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Sources 14

Mahdi Abdeljaouad and Jeffrey Oaks:

A. Conspectus of problems
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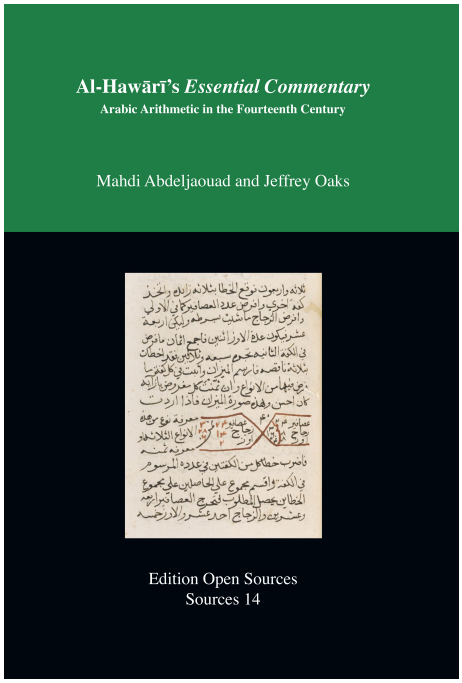

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A. Conspectus of problems

For the following list of calculations and problems in al-Hawārī's book, we have adopted our transcription of the Arabic notation. This is explained in our commentary, especially at 219.1. We had introduced the algebraic notation in our commentary at 211.2, and notation for fractions is explained in our commentary in the chapter on fractions, beginning at 135.1. Here is a quick guide to the notation:

- The notation is a transcription of how it appears, or in some cases would have appeared, in the manuscripts. With the exception of the figures for double false position, we put the notation in red, as it is in many Arabic manuscripts. It should be read right to left.
- The reversed letter "ﻧ" stands for "less", and indicates that the number to its left is removed from the number to its right. So, an apotome that we would write as $\sqrt{5} - \sqrt{3}$ is shown as $\sqrt{3}$ ﻧ $\sqrt{5}$. See our commentary at 86.1. Binomials have no sign for "+", so the modern $5 + \sqrt{21}$ is shown as $\sqrt{21}$ 5.
- Instead of the letter *jīm* above a number to indicate square root, we use the modern "$\sqrt{\ }$" which functions similarly.
- We write "=" for the "equals" in algebraic equations, and for no other purpose. This sign functions like the elongated *lām*, its counterpart in Arabic manuscripts.
- Because the transcribed notation for algebra is entirely different from our notation, we include the versions of the calculations in modern notation as well, even if it sometimes makes little sense (as in the calculations from 219.2 through 222.1).
- An "L" is placed after the reference for examples taken from Ibn al-Bannāʾ's *Lifting the Veil*.

Part 1. On known numbers

Chapter 1. On whole numbers

Passage	Example
65.2	Examples of whole numbers: 15, 18, etc.
65.2	Examples of fractions: $\frac{1}{2}$, $\frac{1}{2}\ \frac{3}{8}$, $\frac{1}{4}\ \frac{1}{9}$, $\frac{7 \bullet 6}{8 \bullet 7}$, $\frac{1}{9}$ ﻧ $\frac{5}{6}$

Passage	Example
65.6, 10, 17, 66.1	Even: 10, 50; evenly-even: 32; evenly-odd: 14; evenly-evenly-odd: 28
66.7	Odd prime: 11, 29; Oddly-odd: 15, composed of 3 by 5
66.17	Even square: 36 is 6 by 6
67.3	Even, composed of two unequal numbers (a surface): 18 is 3 by 6, and also 2 by 9
67.3	Even, composed of three unequal numbers (a surface): 24 is 3 by 4 by 2
67.12	Even cube: 64 is 4 by 4 by 4
67.19	Odd square: 25 is 5 by 5
68.1	Odd, composed of two unequal numbers (a surface): 35 is 5 by 7; 105 is 3 by 5 by 7
68.8	Odd cube: 27 is composed of 3 by 3 by 3
70.8, 70.23	Sample figures for numbers: 9367184225 and 84725
71.6	143 has three places
71.16	The rank of 10000000 is 8
72.4	The name of 1000000000 is thousands thousands of thousands
74.17	Adding from the units place: add 4043 to 2685 to get 6728
75.9	Adding from the highest place: add 978 to 456 to get 1434
75.20	The most one can get by adding is one extra place: add 9 to 9 to get 18
76.15	Chessboard: add the first sixteen squares to get 65535
77.11	Chessboard: add the first eight squares with 4 as the first square to get 1020
78.4	Adding five numbers with a ratio of $\frac{2}{3}$: add 16, 24, 36, 54, 81 to get 211
79.4	Adding six numbers starting at 10, with a difference of three: add 10, 13, 16, 19, 22, 25 to get 105
79.13	Add 1, 2, 3, …, 10 to get 55

