.n.erit vt dupli.d.g.ad.a.f.& ex.18.eiusdem ita erit dupli.x.n.cum simplo.m.n.ad.m.n.vt dupli.d.g.cum simplo.a.f.ad.a.f.Quare solidi



δ iam dicti ad cubum ipsius.a.f.ex. 11. quinti erit vt dupli.x.n.cũ simplo.m.n.ad.m.n. Superius autem vbi. β. demonstratum fuit ita esse ipsius.m.n.ad.n.t.vt cubi.m.n. ad cubum. x. n. & inter. α. et. β. probatum fuit ita esse cubi.a.f. ad cubum. d. g. vt cubi.m.n.ad cubum. x. n. Vnde ex. 11. quinti.m.n.ad.n.t.erit vt cubi.a. f. ad cubum d. g.

Dicit postea quod eadem proportio erit inter cubum. d. g. & corpus illud quod pro basi habeat quadratum ipsius.d.g. altitudinem verò vt dictum est, quæ est inter d.g. & compositum ex duplo.a.f.cum simplo. d. g. quod compositum est altitudo di cta,& verũ dicit ex ratione superius allegata pro reliquo corpore & cubo ipsi us.a.f. Quare etiam quemadmodum. t. n. se habet ad duplum ipsius.o. n.cum simplo.t.n. ex ijsdem rationibus supradictis, vbi loquuti sumus de.x.n.cum.m.n.

Disponantur nũc omnia tali ordine, ita vt. u. primum sit corpus quod pro sua ba si habeat quadratum ipsius.a.f.&c.

Et.y. sit cubus ipsius.a.f.et.s. sit cubus ipsius.d.g.et. z.sit corpus quod basim habet quadratum ipsius.d.g.altitudinem verò vt supradictum est,et. p. sit compositum dupli.n.x.cum simplo.m.n.et. l. sit compositum dupli ipsius. n. o. cum simplo. t. n. Sed.u.locata sit è regione.p.et.y.è regione.m. n.et.s.è regione.n.t.et.z. è regione.l. & habebimus proportionem ipsius.u.ad.y. vt.y.ad.m.n.& ipsius.y.ad.s. vt.m. n. ad. n.t. quod superius iam demonstratum fuit, vbi,δ. et.s.ad.z. ita se habebit vt.n.t.ad. l. vt vltimò probatum fuit. Quare ex.22. quinti ita se habebit.u. ad. z. vt. p. ad. l. quemadmodum dicit Archi.

Et quia vt se habet.u.ad.z.ita facta fuit.h.i.ad.i. K.vbi.R. ideo ex.11. quinti vt se habet. h.i.ad. i.K. ita se habebit.p.ad. l. vt ipse dicit: Et ex.18. quinti ita erit. h. K. ad. K.i. vt.p.l.ad.l. & ex communi conceptu.g.f.se habebit ad. h. K. vt quintuplum ipsius.p. l.ad.p.l. & ex.22.eiusdem ita se habebit.f. g.ad.i.k.vt quintuplum ipsius.p. l.ad.l. quintuplum autem ipsius.p. l.compositum est ex quintuplo ipsius. n. m. cum decuplo ipsius.n.x.cum quintuplo ipsius.n.t. cum decuplo ipsius. n. o. vt à te facilè computare potes.

Verum etiam erit ex communi scientia quod.g.f. ad. f.k.est ut quintuplum ipsius p.l.ad duplum ipsius.p.l.eo quod superius suppositum fuit.h.K.esse quintã mediam, vnde. k. f. relinquebatur pro duabus quintis inferioribus, duplum autem.p. l. compositum est ex duplo ipsius.m.n.cum duplo ipsius.n.t.cum quadruplo ipsius. n. x. & cum quadruplo ipsius. x.o.

Ex conuersa proportionalitate deinde ita se habet,i.K.ad.i.k.ad.f.g.vt.l.ad quintuplum ipsius.p.l.et.k.f. ad.f.g. vt duplum ipsius.p.l.ad quintuplum ipsius.p. l. Vnde ex.24.quinti.i.f.se habebit ad.f.g.vt duplũ ipsius. p.l. cum simplo. l. ad quintuplum θ ipsius. p.l. Deinde ex conuersa proportionalitate quintuplum ipsius.p.l. se habebit ad duplum ipsius.p.l.cum simplo.l.vt.f.g.ad. f. i. Sed compositum dupli ipsius. p. l. cum simplo. l.æquale est duplo ipsius.m.n.cum quadruplo ipsius.x.n. cum sexcuplo ipsius.o.n.cum triplo ipsius.n.t.vt per te computare potes.

Superius enim sumpta fuit.i.r.ad quam ita se haberet. f. h. hoc est tres quintæ ipsius.f.g.vt.m.t.ad.t.n.Quare ex conuersa proportionalitate ita se habebit.i.r.ad tres quintas ipsius.f.g.vt.t.n.ad.t.m. Et quia.o.n.sumpta fuit æqualis ipsi.b.g.et.m. n.ipsi b. f. ideo.m.o.ex communi scientia æqualis erit ipsi.g.f. Vnde proportio.r.i.ad tres quintas ipsius. m.o.erit vt.n.t.ad.t.m.vt inquit Archi.