Passage	Example
79.18	Add the squares of 1, 2, 3, …, 10 to get 385
80.1	Add the cubes of 1, 2, 3, …, 10 to get 3025
80.5	Add 1, 3, 5, 7, 9 to get 25
80.10	Add the squares of 1, 3, 5, 7, 9 to get 165
80.15	Add the cubes of 1, 3, 5, 7, 9 to get 1225
80.20	Add 2, 4, 6, 8, 10 to get 30
81.4	Add the squares of 2, 4, 6, 8, 10 to get 220
81.9	Add the squares of 2, 4, 6, 8, 10, 12 to get 364
81.15	Add the cubes of 2, 4, 6, 8, 10 to get 1800
83.16	Subtracting from the highest place: subtract 4968 from 5035 to get 67
84.13	Subtracting from the units place: subtract 3469 from 6543 to get 3074
85.8	The most one can get by subtracting is one fewer place: subtract 1 from 10 to get 9
86.9 L	2 ↘ 5 ↘ 7 ↘ 8 ↘ 10 is 6
87.19	Casting out nines from 6435 gives nothing
88.5	Casting out eights from 5393 gives 1
89.4	Casting out sevens from 23786435 gives 1
90.7	Casting out sevens from 58064 gives 6
91.4	Add 43 to 64 to get 107 Cast out sevens to check: add 1 to 1 to get 2
91.16	Subtract 74 from 96 to get 22 Cast out sevens to check: subtract 4 from 5 to get 1
92.10	Multiply 12 by 16 to get 192 Cast out sevens to check: multiply 5 by 2 to get 3
92.17 L	Multiply $\frac{1}{3}$ by $\frac{1}{4}14$ to get $\frac{3}{4}4$ Cast out sevens to check: multiply $\frac{1}{3}$ by $\frac{1}{4}$ to get $\frac{1\,0}{3\,4}$

Passage	Example
93.10	Divide 1488 by 12 to get 124 Cast out sevens to check: multiply 5 by 5 to get 4
93.15 L	Divide $\frac{3}{4}\ \frac{5}{6}$ by $\frac{1}{2}$ to get $\frac{1}{6}3$. Cast out sevens to check: multiply $\frac{5}{6}$ by $\frac{1}{2}$, then convert to 4ths of 6ths to get $\frac{3\ 0}{4\ 6}$
94.1	Denominate 11 with 15 to get $\frac{2\ 3}{3\ 5}$. Cast out sevens to check: multiply 4 by 1 to get 4
94.5 L	Denominate $\frac{2\ 2}{3\ 6}$ with $\frac{1\ 5}{3\ 8}$ to get $\frac{2}{3}$. Cast out sevens to check: multiply 2 by 2, adjust for the denominators to get 3
95.10 L	Meanings of multiplication: 3 men, each has 5 dirhams; 5 dirhams, how many thirds?
96.3	Multiplication by shifting: multiply 43 by 54 to get 2322
97.4	Vertical multiplication: multiply 42 by 37 to get 1554
98.15	Multiplication by half-shifting: multiply 463 by itself to get 214369
100.5	Lattice multiplication: multiply 435 by 287 to get 124845
102.1	Vertical multiplication (no shifting): multiply 183 by 347 to get 63501
104.10	Sleeper multiplication (no shifting): multiply 253 by 987 to get 249711
107.6	Multiply 444 by 333 to get 147852
108.13	Multiplication by excess: multiply 12 by 15 to get 180
109.1	Multiply 13 by 17 to get 221
109.10	Multiplication by denomination: multiply 6 by 12 to get 72
110.6	Another method of multiplication by denomination: multiply 24 by 8 to get 192
110.12	Multiply 12 by 15 to get 180
111.1	Multiply 3 by 15 to get 45
111.15	Multiplication by nines: multiply 444 by 999 to get 443556

Passage	Example
112.10	Another method of multiplication by nines: multiply 999 by 9354 to get 9344646
113.1	Multiplication by squaring: multiply 17 by 19 to get 323
113.9	Another squaring method: multiply 25 by 15 to get 375
113.19	Another squaring method: multiply 36 by 14 to get 504
114.8	Multiplication with zeros: multiply 30 by 140 to get 4200
117.16 L, 118.1 L	Meanings of division: Divide 15 dirhams among 3 men; Divide a piece of wood of 15 spans by a piece of wood of 3 spans
119.1	Divide 245 by 12 to get $\frac{1}{2}\ \frac{2}{6}$ 20
120.5	Divide 44 among 11 men to get 4
120.10	Divide 96 by 12 to get 8
120.16	Divide 35 by 15 to get $\frac{1}{3}$2
121.4	Apportionment. Wealth of donors: 4, 5, 6 dinars, amount of loan: 10 dinars
122.5	Apportionment. Wealth of donors: $\frac{1}{3}$4, $\frac{1}{4}$5, $\frac{1}{6}$6 dinars, amount of loan: 12 dinars
123.22	Common denomination: denominate 11 with 15 to get $\frac{2}{3}\ \frac{3}{5}$
124.12,14,17	Other denominations: denominate 4 with 12 to get $\frac{1}{3}$; denominate 9 with 15 to get $\frac{3}{5}$; denominate 10 with 16 to get $\frac{5}{8}$
124.20ff	Finding divisors: 50, 36, 66, 42, 64, 68, 14, 26, 81, 39, 123, 77, 221
129.6	Restore 8 to 19; reduce 50 to 6
129.8	Restore 3 to 6: divide 6 by 3 to get 2
129.12	Reduce 8 to 3. Denominate 3 with 8 to get $\frac{3}{8}$

Part 1, Chapter 2. On fractions

Passage	Example
134.2 L	Language of parts: $\frac{1}{11}$, $\frac{1}{17}$
134.8	Numerator < denominator: $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$; and $\frac{1}{7}$ through $\frac{6}{7}$
135.1	Fractions with two or more names: $\frac{1}{7}\,\frac{2}{8}$; $\frac{1}{3}\,\frac{2}{6}\,\frac{4}{7}\,\frac{8}{9}$
135.10 L	A related fraction: $\frac{2}{3}\,\frac{4}{5}\,\frac{5}{6}$
136.8 L	A distinct fraction: $\frac{4}{5}\,\frac{5}{6}$
137.1 L	A portioned fraction: $\frac{5 \bullet 3}{6 \bullet 4}$
137.13	The numerator of $\frac{1}{7}$ is 1
138.5	The numerator of $\frac{2}{3}\,\frac{3}{5}\,\frac{4}{7}\,\frac{5}{8}$ is 596
139.2	The numerator of $\frac{4}{6}\,\frac{1}{2}\,\frac{5}{7}$ is 122
139.11	The numerator of $\frac{3 \bullet 5 \bullet 7}{10 \bullet 6 \bullet 9}$ is 105
140.8	The numerator of $\frac{1}{9} \diagdown \frac{6}{8}$ is 46
141.1	The numerator of $\frac{1}{3} \diagdown \frac{1}{2}\,\frac{6}{7}$ is 26
141.11	The numerator of the connected fraction $\frac{1}{5} \diagdown \frac{1}{7} \diagdown \frac{1}{4} \diagdown \frac{1}{3}\,5$, also written $\frac{1}{5}\,\frac{1}{7}\,\frac{1}{4} \diagdown \frac{1}{3}\,5$, is 912
142.6	The numerator of the distinct fractions $\frac{1}{5} \diagdown \frac{1}{7} \diagdown \frac{1}{4} \diagdown \frac{1}{3}\,5$ is 1991
143.5	The numerator of $\frac{3}{4}\,\frac{5}{6}\,5$ is 143
143.13	The numerator of $10\,\frac{6}{8}\,\frac{4}{7}$ is 740
144.10	The numerator of $\frac{3}{6}\,5\,\frac{4}{9}$ is 147
145.4	The numerator of $\frac{4}{7}\,7\,\frac{2}{3}$ is 106
147.4	Add $\frac{6}{8}\,\frac{4}{5}\,3$ to $\frac{1}{2}\,\frac{3}{8}\,\frac{4}{10}$ to get $\frac{1}{2}\,\frac{7}{8}\,\frac{9}{10}4$
147.15	Subtract $\frac{1}{3} \diagdown 2\,\frac{7}{10}$ from $\frac{5 \bullet 3}{6 \bullet 4}\,4$ to get $\frac{1}{2}\,\frac{3}{6}\,\frac{5}{10}3$
149.8	Multiply $\frac{1}{3}\,\frac{3}{4}$ by $\frac{1}{5}\,\frac{4}{6}\,\frac{3}{9}$ to get $\frac{1}{4}\,\frac{0}{5}\,\frac{0}{6}\,\frac{4}{9}$
150.2	Multiply $\frac{1}{8}\,4\,\frac{1}{3}$ by $10\,\frac{2 \bullet 1}{3 \bullet 5}$ to get $\frac{5}{6}1$
151.4	Divide $\frac{1}{3}\,6$ by $3\,\frac{7 \bullet 4}{8 \bullet 5}$ to get $\frac{1}{7}\,\frac{0}{9}\,3$

Passage	Example
151.14	Denominate $\frac{2}{9} \searrow \frac{1}{4} 3$ with $\frac{3}{5} \frac{2}{8} 6$ to get $\frac{5 \quad 50}{9 \; 137}$
152.11	Divide $\frac{2 \quad 9}{3 \quad 10} 8$ by $\frac{1 \quad 5}{3 \quad 10}$ to get $\frac{1 \; 6}{2 \; 8} 16$
153.3	Denominate $\frac{1}{3} 2$ with $\frac{2}{3} 6$ to get $\frac{1 \quad 3}{2 \quad 10}$
153.12 L	Divide 5 by $\frac{5}{6}$ to get 6; denominate $\frac{5}{6}$ with 5 to get $\frac{1}{6}$
154.6ff	Restore $\frac{1}{2}$ to $\frac{9}{10}$; $\frac{1 \; 2}{2 \; 7}$ to $\frac{1}{2}5$; $\frac{5 \bullet 2}{7 \bullet 3}$ to 10; 5 to $\frac{4}{6}10$; $\frac{1 \quad 3}{2 \quad 10}4$ to 8; $\frac{1}{3} \searrow \frac{3}{5}3$ to $\frac{3}{5}12$
155.11ff	Reduce $\frac{7}{10}$ to $\frac{1}{3}$; 8 to $\frac{1}{2}2$; 10 to $\frac{3}{4}$; $\frac{1}{4}7$ to $\frac{4}{6}3$; $\frac{4 \quad 9}{7 \; 10}11$ to 5; $\frac{1}{3}2$ to $\frac{1}{9}$
157.2 L	Convert $\frac{3}{4} \frac{5}{6}$ to tenths. Answer: $\frac{5 \quad 5}{6 \; 10}1$
157.12 L	How many tenths are in $\frac{3}{4} \frac{5}{6}$? Again, it is $\frac{5 \quad 5}{6 \; 10}1$
158.1 L	How many tenths are in 5? Answer: 50
158.11	How many ninths are in $\frac{4}{10} \frac{6}{8}$? Answer: $\frac{4 \quad 3 \quad 1}{8 \; 10 \; 9}1$