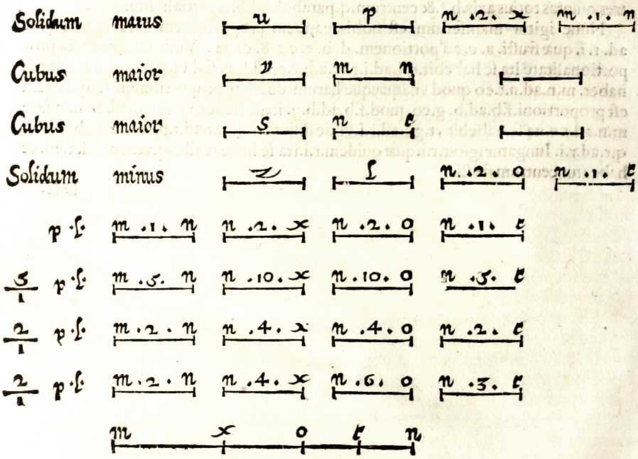
Sed vbi.θ. iam probauimus ita se habere.i.f.ad.f.g. vt duplum ipsi⁹.p. l.cum simplo.l.se habet ad quintuplum ipsius.p.l.hoc est.i.f.ad.m.o.vt duplum ipsius.p.l.cum simplo.l.ad quintuplum ipsius.p.l.

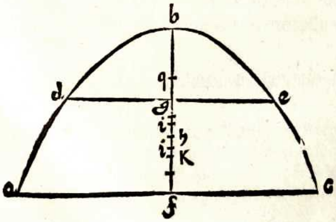
Habemus

Habemus igitur nunc omnes illas conditiones quas Archimedes in præcedenti propositione supponit. Vnde ex rationibus ibi allegatis sequitur.f.r.esse duas quintas ipsius.m.n. hoc est ipsius.f. b. Quapropter punctum.r.centrum erit ponderis totius sectionis parabolæ ex.8. secundi lib.de ponderibus eiusdem Archimedis.

Inquit nunc Archimedes,quod existente. q. centro ponderis ipsius parabolæ. d. b.e. partialis, centrum frusti erit in linea recta. q. r. f. ita remotum à centro. r. quod proportio.q.r.ad partem illam ipsius.r.f.quæ reperitur inter centrum. r. & centrum huius frusti æqualis est proportioni totius parabolæ ad partialem. Quod quidem verum est ex.8.primi libri eiusdem.

Inquit etiam punctum.i.illud esse,eo quod cum probatum sit.f. r. duas quintas esse ipsius.f.b.ideo.b.r.tres quintas erit ipsius.b.f. vt ipse dicit.





Sed.q.b. similiter tres quintæ est ipsius.d.b.ex.8.prædicta. Quare.q.r.tres quintæ erit ipsius.f.g.ex. 19. quinti.

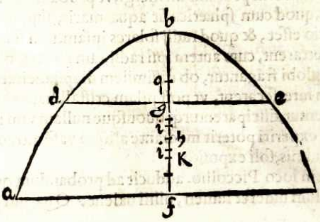
Dicamus igitur hoc modo cum. f.b.totum ad totum.b. r. ita se habeat vt abscissum.b.g.ad abscissum.q.b.ex.7.et.8. dicti primi libri eiusdem ideo residuum.f.g.ex f.b.ad residuum.r.q.ex.r.b.erit vt totum.f.b.ad.totum.r.b.ex. 19.quinti Eucli.

Sed iam sub. A. probauimus ita se habere frustum.a.d.e.c.ad parabolam.d.b.e. vt m.t.ad.t.n.sed vt.m.t.ad.t.n.ita assumpta fuit (vbi.A.).i.r.ad quam sic se haberet. f. h.hoc est tres quintæ ipsius.f. g.hoc est.q.r.quare ex.11. quinti proportio frusti. a. d.e.c.ad parabolam partialem erit vt.q.r.ad.r.i. Existente igitur.r. centro totius parabolæ et.q. centro partialis,ergo.i.centrum erit frusti propositi.

Sed si nullo solido intercedente,voluerimus centrum.i.frusti.a.e. citius inuenire, inueniemus primò centrum.r.totius figuræ ex.8.secundi eiusdem constituendo.b.r. tres quintas totius axis.b.f.& centrum.q.parabolæ.d.b.e.partialis similiter.

Nunc igitur manifestum est nobis, eandem proportionem fore ipsius.q.r. ad. r. i. quæ frusti. a. e. ad portionem. d. b. e. ex. 8. dicta. Vnde ex coniuncta proportionalitate ita se habebit.q.i.ad.i.r.vt.a.b.c.ad.d.b.e. sed vt.a.b.c.ad.d.b.e.ita se habet. m.n.ad.n.t.eo quod vnaquæque harum duarum proportionum sesquialtera est proportioni.f.b.ad.b. g.eo quod.f.b.ad.b.g. ita se habet.vt. m. n. ad. o. n. quare m.n.ad.t.n.ita se habebit vt.g.i.ad.r.i.vnde disiunctim.m.t.ad.t.n. ita se habebit vt q.r.ad.r.i. Iungatur igitur.r.i.quæ quidem.r.i.ita se habeat ad.r.q.vt.t. n. ad. t. m. vt habeatur centrum frusti.

DE-



DEFENSIO NOSTRA CONTRA ANTONIVM Bergam, & Alexandrum Piccolhomineum.

Illustri Domino Horatio Muto.

INTER ea quæ olim contra Antonium Bergam, ſermone Italico ſcripſi, hoc vnum erat, quod ipſe Berga non viderat quendam notatu dignum errorem ipſius Piccolhominei, vbi ipſe Alexander arguit quendam authorem in tractatu de magnitudine terræ & aquæ pag. 37. linea. 26. ita di „ cens, & erit maior aqua.

Quo