Part 1, Chapter 3. On roots

Passage	Example
163.4 L	Examples: $\sqrt{10}$, $\sqrt{\frac{1}{2}}$, $\sqrt{\frac{1}{2}10}$, $\sqrt{\sqrt{10}}$
166.13	$\sqrt{625}$ is 25
167.8	$\sqrt{20}$ is approximately $\frac{1}{2}4$
167.14	$\sqrt{54}$ is approximately $\frac{1 \; 2}{2 \; 7}7$
168.1	$\sqrt{92}$ is approximately $\frac{3}{5}9$
168.16	$\sqrt{92}$ is approximately $\frac{1 \quad 5 \quad 5}{2 \; 6 \; 10} 9$
169.4	$\sqrt{12}$ is approximately $\frac{1 \; 3}{4 \; 7}3$
169.10,17 L	$\sqrt{625}$ is 25; $\sqrt{729}$ is 27
170.6	$\sqrt{100}$ is 10
170.16	$\sqrt{\frac{1 \; 4}{6 \; 6}}$ is $\frac{5}{6}$

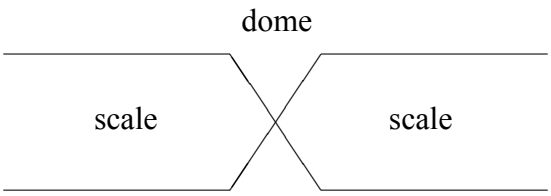
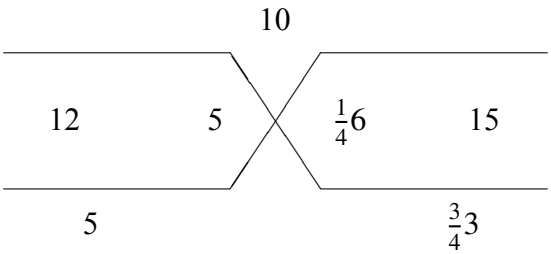
Passage	Example
171.1	$\sqrt{\frac{1}{4}12}$ is $\frac{1}{2}3$
171.6	$\sqrt{\frac{3\ 4}{6\ 9}}$ is $\frac{1\ 5\ 3\ 13}{2\ 6\ 9\ 19}$
171.15	$\sqrt{\frac{1\ 7}{2\ 8}10}$ is approximately $\frac{1\ 2\ 16}{4\ 8\ 53}3$
172.6	$\sqrt{\frac{1\ 7}{2\ 8}10}$ is approximately $\frac{4}{13}3$
172.12	$\sqrt{\frac{1\ 4}{2\ 7}}$ is approximately $\frac{3\ 5\ 8}{4\ 7\ 11}$
173.13 L	Binomials: $\sqrt{3}\ 5$; $\sqrt{3}\ \sqrt{5}$
173.16 L	Apotomes: $\sqrt{3} \diagdown 5$; $\sqrt{3} \diagdown \sqrt{5}$
174.5	1st & 4th binomials: $\sqrt{21}\ 5$, $\sqrt{2}\ 2$
174.8	2nd & 5th binomials: $\sqrt{45}\ 5$, $\sqrt{72}\ 5$
174.11	3rd & 6th binomials: $\sqrt{18}\ \sqrt{10}$, $\sqrt{8}\ \sqrt{7}$
175.11	$\sqrt{\sqrt{60}\ 8}$ is $\sqrt{3}\ \sqrt{5}$
175.19	$\sqrt{\sqrt{60} \diagdown 8}$ is $\sqrt{3} \diagdown \sqrt{5}$
176.6	$\sqrt{\sqrt{55}\ 8}$ is $\sqrt{\frac{1}{2}2}\ \sqrt{\frac{1}{2}5}$; $\sqrt{\sqrt{55} \diagdown 8}$ is $\sqrt{\frac{1}{2}2} \diagdown \sqrt{\frac{1}{2}5}$
176.10	$\sqrt{\sqrt{112}\ 7}$ is $\sqrt{\sqrt{\frac{3}{4}1}\ \sqrt{\sqrt{\frac{3}{4}85}}}$; $\sqrt{\sqrt{112} \diagdown 7}$ is $\sqrt{\sqrt{\frac{3}{4}1} \diagdown \sqrt{\sqrt{\frac{3}{4}85}}}$
176.15	$\sqrt{\sqrt{14}\ \sqrt{32}}$ is $\sqrt{\sqrt{\frac{1}{2}}\ \sqrt{\sqrt{\frac{1}{2}24}}}$; $\sqrt{\sqrt{14} \diagdown \sqrt{32}}$ is $\sqrt{\sqrt{\frac{1}{2}} \diagdown \sqrt{\sqrt{\frac{1}{2}24}}}$
176.20	$\sqrt{\sqrt{30}\ 7}$ is $\sqrt{\sqrt{\frac{3}{4}4} \diagdown \frac{1}{2}3\ \sqrt{\sqrt{\frac{3}{4}4}\ \frac{1}{2}3}}$; $\sqrt{\sqrt{30} \diagdown 7}$ is $\sqrt{\sqrt{\frac{3}{4}4} \diagdown \frac{1}{2}3 \diagdown \sqrt{\sqrt{\frac{3}{4}4}\ \frac{1}{2}3}}$
177.5	$\sqrt{\sqrt{20}\ 3}$ is $\sqrt{\sqrt{\frac{1}{4}\ \frac{1}{2}2} \diagdown \sqrt{5}\ \sqrt{\sqrt{\frac{1}{4}\ \frac{1}{2}2}\ \sqrt{5}}}$; $\sqrt{3 \diagdown \sqrt{20}}$ is $\sqrt{\sqrt{\frac{1}{4}\ \frac{1}{2}2} \diagdown \sqrt{5} \diagdown \sqrt{\sqrt{\frac{1}{4}\ \frac{1}{2}2}\ \sqrt{5}}}$
177.11	$\sqrt{\sqrt{11}\ \sqrt{10}}$ is $\sqrt{\frac{1}{2} \diagdown \sqrt{\frac{3}{4}2}\ \sqrt{\sqrt{\frac{3}{4}2}\ \frac{1}{2}}}$; $\sqrt{\sqrt{10} \diagdown \sqrt{11}}$ is $\sqrt{\frac{1}{2} \diagdown \sqrt{\frac{3}{4}2} \diagdown \sqrt{\sqrt{\frac{3}{4}2}\ \frac{1}{2}}}$

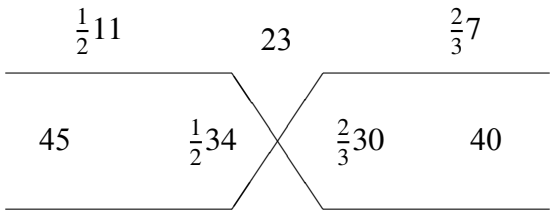
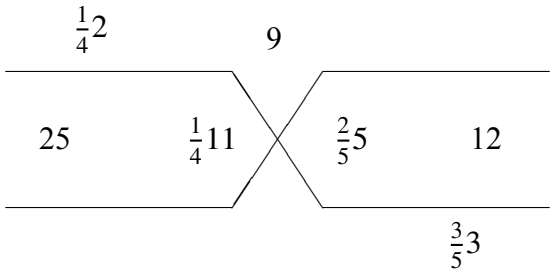
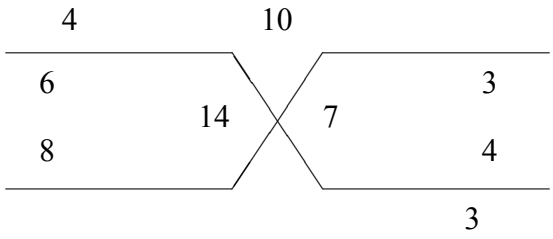
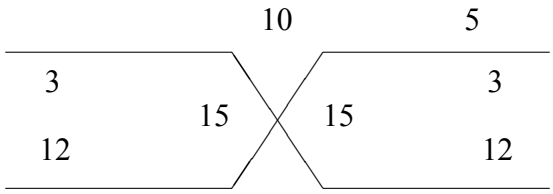
Passage	Example
179.7,11	Add $\sqrt{3}$to $\sqrt{27}$ to get $\sqrt{48}$
179.16	Add $\sqrt{2}$ to $\sqrt{8}$ to get $\sqrt{18}$
179.20	Add half of $\sqrt{20}$ to two $\sqrt{5}$s to get $\sqrt{45}$
180.10	Add $\sqrt{3}$ to $\sqrt{15}$ to get $\sqrt{15}$ $\sqrt{3}$
180.15	Add half of $\sqrt{\sqrt{80}}$ to $\frac{1 \bullet 1}{4 \bullet 3}$ of $\sqrt{684}$ to get $\sqrt{\sqrt{\frac{1\,4}{2\,8}}\,22}$ $\sqrt{\sqrt{5}}$
181.6	Subtract $\sqrt{8}$ from $\sqrt{32}$ to get $\sqrt{8}$
181.12,16	Subtract $\sqrt{12}$ from $\sqrt{27}$ to get $\sqrt{3}$
182.4	Subtract $\sqrt{8}$ from $\sqrt{10}$ to get $\sqrt{8} \diagdown \sqrt{10}$
183.4	Multiply $\sqrt{8}$ by $\sqrt{9}$ to get $\sqrt{72}$
183.7	Multiply $\sqrt{\sqrt{5}}$ by $\sqrt{\sqrt{7}}$ to get $\sqrt{\sqrt{35}}$
183.11	Multiply $\sqrt{\sqrt{\sqrt{3}}}$ by $\sqrt{\sqrt{\sqrt{8}}}$ to get $\sqrt{\sqrt{\sqrt{24}}}$
183.15	Multiply 3 by $2 \diagdown \sqrt{7}$ to get $6 \diagdown \sqrt{63}$
184.1	Multiply 3 by $\sqrt{7}$ to get $\sqrt{63}$
184.4	Multiply 2 by $\sqrt{\sqrt{3}}$ to get $\sqrt{\sqrt{48}}$
184.11	Multiply 2 by two $\sqrt{7}$s to get $\sqrt{112}$
184.18	Multiply 5 by three $\sqrt{\sqrt{2}}$s to get $\sqrt{\sqrt{101250}}$
185.6	Multiply $\frac{2}{3}$ by half of $\sqrt{20}$ to get $\sqrt{\frac{2}{9}2}$
185.12	Multiply $\sqrt{5}$ by half of $\sqrt{\sqrt{40}}$ to get $\sqrt{\frac{1}{2}62}$
186.1	Duplicate $\sqrt{3}$ twice to get $\sqrt{12}$
186.4	Duplicate $\sqrt{7}$ five times to get $\sqrt{175}$
186.8	Half of $\sqrt{10}$ is $\sqrt{\frac{1}{2}2}$
186.11	$\frac{4 \bullet 1}{8 \bullet 3}$ of $\sqrt{\sqrt{60}}$ is $\sqrt{\sqrt{\frac{1\,2\,0}{2\,6\,9}}}$
187.4	Divide $\sqrt{20}$ by $\sqrt{3}$ to get $\sqrt{\frac{2}{3}6}$
187.7	Denominate $\sqrt{3}$ with $\sqrt{8}$ to get $\sqrt{\frac{3}{8}}$

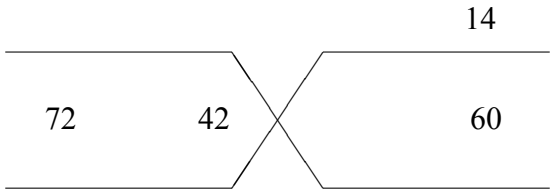
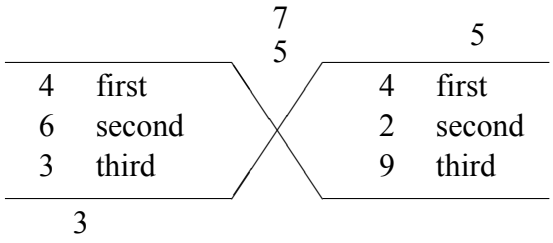
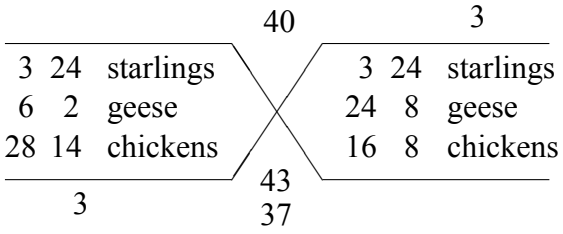
Passage	Example
187.10	Divide $\sqrt{\sqrt{6}}$ by $\sqrt{\sqrt{2}}$ to get $\sqrt{\sqrt{3}}$
187.14	Denominate $\sqrt{\sqrt{18}}$ with $\sqrt{\sqrt{32}}$ to get $\sqrt{\sqrt{\frac{1}{2}\frac{4}{8}}}$
188.6	Divide $\sqrt{\sqrt{14}}$ by $\sqrt{2}$ to get $\sqrt{\sqrt{\frac{1}{2}3}}$
188.11	Divide two $\sqrt{15}$s by 2 to get $\sqrt{15}$
188.15	Divide half of $\sqrt{24}$ by $\sqrt{2}$ to get $\sqrt{3}$
189.1	Divide 12 by $\sqrt{3}$ 5 to get $\sqrt{\frac{9}{11}\frac{9}{11}}$ ↘ $\frac{8}{11}2$
189.11	Divide 10 by $\sqrt{7}$ ↘ 3 to get $\sqrt{175}$ 15

Part 2. Finding unknown numbers

Chapter 1. Solving problems by proportion

Passage	Example
195.16	Example of four proportional numbers: 3 : 6 :: 4 : 8
198.7 L	
199.1	

Passage	Example
200.1	
201.1	
202.3	
203.1	

Passage	Example
203.11 L	
205.5 L	
207.6 L	

Part 2, Chapter 2. Solving problems by algebra

Passage	Example
211.15	Simple equations: $\frac{t}{7} = \frac{m}{3}$; $20 = \frac{m}{5}$; $12 = \frac{t}{3}$ ($3x^2 = 7x$; $5x^2 = 20$; $3x = 12$)
212.6	Composite equations: $24 = \frac{t}{10}\frac{m}{1}$; $\frac{t}{5} = 4\frac{m}{1}$; $5\frac{t}{4} = \frac{m}{1}$ ($x^2 + 10x = 24$; $x^2 + 4 = 5x$; $x^2 = 4x + 5$)
213.7	Solve $\frac{t}{15} = \frac{m}{3}$ to get $\frac{t}{1}$ is 5 and $\frac{m}{1}$ is 25 ($3x^2 = 15x \Rightarrow x = 5$, $x^2 = 25$)

Passage	Example
213.13	Solve $18 = {}^{m}_{2}$ to get ${}^{m}_{1}$ is 9 and ${}^{t}_{1}$ is 3 $(2x^2 = 18 \Rightarrow x^2 = 9, x = 3)$
214.1	Solve $20 = {}^{t}_{5}$ to get ${}^{t}_{1}$ is 4 and ${}^{m}_{1}$ is 16 $(5x = 20 \Rightarrow x = 4, x^2 = 16)$
214.9	Solve $15 = {}^{t}_{2}\ {}^{m}_{1}$ to get ${}^{t}_{1}$ is 3 and ${}^{m}_{1}$ is 9 $(x^2 + 2x = 15 \Rightarrow x = 3, x^2 = 9)$
215.6	Solve ${}^{t}_{6} = 8\ {}^{m}_{1}$ to get ${}^{t}_{1}$ is 4 and ${}^{m}_{1}$ is 16, or ${}^{t}_{1}$ is 2 and ${}^{m}_{1}$ is 4 $(x^2 + 8 = 6x \Rightarrow x = 4, x^2 = 16$ or $x = 2, x^2 = 4)$
215.14	Solve ${}^{t}_{6} = 9\ {}^{m}_{1}$ to get ${}^{m}_{1}$ is 9 and ${}^{t}_{1}$ is 3 $(x^2 + 9 = 6x \Rightarrow x^2 = 9, x = 3)$
216.13	Solve $3\ {}^{t}_{2} = {}^{m}_{1}$ to get ${}^{t}_{1}$ is 3 and ${}^{m}_{1}$ is 9 $(x^2 = 2x + 3 \Rightarrow x = 3, x^2 = 9)$
217.10	$36 = {}^{t}_{6}\ {}^{m}_{2}$ simplifies to $18 = {}^{t}_{3}\ {}^{m}_{1}$ $(2x^2 + 6x = 36 \Rightarrow x^2 + 3x = 18)$
218.1	$6 = {}^{t}_{2}\ {}^{m}_{\frac{1}{2}}$ simplifies to $12 = {}^{t}_{4}\ {}^{m}_{1}$ $(\frac{1}{2}x^2 + 2x = 6 \Rightarrow x^2 + 4x = 12)$
219.2	Add ${}^{m}_{1}$, ${}^{t}_{6}$, and 10 to get $10\ {}^{t}_{6}\ {}^{m}_{1}$ $(6x + x^2 + 10 \Rightarrow x^2 + 6x + 10)$
219.5 L	Add ${}^{t}_{1} \searrow {}^{m}_{1}$ to 10 to get ${}^{t}_{1} \searrow 10\ {}^{m}_{1}$ $((x^2 - x) + 10 \Rightarrow x^2 + 10 - x)$
219.7 L	Add ${}^{m}_{1} \searrow {}^{m}_{2}$ to 10 to get $10\ {}^{m}_{1}$ $((2x^2 - x^2) + 10 \Rightarrow x^2 + 10)$
219.10 L	Add ${}^{t}_{2} \searrow {}^{m}_{1}$ to ${}^{t}_{10}$ to get ${}^{t}_{8}\ {}^{m}_{1}$ $((x^2 - 2x) + 10x \Rightarrow x^2 + 8x)$
219.12 L	Add ${}^{t}_{5} \searrow {}^{m}_{1}$ to ${}^{t}_{10}$ to get ${}^{t}_{5}\ {}^{m}_{1}$ $((x^2 - 5x) + 10x \Rightarrow x^2 + 5x)$
220.1	Subtract ${}^{t}_{1}$ from ${}^{m}_{1}$ to get ${}^{t}_{1} \searrow {}^{m}_{1}$ $(x^2 - x \Rightarrow x^2 - x)$
220.3	Subtract 10 from ${}^{t}_{3}\ {}^{m}_{2}$ to get $10 \searrow {}^{t}_{3}\ {}^{m}_{2}$ $((2x^2 + 3x) - 10 \Rightarrow 2x^2 + 3x - 10)$
220.9	Subtract ${}^{t}_{1}\ 2$ from ${}^{t}_{3} \searrow {}^{m}_{1}$ to get $2 \searrow {}^{t}_{4} \searrow {}^{m}_{1}$ $((x^2 - 3x) - (2 + x) \Rightarrow x^2 - 4x - 2)$
221.1	Subtract ${}^{t}_{5} \searrow 52$ from $30\ {}^{c}_{2}$ to get ${}^{t}_{22} \searrow {}^{t}_{5}\ {}^{c}_{2}$ $((2x^3 + 30) - (52 - 5x) \Rightarrow 2x^3 + 5x - 22)$

Passage	Example
221.13	Subtract $\overset{t}{4}\diagdown 12$ from $\overset{t}{2}\diagdown\overset{m}{3}$ to get $12\diagdown\overset{t}{2}\ \overset{m}{3}$ $((3x^2-2x)-(12-4x)\Rightarrow 3x^2+2x-12)$
222.1	Subtract $\overset{m}{2}\diagdown\overset{c}{1}$ from $\overset{t}{4}\diagdown 30$ to get $\overset{t}{4}\diagdown\overset{c}{1}\diagdown\overset{m}{2}\ 30$ $((30-4x)-(x^3-2x^2)\Rightarrow 30+2x^2-x^3-4x)$
223.2	$\overset{t}{1}\ 2=\overset{t}{3}\diagdown\overset{m}{1}$ simplifies to $\overset{t}{4}\ 2=\overset{m}{1}$ $(x^2-3x=2+x\Rightarrow x^2=2+4x)$
223.7	$\overset{t}{5}\diagdown 24=\overset{t}{3}\diagdown\overset{m}{1}$ simplifies to $24=\overset{t}{2}\ \overset{m}{1}$ $(x^2-3x=24-5x\Rightarrow x^2+2x=24)$
223.14	$\overset{t}{\frac{1}{2}2}\diagdown\overset{m}{1}=10\diagdown\overset{m}{1}$ simplifies to $\overset{t}{\frac{1}{2}2}=10$ $(x^2-10=x^2-2\frac{1}{2}x\Rightarrow 10=2\frac{1}{2}x)$
223.17	$\overset{t}{4}\diagdown 51=10\ \overset{m}{1}$ simplifies to $41=\overset{t}{4}\ \overset{m}{1}$ $(x^2+10=51-4x\Rightarrow x^2+4x=41)$
224.1	$\overset{t}{1}\diagdown\overset{m}{2}\ 10=\overset{t}{5}\ \overset{m}{1}$ simplifies to $10\ \overset{m}{1}=\overset{t}{6}$ $(x^2+5x=10+2x^2-x\Rightarrow 6x=x^2+10)$
224.4	$\overset{t}{7}\diagdown 7\ \overset{m}{3}=\overset{t}{2}\diagdown 10\ \overset{m}{2}$ simplifies to $\overset{t}{5}\ 3=\overset{m}{1}$ $(2x^2+10-2x=3x^2+7-7x\Rightarrow x^2=3+5x)$
225.8	The power of the *māl māl* is 4; of the *māl* cube is 5; of the *māl māl māl* is 6; of the *māl* cube *māl* cube is 10
225.13	The power of the *māl* cube *māl māl* is 9; of the cube *māl* cube cube *māl māl* is 15
226.1	A term for 4 is a *māl māl*; for 7 is a cube *māl māl*; for 6 is a *māl māl māl* or a cube cube
226.3	A term for 8 is a *māl māl māl māl* or a cube *māl* cube, or a cube cube *māl*, or a *māl* cube cube, etc.
226.7	A term for 9 is a cube cube cube or a cube *māl māl māl*, etc.
226.12	Multiply $\overset{t}{5}$ by $\overset{t}{7}$ to get $\overset{m}{35}$ $(5x\times 7x\Rightarrow 35x)$
226.17	Multiply $\overset{t}{10}$ by $\overset{m}{6}$ to get $\overset{c}{60}$ $(10x\times 6x^2\Rightarrow 60x^3)$
227.4	Multiply $\overset{t}{1}$ by $\overset{c}{1}$ to get $\overset{mm}{1}$ $(x\times x^3\Rightarrow x^4)$
227.7	Multiply 6 by $\overset{m}{4}$ to get $\overset{m}{24}$ $(6\times 4x^2\Rightarrow 24x^2)$

Passage	Example
227.10	Multiply 7 by $\overset{mc}{3}$ to get $\overset{mc}{21}$ $(7 \times 3x^5 \Rightarrow 21x^5)$
227.17	$\overset{m}{10}\ \overset{c}{4} = \overset{mm}{3}$ simplifies to $10\ \overset{t}{4} = \overset{m}{3}$ $(3x^4 = 4x^3 + 10x^2 \Rightarrow 3x^2 = 4x + 10)$
228.1	$\overset{t}{20}\ \overset{m}{10} = \overset{c}{3}$ simplifies to $20\ \overset{t}{10} = \overset{m}{3}$ $(3x^3 = 10x^2 + 20x \Rightarrow 3x^2 = 10x + 20)$
228.4	$\overset{t}{39} = \overset{m}{10}\ \overset{c}{1}$ simplifies to $39 = \overset{t}{10}\ \overset{m}{1}$ $(x^3 + 10x^2 = 39x \Rightarrow x^2 + 10x = 39)$
228.11	Multiply $\overset{t}{5}$ by $\overset{t}{4} \searrow 13$ to get $\overset{m}{20} \searrow \overset{t}{65}$ $(5x \times (13 - 4x) \Rightarrow 65x - 20x^2)$
228.15	Multiply $\overset{t}{2} \searrow 8$ by $\overset{m}{4} \searrow 7$ to get $\overset{m}{32} \searrow \overset{t}{14} \searrow 56\ \overset{c}{8}$ $((8 - 2x) \times (7 - 4x^2) \Rightarrow 8x^3 + 56 - 14x - 32x^2)$
229.4	Divide $\overset{m}{10}$ by $\overset{t}{2}$ to get $\overset{t}{5}$ $(10x^2 \div 2x \Rightarrow 5x)$
229.8	Divide $\overset{c}{15}$ by $\overset{t}{3}$ to get $\overset{m}{5}$ $(15x^3 \div 3x \Rightarrow 5x^2)$
229.13	Divide $\overset{m}{12}$ by $\overset{m}{3}$ to get 4 $(12x^2 \div 3x^2 \Rightarrow 4)$
229.17	Divide $\overset{t}{12}$ by 4 to get $\overset{t}{3}$ $(12x \div 4 \Rightarrow 3x)$
230.4	Divide $\overset{m}{3} \searrow \overset{c}{12}$ by $\overset{t}{2}$ to get $\overset{t}{1\frac{1}{2}} \searrow \overset{m}{6}$ $((12x^3 - 3x^2) \div 2x \Rightarrow 6x^2 - 1\frac{1}{2}x)$
230.8	Divide $\overset{t}{3} \searrow \overset{m}{10}$ by 2 to get $\overset{t}{1\frac{1}{2}} \searrow \overset{m}{5}$ $((10x^2 - 3x) \div 2 \Rightarrow 5x^2 - 1\frac{1}{2}x)$
230.13	Divide $\overset{m}{6}$ by $\overset{c}{3}$ to get $\overset{2}{\underset{1}{t}}$ $(6x^2 \div 3x^3 \Rightarrow \frac{2}{x})$
230.17	Divide $\overset{m}{10}$ by $\overset{t}{1} \searrow 3$ to get $\overset{m}{\underset{1}{\overset{10}{t}}} \searrow 3$ $(10x^2 \div (3 - x) \Rightarrow \frac{10x^2}{3-x})$